American Journal of Fluid Dynamics 2018, 8(4): 105-111
DOI: 10.5923/j.ajfd.20180804.01

Onset of Magnetoconvection in a Rotating Darcy Porous Layer Heated from Below with Temperature Dependent Heat Source

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Abstract The onset of stationary and oscillatory magnetoconvection in a rotating infinitely horizontal porous layer filled with electrically conducting Newtonian fluid heated from below with temperature – dependent heat source within the Darcy limit using linear stability analysis is investigated for free – free boundaries. The effects of heat source, magnetic field and rotation parameters on the onset of convection are presented graphically and analyzed in detail. It is found that increases in magnetic and rotation parameters delayed the onset of stationary and oscillatory convection, thereby stabilizing the system. The heat source parameter increment accelerates the onset of convection and the system is more unstable; while Prandtl number slowed the onset of oscillatory convection.

Keywords Magnetoconvection, Rotating Darcy porous layer, Temperature– dependent heat source, Free- free boundaries

1. Introduction

The study of fluid convection in a rotating porous layer heated from below has received considerable attention due to many applications in its many applications in geophysics, oceanic flows, astrophysics, and in engineering. Such applications in engineering include among others chemical processing, food processing, solidification and centrifugal casting of metals, and rotating machinery. Excellent reviews of the problem of thermal convection in a rotating porous medium can be found in [1-7]. In particular, Vadasz [5] investigated the effect of the Coriolis force in the thermal convection when the Darcy model is extended by including the acceleration term in the momentum equatiom. Vadasz and Govender [8] considered the influence of gravity and centrifugal forces on the onset of convection in a rotating porous layer. Recently, Falsaperla *et al.* [9] considered the problem of classical Benard system, with and without rotation under the Newton-Robin boundary conditions and fixed heat fluxes for temperature field. More recently, Kang *et al.* [10] investigated high Rayleigh number steady state thermal convection in a rotating porous half space in which rotation led to rise in downward flow in contrast to the upward thermal convection.

It is important to recognize that many convective instability problems of practical importance involve electrically conducting fluids. In such cases, the influence of external fields like electric and magnetic fields become important. Comprehensive account of the linear theory of Rayleigh-Benard magnetoconvection can be found in Chandrasekhar [1], where the onset of instability affected by vertically imposed magnetic field. In particular, the effects of magnetic field become dominant when the fluid is highly electrically conducting. The study of the effects of externally imposed magnetic field is important in understanding the dynamics of the system under consideration. In such situations the electric current, $\boldsymbol{J}$ is induced when a flow of electrically conducting liquid crosses a magnetic field $\boldsymbol{B}$ which in then generates a Lorentz force proportional to $\boldsymbol{J} \times \boldsymbol{B}$ [11, 12]. It is important to mention that the Lorentz force changes the force balance in the momentum equation, making the behaviour different from flows in situations without external magnetic fields.

There are a number of practical situations in which the onset of convection is induced by internal heating in areas such as geophysics and engineering under situations in nuclear heat cores, nuclear waste disposal, oil extraction, weak exothermic reactions and crystal growth within the porous media [13-18]. Alex and Patil [19] investigated the problem of thermal instability with combined effects of centrifugal acceleration and anisotropy for both Darcy and Brinkman limits. Although sufficient literature exists on thermal convection in porous medium, very little has been

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Published online at http://journal.sapub.org/ajfd

devoted to the study of the effects of internal heating and magnetic field in a rotating Darcy layer, nor in the case of combined effects rotation and internal heating in isotropic porous medium. Yu and Shih [20] investigated the problem of convection induced by internal heating in electrically conducting fluid in the presence of a magnetic field, and found that magnetic field increases the stability of the fluid layer.

To the best of our knowledge, in all the investigations mentioned above, there is no study on the combined effects of magnetic field and heat source on the onset of thermal convection in a rotating porous medium with an externally imposed magnetic field. Therefore, in this present paper, we analyze the onset of stationary and oscillatory convection in a rotating fluid layer heated from below under simultaneous effects of magnetic field and temperature dependent heat source.

2. Mathematical Formulation

We consider an infinite electrically conducting horizontal fluid saturated porous layer confined between two parallel boundaries located at $z^* = 0$ and $z^* = d,$ and are maintained at temperatures $T_h^*,$ and T_c^* $(T_h^* > T_c^*)$, respectively. The porous layer is rotating uniformly about the vertical axis at a constant angular velocity $\boldsymbol{\omega}^* = \omega\boldsymbol{k}$. A uniform magnetic field $\boldsymbol{B}^* = B_0^*\boldsymbol{k}$ is applied across the fluid layer in the vertically upward direction, where the induced magnetic field is neglected on the account of small magnetic Reynolds number. A Cartesian coordinate system (x^*, y^*, z^*) is chosen such that the origin is at the bottom of the porous layer and the gravity force acting in the negative $z^* -$ direction. The schematic diagram of the system considered is shown in Fig. 1.

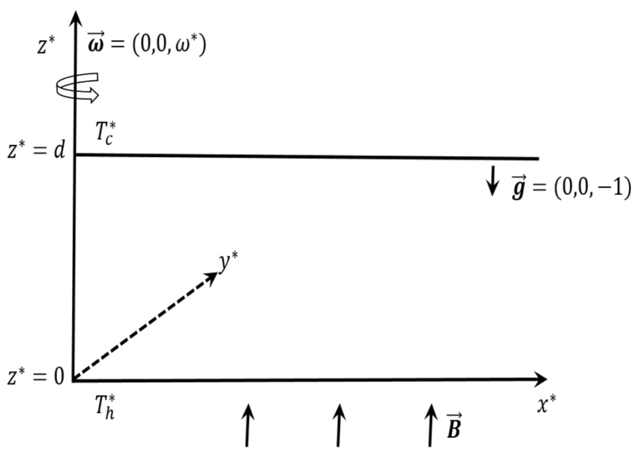

Figure 1. Physical model and coordinate system

Assuming that the heat source is linearly dependent on the temperature, T^*, and taking into account the Coriolis force and acceleration coefficient terms, the governing equations of the fluid motion in a homogeneous and isotropic medium follow Darcy model under Bousssinesq approximation together with Lorentz force are [1, 6, 18, 21, 22]

$$\nabla^* \cdot \boldsymbol{V} = 0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \boldsymbol{V}}{\partial t} + \nabla^* P^* = -\rho_f g\boldsymbol{k} - \frac{\mu}{K}\boldsymbol{V} - 2\frac{\rho_0}{\varepsilon}\boldsymbol{\omega}^* \times \boldsymbol{V} + \boldsymbol{J}^* \times \boldsymbol{B}^* \tag{2}$$

$$A\frac{\partial T^*}{\partial t^*} + \boldsymbol{V}^*.\nabla^* T^* = \alpha_m \nabla^{*2} T^* + Q(T^* - T_0) \tag{3}$$

$$\rho_f = \rho_0[1 - \beta_T(T^* - T_0)] \tag{4}$$

$$\boldsymbol{J}^* = \sigma_c(\boldsymbol{E}^* + \boldsymbol{V}^* \times \boldsymbol{B}^*), \nabla^*.\boldsymbol{J}^* = 0 \tag{5}$$

where $\boldsymbol{V}^* = (u^*, v^*, w^*)$ is the velocity, g is the acceleration due to gravity, $\boldsymbol{k}$ is the unit vector in the vertical direction, ρ_0 is the reference temperature, β_T is the cofficient of thermal expansion, P^* is the pressure, K is the permeability of the porous medium, μ is the fluid viscosity, $\alpha_m = \frac{\kappa}{(\rho c_p)_f}$ is the thermal diffusivity, where $(\rho c_p)_f$ is the volumetric heat capacity of the fluid, $A = \frac{(\rho c_p)_m}{(\rho c_p)_f}$ is the ratio of heat capacities, $(\rho c_p)_m = (1-\varepsilon)(\rho c_p)_s + \varepsilon(\rho c_p)_f$ is volumetric heat capacity of the porous medium, c_p is the specific heat capacity, ε is the porosity parameter where the subscripts *f*, *s* and *m* denotes the properties of the fluid, solid and porous matrix, respectively. Further, $\boldsymbol{J}^*$ is the current density, $\boldsymbol{E}^*$ is the electric field, σ_c is the electrical conductivity, and $Q = \frac{Q_0}{(\rho c_p)_f}$ is constant of proportionality.

The electric field given in Eq. (5) can be written in terms of the electrostatic potential, ϕ as $\boldsymbol{E}^* = -\nabla^* \phi$. Now assuming that the boundaries are electrically insulated for which ϕ is a constant. Based on this assumption, the current density, $\boldsymbol{J}^*$ reduces to

$$\boldsymbol{J}^* = \sigma_c(\boldsymbol{V}^* \times \boldsymbol{B}^*) \tag{6}$$

and consequently, the Lorentz force takes the form

$$\boldsymbol{J}^* \times \boldsymbol{B}^* = \sigma_c(\boldsymbol{V}^* \times \boldsymbol{B}^*) \times \boldsymbol{B}^* \tag{7}$$

2.1. Boundary Conditions

Since the field is bounded by two horizontal parallel plates located at $z^* = 0$ and $z^* = d$, the following boundary condition are assumed for the velocity field

$$\boldsymbol{V}^* = 0 \quad \text{on } z^* = 0, d \tag{8a}$$

while for the temperature field

$$T^* = T_0 + \Delta T, c^* = c_0 + \Delta c \quad \text{on } z^* = 0 \tag{8b}$$

$$T^* = T_0, c^* = c_0 \quad \text{on } z^* = d \tag{8c}$$

2.2. Non - dimensionalization

Using Eqs. (4) and (7) together with the following scaling $d, \frac{\alpha_m}{d}, \frac{Ad^2}{\alpha_m}, \frac{T^* - T_0}{\Delta \mathrm{T}}, \frac{\alpha_m \mu}{K}$ for length, velocity, time, temperature, and pressure respectively, where $\varepsilon = \frac{\phi}{A}$, $d\nabla^* = \nabla$, respectively, the governing equations and the boundary conditions (8) in dimensionless form are

$$\nabla.\boldsymbol{V} = 0 \tag{9}$$

$$\left(\frac{1}{Pr_D}\frac{\partial}{\partial t} + 1\right)\boldsymbol{V} + \nabla P = Ra_D\, T\, \boldsymbol{k} - \sqrt{T_D}\,\boldsymbol{k} \times \boldsymbol{V} - Ha^2(u, v, 0) \tag{10}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{V}.\boldsymbol{\nabla}T = (\boldsymbol{\nabla}^2 + \gamma)T \tag{11}$$

with boundary conditions

$$\boldsymbol{V} = 0, T = 1 \text{ on } z = 0 \tag{12a}$$

$$\boldsymbol{V} = 0, T = 0 \text{ on } z = 1 \tag{12b}$$

2.3. Basic State

The basic state of the system is assumed to be independent of time (quiescent). They vary only in the $z-$direction, and are described by

$$\boldsymbol{V} = 0,\ T = T_b(z), p = p_b(z) \tag{13}$$

where the subscript "b" denotes the basic state. The substitution of Eq. (13) into Eqs. (9) – (11) and the (12) yield the equations governing the basic state as

$$\frac{dp_b}{dz} = Ra\, T_b \tag{14}$$

$$\frac{d^2T_b}{dz^2} + \gamma T_b = 0 \tag{15}$$

The boundary conditions for T_b are

$$T_b = 1 \text{ on } z = 0$$

$$T_b = 0 \text{ on } z = 1 \tag{16}$$

On solving Eqs. (14) – (15) subject to conditions (16) yield the basic state pressure and temperature profiles as

$$p_b(z) = Ra\ \frac{Cos\,[(1-z)\sqrt{\gamma}]}{\sqrt{\gamma}\sin[\sqrt{\gamma}]},\quad T_b(z) = \frac{sin\,[(1-z)\sqrt{\gamma}]}{\sin[\sqrt{\gamma}]}, \tag{17}$$

2.4. Perturbation Equations

To study the stability of the basic state, we superimpose perturbations on the basic state in the form

$$\boldsymbol{V} = \boldsymbol{v}, P = p_b + p, T = T_b(z) + \theta \tag{18}$$

where $v, p,$ and $\theta,$ are the perturbed quantities over their equilibrium counterparts and are assumed small. On substituting Eq. (18) into Eqs. (9) – (12), and linearize by neglecting products of perturbed quantities yield, the following equations

$$\boldsymbol{\nabla}.\boldsymbol{v} = 0 \tag{19}$$

$$\left(\frac{1}{Pr_D}\frac{\partial}{\partial t} + 1\right)\boldsymbol{v} + Ha^2(u, v, 0) + \nabla p$$

$$-Ra_D\,\theta\,\boldsymbol{k} + \sqrt{T_{\boldsymbol{D}}}\,\boldsymbol{k}\times\boldsymbol{v} = 0 \tag{20}$$

$$\frac{\partial\theta}{\partial t} - (\nabla^2 + \gamma)\theta + f(z)\,w = 0 \tag{21}$$

where $f(z) = \frac{\partial T_b}{\partial z} - \frac{\sqrt{\gamma}}{\sin[\sqrt{\gamma}]}\cos[\sqrt{\gamma}(1-z)]$ is the basic temperature gradient distribution.

Since the flow is confined between two parallel plates, located at $z = 0$ and 1, the temperature is maintained at fixed values at the plate walls. Therefore, the temperature perturbations vanish at the plate walls. Hence, the boundary conditions for the temperature perturbation is now

$$\theta = 0 \text{ on } z = 0, 1 \tag{22a}$$

Also, because of no-slip at the walls, the boundary conditions for the velocity perturbation is

$$w = 0 \text{ on } z = 0, 1 \tag{22b}$$

3. Linear Stability Analysis

To facilitate the analysis, we perform the operations $curl$ and $curl\,curl$ on Eq. (20), using the continuity equation (Eq. (19)), the identity $curl\,curl\,\boldsymbol{A} = grad\ div\boldsymbol{A} - \boldsymbol{\nabla}^2\boldsymbol{A}$ and keeping only the z components yield

$$\frac{1}{Pr_D}\frac{\partial\xi}{\partial t} + (1 + Ha^2)\xi - \sqrt{T_D}\,Dw = 0 \tag{23}$$

$$\left(\frac{1}{Pr_D}\frac{\partial}{\partial t} + 1\right)\nabla^2 w + Ha^2D^2w + \sqrt{T_D}\,D\xi - Ra_D\,\nabla_h^2\theta = 0 \tag{24}$$

where $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z – component of the vorticity, $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian in the horizontal plane and $D = \frac{\partial}{\partial z}$.

In order to predict the threshold of convection, we assume all fields to be two dimensional time periodic wave with wave number a in the horizontal plane. Then, we set

$$\begin{pmatrix} w \\ \xi \\ \theta \end{pmatrix} = \begin{pmatrix} W(z) \\ Z(z) \\ \Theta(z) \end{pmatrix} f(x,y)e^{s\,t} \tag{25}$$

where $\nabla_h^2 f(x,y) + a^2 f(x,y) = 0$ and $s(=\omega_r + i\omega_i)$ is the growth rate of disturbances. The substitution of Eq. (25) into Eqs. (21), (23) and (24) yield

$$(D^2 - a^2 + \gamma - s)\Theta + f(z)W = 0 \tag{26}$$

$$\left(\frac{s}{Pr_D} + 1 + Ha^2\right)Z - \sqrt{T_D}\,DW = 0 \tag{27}$$

$$\left(\frac{S}{Pr_D} + 1\right)(D^2 - a^2)W + Ha^2D^2W$$

$$+a^2Ra_D\Theta + \sqrt{T_D}\,DZ = 0 \tag{28}$$

Now subject to

$$W = D^2W = \Theta = Z = 0 \quad \text{on } z = 0,1 \tag{29}$$

Equations (26) – (28) together with conditions (29) constitute a linear eigenvalue problem of the system. The examination of the boundary conditions and for the stability of system suggest required solutions of the form

$$W = W_n Cos(n\pi z), \Theta = \Theta_n Cos(n\pi z), Z = Z_n Sin(n\pi z) \tag{30}$$

for arbitrary W_n, Θ_n, Z_n and n an integer. From now on, we restrict our analysis to the lowest mode (idealized mode) $n = 1$, which corresponds to the most dangerous mode. On using Eq. (30) the eigenvalue problem Eqs. (26) – (28) in matrix form become

$$H\bar{X} = 0 \tag{31}$$

where

$$H = \begin{pmatrix} 2F & 0 & \pi^2 + a^2 - \gamma + s \\ \pi\sqrt{T_D} & \frac{s}{Pr_D} + 1 + Ha^2 & 0 \\ \left(\frac{s}{Pr_D} + 1\right)(\pi^2 + a^2) + \pi^2Ha^2 & -\pi\sqrt{T_D} & -a^2\,Ra_D \end{pmatrix},$$

$\bar{X} = (W_1, \Theta_1, Z_1)$ and

$$F = \frac{\sqrt{\gamma}}{Sin[\sqrt{\gamma}]}\int_0^1 Cos[\sqrt{\gamma}\,(1-z)]Sin^2(\pi z)\,dz = \frac{2\pi^2}{4\pi^2 - \gamma}.$$

The solvability of the system Eq. (31) requires that the determinant of H vanish, that is $|H| = 0$. This condition yields the expression for the Rayleigh number, Ra_D as

$$Ra_D = (\delta - \gamma + s)\left[\frac{\left(\frac{s}{Pr_D}+1\right)\delta}{2\,a^2F} + \frac{\pi^2 Ha^2}{2\,a^2F} + \frac{\pi^2 T_D}{2\,a^2F\left(\frac{s}{Pr_D}+1+Ha^2\right)}\right] \quad (32)$$

where $\delta = \pi^2 + a^2$.

Since the growth rate $s(= \omega_r + i\omega_i,\ \omega_r,\ \omega_i)$ is in general a complex quantity. The system with $\omega_r < 0$ is always stable, while for $\omega_r < 0$, it will be unstable. For neutral (marginal) stability $s = 0$.

3.1. Onset of Stationary Convection

For the validity of the principle of exchange of stabilities to hold, the growth rate must vanish. That is, $\omega_r = \omega_i = 0$ at the marginal stability. Setting $s = 0$ in Eq. (32) and simplifying yields the Rayleigh number, $Ra_D^{(st)}$ for the onset of stationary convection as

$$Ra_D^{(st)} = (\delta - \gamma)\left[\frac{\delta}{2\,a^2F} + \frac{\pi^2 Ha^2}{2\,a^2F} + \frac{\pi^2 T_D}{2\,a^2F\left(\frac{s}{Pr_D}+1+Ha^2\right)}\right] \quad (33)$$

Next, we compute the critical (minimum) values of the critical wave number, a_c and the corresponding critical Rayleigh number, $Ra_{D,c}^{(st)}$ for the onset of convection. By setting $a = a_c$ and $Ra_D^{(st)} = Ra_{D,c}^{(st)}$ in Eq. (33) and minimizing according to [1]

$$\frac{\partial Ra_{D,c}^{(st)}}{\partial a_c^2} = 0 \quad (34)$$

This condition yields the following fourth order polynomial in a_c given by

$$a_c^4 - (\pi^2 - \gamma)\pi^2\left(M^2 + \frac{T_D}{M}\right) = 0 \quad (35)$$

where $M = 1 + Ha^2$.

The solution of Eq.(35) yields the critical wave number, a_c as

$$a_c = \left(\pi^2(\pi^2 - \gamma)\left(M^2 + \frac{T_D}{M}\right)\right)^{1/4} \quad (36)$$

and the corresponding critical Rayleigh number, $Ra_{D,c}^{(st)}$ for the onset of stationary convection as

$$Ra_{D,c}^{(st)} = (\pi^2 + a_c^2 - \gamma)\left[\frac{\pi^2 + a_c^2}{2\,a_c^2F} + \frac{\pi^2 Ha^2}{2\,a_c^2F} + \frac{\pi^2 T_D}{2\,a_c^2F\left(\frac{s}{Pr_D}+1+Ha^2\right)}\right] \quad (37)$$

To validate our results with those in literature, we set the internal heat source parameter, $\gamma = 0$ and magnetic field parameter $Ha = 0$ in Eq. (36), we obtain the critical wave number as

$$a_c = \pi(1 + T_D)^{1/4} \quad (38)$$

and the corresponding critical Rayleigh number as

$$Ra_{D,c}^{(st)} = \left(\frac{\pi^2 + a_c^2}{a_c}\right)^2 + \pi^2 T_D\left(\frac{\pi^2 + a_c^2}{a_c}\right) \quad (39)$$

Equations (38) and (39) are the exact results obtained for the onset of stationary convection in a rotating Darcy porous layer by [3, 5]. Further, in the absence of rotation (Taylor number, T_D), Eqs. (38) and (39) reduce to

$$a_c = \pi \quad (40)$$

and

$$Ra_{D,c}^{(st)} = 4\pi^2 \quad (41)$$

These results are identical to those of Horton and Rogers [23], and Lapwood [24].

3.2. Onset of Oscillatory Convection

For the onset of oscillatory convection, we set $s = i\omega$ and $Ra_D^{(st)} = Ra_D^{(os)}$ in Eq. (32). The result gives the oscillatory Raleigh number, $Ra_D^{(os)}$ as

$$Ra_D^{(os)} = \Delta_1 + i\,\omega_i\Delta_2 \quad (42)$$

where

$$\Delta_1 = \frac{Pr_D\delta(\delta-\gamma) - \delta\omega_i^2}{2a^2F\,Pr_D} + \frac{\pi^2 Ha^2(\delta-\gamma)}{2a^2F} + \frac{\pi^2 Pr_D^2 T_D[MPr_D(\delta-\gamma)+\omega_i^2]}{2a^2F\,Pr_D(M^2Pr_D^2+\omega_i^2)} \quad (43)$$

$$\Delta_2 = \frac{(Pr_D+\delta-\gamma)\delta}{2a^2F\,Pr_D} + \frac{\pi^2 Ha^2}{2a^2F} + \frac{\pi^2 Pr_D T_D[MPr_D-\delta+\gamma]}{2a^2F\,Pr_D(M^2Pr_D^2+\omega_i^2)} \quad (44)$$

Now, since the oscillatory Rayleigh number, $Ra_D^{(os)}$ is a physical quantity, it must be real. Hence, for the validity of oscillatory convection $\omega_i \neq 0$ and $\Delta_2 = 0$. Setting $\Delta_2 = 0$ in Eq. (42) and simplifying gives the expression for the frequency, ω_i^2 as

$$\omega_i^2 = Pr_D^2\left[\frac{(\delta - MPr_D - \gamma)\pi^2 T_D}{\pi^2 Ha^2 Pr_D + \delta(\delta + Pr_D - \delta)} - M^2\right] > 0 \quad (45)$$

Hence, from Eq. (45) oscillatory convection can occur for a particular wave number if the following inequality is satisfied

$$T_D > \frac{M^2}{\pi^2}\left(\frac{\pi^2 Ha^2 Pr_D + \delta(\delta + Pr_D - \delta)}{(\delta - MPr_D - \gamma)}\right) \quad (46)$$

Using Eq.(45) in Eq.(42) together with $\Delta_2 = 0$ yields the expression for oscillatory number, $Ra_D^{(os)}$ as

$$Ra_D^{(os)} = \Delta_1(\omega_i^2) \quad (47)$$

4. Results and Discussion

The onset of magnetoconvection in a rotating Darcy porous layer saturated with electrically conducting fluid heated from below with temperature dependent heat source with free – free boundaries has been studied analytically. In this section, we discuss the effects of the parameters in the governing equations on the basic temperature distribution, basic temperature gradient and on the onset of both stationary and oscillatory convections numerically and graphically. Figure 2 shows the effect of heat source parameter γ on the distribution of basic temperature, $T_b(z)$ of the system across the fluid layer. It is observed that increases in the heat source γ amounts to increase in the basic temperature distribution, $T_b(z)$. This is an indication that increase in γ may lead to some instabilities in the system. To know effect of the heat source γ on the basic temperature gradient of the system, the calculated basic temperature gradient distribution, $\frac{dT_b}{dz}$ for different values of heat source γ is shown in Fig.3. As expected, it is observed that for the stability problem all the values are negative. Also, it is observed from Fig. 3 that increase in heat source amounts to increase in the basic temperature profile which in turn enhances the disturbances in the fluid layer and the system becomes more unstable.

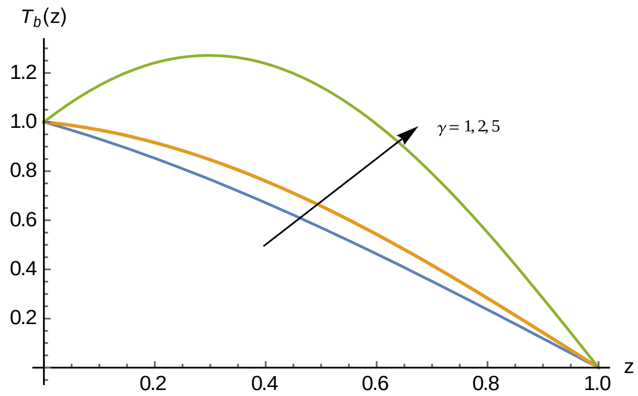

Figure 2. Effect of heat source, γ on the distribution of basic temperature, $T_b(z)$

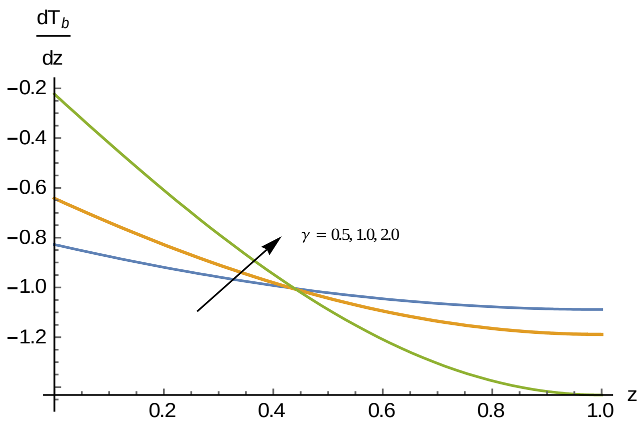

Figure 3. Effect of heat source, γ on the distribution of basic temperature gradient, $\frac{dT_b}{dz}$

To study the effects of heat source, magnetic field and rotation parameters on the onset of stationary convection, we examine the behaviours of $\frac{\partial Ra_D^{(st)}}{\partial \gamma}$, $\frac{\partial Ra_D^{(st)}}{\partial Ha}$, and $\frac{\partial Ra_D^{(st)}}{\partial T_D}$ analytically. From Eq. (33), it follows that

$$\frac{\partial Ra_D^{(st)}}{\partial \gamma} = -\frac{1}{2a^2F}\left(\delta + \pi^2 Ha^2 + \frac{\pi^2 T_D}{1+Ha^2}\right) < 0 \qquad (48a)$$

$$\frac{\partial Ra_D^{(st)}}{\partial Ha} = 2(\delta - \gamma)\pi^2 + Ha^2\left(\frac{2+Ha^2}{(1+Ha^2)^2}\right) > 0 \qquad (48b)$$

and

$$\frac{\partial Ra_D^{(st)}}{\partial T_D} = \frac{(\delta-\gamma)\pi^2}{2a^2\, F\, M} > 0 \qquad (48c)$$

for $\delta > \gamma$.

From Eq. (48a), it worth noting that $\frac{\partial Ra_D^{(st)}}{\partial \gamma} < 0$, which indicates that the heat source parameter, γ has destabilization effect on the system. This is in agreement with the earlier result obtained by Israel-Cookey et al. [22] in the absence of solute concentration Rayleigh number. Also, from Eqs. (48b) and (48c), it follows that magnetic field and rotation parameters inhibit the onset of stationary convection. Hence, magnetic field and rotation have stabilizing effects to make the system more stable. This may be due to the fact that magnetic field and rotation tend to suppress the vertical motion and hence convection, by restricting the motion to the horizontal plane. These results are in agreement with Vadasz [5, 22].

The stability curves in $Ra_D - a$ plane for the onset of both stationary and oscillatory convection for various values of heat source, magnetic field, and rotation parameters are depicted in Figs. (4) – (7), respectively. From Figs. (4a) and (4b) it is observed that the Raleigh number for both stationary and oscillatory convections decreases with increase in heat source parameter, which indicates that the heat source parameter has a destabilizing effect to make the system more unstable.

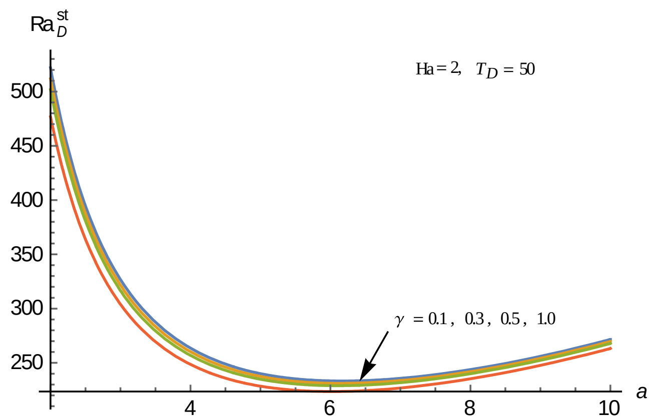

Figure 4(a). Effect of heat source, γ on thermal Rayleigh number, Ra_D^{st} with respect to wave number, a for stationary convection for different values of parameters

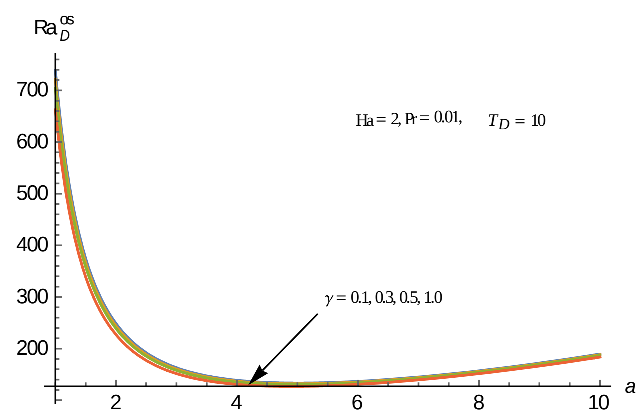

Figure 4(b). Effect of heat source, γ on thermal Rayleigh number, Ra_D^{os} with respect to wave number, a for oscillatory convection for different values of parameters

The effects of imposed magnetic field and rotation on the stability of the system are depicted in Figs. (5) and (6). From Figs. (5a) and (5b) it observed that increases in the magnetic field parameter, Ha, increases the values of critical Rayleigh numbers for the onset of both stationary and oscillatory convection. Hence, magnetic field parameter, Ha delays the onset of convection in the electrically saturated – rotating porous layer. This is because, the Lorentz force suppresses the vertical motion and hence convection, by restricting the motion in the horizontal plane.

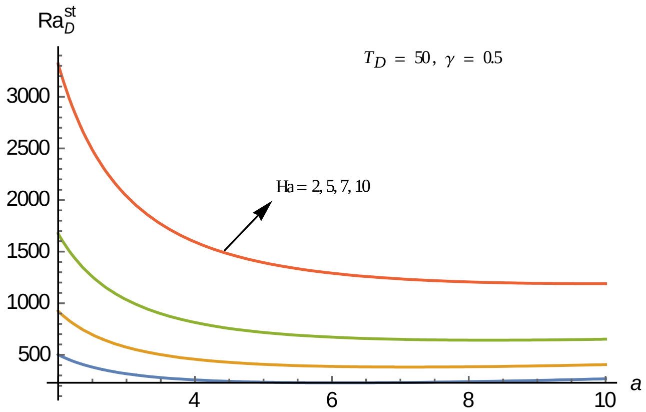

Figure 5(a). Effect of magnetic field, *Ha* on thermal Rayleigh number, Ra_D^{st} with respect to wave number, a for stationary convection for different values of parameters

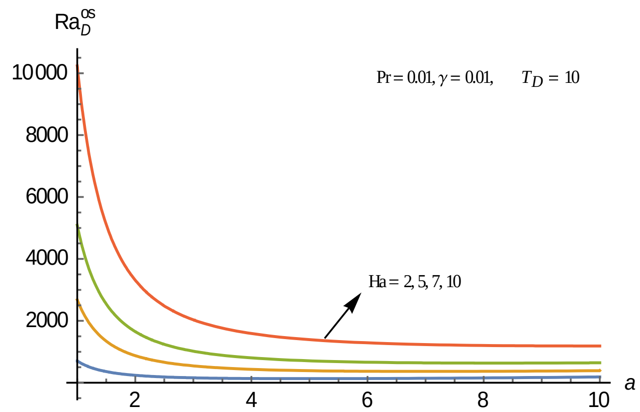

Figure 5(b). Effect of magnetic field, *Ha* on thermal Rayleigh number, Ra_D^{os} with respect to wave number, a for oscillatory convection for different values of parameters

Figures (6a) and (6b) show the effect of rotation parameter, T_D on the onset of both stationary and oscillatory convection on the system. It is observed from these figures that increases in the rotation parameter, T_D results in increase in the Rayleigh number for both stationary and oscillatory convection, which is an indication that the effect of rotation is to enhance the stability of the system. This results are in good agreement with Vadasz [5].

To access the effect Prandtl number, Pr on the stability of the system, the variation of oscillatory convection as a function of wave number, a for fixed values of $Ha = 2$, $T_D = 10$ and $\gamma = 0.3$ is depicted in Fig. 7. It is observed, the Prandtl number affects only the oscillatory convection and from Fig. 7 it is observed that the oscillatory Rayleigh number, Ra_D^{os} is increased with increase in the Prandtl number, Pr which means that Prandtl number has a stabilizing effect on the system.

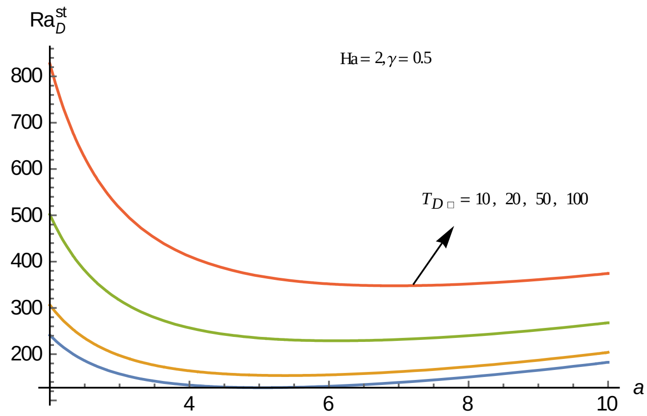

Figure 6(a). Effect of rotation, T_D on thermal Rayleigh number, Ra_D^{st} with respect to wave number, a for stationary convection for different values of parameters

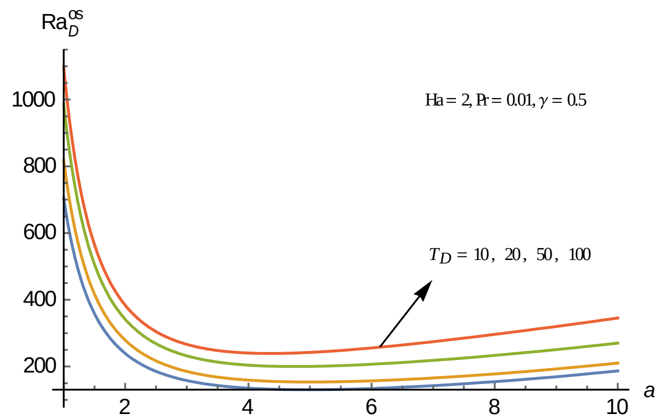

Figure 6(b). Effect of rotation, T_D on thermal Rayleigh number, Ra_D^{os} with respect to wave number, a for oscillatory convection for different values of parameters

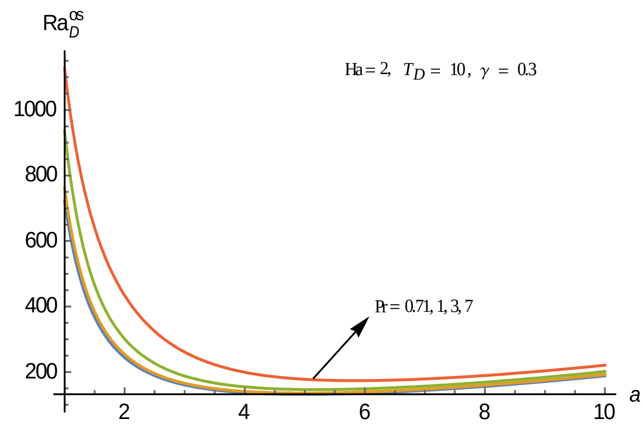

Figure 7. Effect of Prandtl number, Pr on thermal Rayleigh number, Ra_D^{os} with respect to wave number, a for oscillatory convection for different values of parameters

5. Conclusions

The criterion for the onset of magnetoconvection in a rotating Darcy layer filled with electrically conducting fluid with temperature – dependent heat source, which is heated from below for free – free boundaries has been investigated.

The linear stability analysis is used for establishing the criteria for onset of both stationary and oscillatory convections in the system. The effects of physical parameters in the governing equations, such as heat source parameter, γ , magnetic field parameter, Ha , rotation parameter, T_D , and the Prandtl number, Pr are shown graphically. The effects of increasing magnetic field and rotation parameters slow down the onset of both stationary and oscillatory convection, while the Prandtl number delays the onset of oscillatory convection. This means that magnetic field parameter, Ha, rotation parameter, T_D, and the Prandtl number, Pr are stabilizing factors. On the other hand, the heat source parameter, γ accelerates the onset of convection.

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