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Some Additive Failure Rate Models Related with MOEU Distribution

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Abstract In reliability theory, a combination of two distributions failure rate model for reliability studies is paid much attention. In this paper, we will derive the failure rate model of (Marshall-Olkin Extended Uniform distribution) MOEU (α,θ) and every one of MOEU (a,b), MOEU (a,θ), uniform(θ), truncated exponential (λ,θ), truncated Weibull (λ,k,θ), truncated Frechet (a,b,θ), truncated Rayleigh (σ^2,θ), doubly truncated Cauchy (a,b,θ) and doublytruncated Gumbel (a,b,θ) distributions.

Keywords Reliability, MOEU, Additive rate model, Truncated distribution

1. Introduction

In reliability studies, combinations of components forming series, parallel and k out of n systems are quite popular. The reliability probabilities of such systems are evaluated either by the system as a whole or through the reliability probabilities of the components that define the system. It is well known that in a series system of a finite number of components with independent life time random, the system reliability is equal to the product of the component reliabilities.

If $f(x), F(x), h(x)$ respectively indicate the failure density, failure probability and failure rate of a component with life time random variable x, then we know that the reliability is given by,

$$R(x) = 1 - F(x) = Exp\{-\int_0^x h(x)dx\} \qquad (1)$$

Where

$$F(x) = p(X \le x) = \int_0^x f(x)dx \qquad (2)$$

If a series system has two component with independent but non-identical life patterns explained by two distinct random variables say x_1 and x_2 with respective failure densities, failure probabilities and failure rates as $f_1(x), f_2(x)$; $F_1(x), F_2(x)$; $h_1(x), h_2(x)$, then the system reliability is given by,

$$R(x) = Exp\{-\int_0^x [h_1(x) + h_2(x)]dx\} \qquad (3)$$

From the above expression we can get the failure density and the failure rate of the series system whose reliability is given by (3), such models are already studied in the past with different choices of $h_1(x)$ and $h_2(x)$ by Rao, Nagendram and Rosaiah (2013), Rao, Kantam, Rosaiah and Baba (2013) [4] and Rosaiah, Nagarjuna, Kumar and Rao (2014) [3]. In this paper a combination of $\text{MOEU}(\alpha,\theta)$ and some other distributions will studied.

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2. MOEU Distribution and Its Properties

Marshall and Olkin (1997) [2] introduced a new family of distributions in an attempt to add a parameter to a family of distributions. Let $\bar{G}(x) = P(X > x)$ be the reliability function of a random variable X and $\alpha > 0$ be a parameter. Then

$$\bar{F}(x,\alpha) = \frac{\alpha\bar{G}(x)}{1-(1-\alpha)\bar{G}(x)}, -\infty < x < \infty, \alpha > 0, \quad (4)$$

is a proper reliability function. $\bar{F}(x,\alpha)$ is called Marshall-Olkin family of distributions. The probability density function (p.d.f) corresponding to (4) is given by

$$f(x,\alpha) = \frac{\alpha\, g(x)}{[1-(1-\alpha)\bar{G}(x)]^2}, -\infty < x < \infty, \alpha > 0, \quad (5)$$

where $g(x)$ is the p.d.f. corresponding to $\bar{G}(x)$. The hazard (failure) rate function is given by

$$h(x,\alpha) = r(x)/[1-(1-\alpha)\bar{G}(x)],$$

where $r(x) = g(x)/\bar{G}(x)$.

Now, Let X follows $U(0,\theta)$ distribution, where $\theta > 0$. Then $\bar{G}(x) = 1 - (x/\theta)$. Substituting in (1) we get a new distribution denoted by MOEU (α,θ) with reliability function [1].

$$\bar{F}(x,\alpha,\theta) = \alpha(\theta - x)/(\alpha\theta + (1-\alpha)x), \quad 0 < x < \theta, \alpha > 0. \quad (6)$$

The corresponding pdf is obtained as

$$f(x,\alpha,\theta) = \alpha\theta/(\alpha\theta + (1-\alpha)x)^2, 0 < x < \theta, \alpha > 0. \quad (7)$$

and the corresponding cumulative distribution function is,

$$F(x,\alpha,\theta) = 1 - \bar{F}(x,\alpha,\theta) = x/(\alpha\theta + (1-\alpha)x) \ ,0 < x < \theta, \alpha > 0. \qquad (8)$$

Note that α is the shape parameter and θ is the scale parameter of the distribution. The hazard rate function of a random variable X with MOEU(α,θ) distribution is

$$h(x,\alpha,\theta) = \theta/[\alpha\theta + (1-\alpha)x](\theta - x) \qquad (9)$$

The higher-order moments is [1],

$$E(X^r) = \int_0^\theta x^r \cdot \frac{\alpha\theta}{(\alpha\theta + (1-\alpha)x)^2}\, dx$$

$$= \frac{\alpha\theta}{(1-\alpha)^{r+1}} \left[\sum_{s=0}^{r} \frac{r!\,(-\alpha\theta)^s (\alpha\theta + (1-\alpha)x)^{r-s-1}}{(r-s)!\ \ s!\ \ (r-s-1)}\right]_0^\theta$$

$$= \frac{\alpha\theta}{(1-\alpha)^{r+1}} \sum_{s=0}^{r} \frac{r!(-\alpha\theta)^s}{(r-s)!\ s!\ (r-s-1)} \cdot [(\theta)^{r-s-1} - (\alpha\theta)^{r-s-1}] \qquad (10)$$

Specially, the mean and the variance of a random variable X with MOEU(α,θ) distribution are, respectively [1], $\acute{\mu}_1 = \frac{\alpha\theta}{(1-\alpha)^2}(\alpha - \log\alpha - 1)$, $\mu_2 = \frac{\alpha\theta^2}{(1-\alpha)^4}[(1-\alpha)^2 - \alpha(\log\alpha)^2]$.

So, the coefficient of variation is, $Cv = \sqrt{(1-\alpha)^2 - \alpha(\log\alpha)^2}/\left(\sqrt{\alpha}\ (\alpha - \log\alpha - 1)\right)$, $\alpha > 0$. The q_{th} quantile of a random variable X with MOEU(α,θ) distribution is given by [1],

$x_q = F^{-1}(q) = q\alpha\theta/(1 - q(1-\alpha))$, $0 \le q \le 1$, Where $F^{-1}(\cdot)$ is the inverse distribution function.

The **median** is $Me = \alpha\theta/(\alpha+1)$, and the **mode** is $Mo = x = \frac{\alpha\theta}{(1-\alpha)}$. The **skewness** is

$S_k = \frac{\mu_x - mo}{\sigma_x} = \frac{\sqrt{\alpha}[-\ln\alpha + 2(\alpha-1)]}{\sqrt{(1-\alpha)^2 - \alpha(\ln\alpha)^2}}$ and the **kurtosis** is

$$K_r = \frac{\mu_4}{{\mu_2}^2} - 3$$

$$= \frac{\left\{\frac{\alpha\theta^4}{(1-\alpha)^5}\left[4\alpha^3\ln\alpha + \alpha^3\left(\alpha + \frac{16}{3}\right) + 6\alpha^2 - 2\alpha + \frac{1}{3}\right] - 4\frac{\alpha^2\theta^4}{(1-\alpha)^5}\left[-3\alpha^2\ln\alpha + \alpha^2\left(\alpha + \frac{3}{2}\right) - 3\alpha + \frac{1}{2}\right][-\ln\alpha + \alpha - 1] + 6\frac{\alpha^3\theta^4}{(1-\alpha)^7}[2\alpha\ln\alpha - \alpha^2 + 1][-\ln\alpha + \alpha - 1]^2 - 3\frac{\alpha^4\theta^{16}}{(1-\alpha)^{20}}[-\ln\alpha + \alpha - 1]^4\right\}}{\left(\frac{\alpha\theta^2}{(1-\alpha)^4}[(1-\alpha)^2 - \alpha(\ln\alpha)^2]\right)^2} - 3$$

In the following sections we will derive some additive failure rate models related with MOEU distribution.

3. MOEU-MOEU Additive Failure Rate Model

Here we choice $MOEU(\alpha,\theta)$ for $h_1(x)$ and $MOEU(a,b)$ for $h_2(x)$, then

$$\int_0^t [h_1(x) + h_2(x)]\, dx = \int_0^t \frac{\theta}{(\theta - x)(\theta\alpha + (1-\alpha)x)}\, dx + \int_0^t \frac{b}{(b-x)(ba + (1-a)x)}\, dx\ . \qquad (11)$$

$$since \int_0^t \frac{\theta}{(\theta - x)(\theta\alpha + (1-\alpha)x)}\, dx = \int_0^t \frac{\theta - (\alpha\theta + (1-\alpha)x) + (\alpha\theta + (1-\alpha)x)}{(\theta - x)(\alpha\theta + (1-\alpha)x)} dx$$

$$= \int_0^t \frac{\theta(1-\alpha) - (1-\alpha)x}{(\theta - x)(\alpha\theta + (1-\alpha)x)} + \frac{(\alpha\theta + (1-\alpha)x)}{(\theta - x)(\alpha\theta + (1-\alpha)x)} dx$$

$$= \int_0^t \frac{(1-\alpha)}{(\alpha\theta + (1-\alpha)x)} dx + \int_0^t \frac{1}{(\theta - x)} dx$$

$$= [\ln|(\alpha\theta + (1-\alpha)x)|]_0^t + [-\ln|(\theta - x)|]_0^t$$

$$= \ln|(\alpha\theta + (1-\alpha)t)| - \ln|\theta\alpha| - \ln|(\theta - t)| + \ln|\theta|$$

$$= \ln\left|\frac{(\alpha\theta + (1-\alpha)t)}{\alpha(\theta - t)}\right|$$

so one can write (11)as $\ln\left|\frac{(\alpha\theta + (1-\alpha)t)}{\alpha(\theta - t)}\right| + \ln\left|\frac{(ba + (1-a)t)}{a(b-t)}\right|$, and then by (3) can get,

$$R(x)=e^{-\int_0^t h(x)dx} = e^{-\left(\ln\left|\frac{(\alpha\theta+(1-\alpha)t)}{\alpha(\theta-t)}\right|+\ln\left|\frac{(ba+(1-a)t)}{a(b-t)}\right|\right)}$$
$$=\left(\frac{\alpha(\theta-t)}{(\theta\alpha+(1-\alpha)t)}\cdot\frac{a(b-t)}{(ba+(1-a)t)}\right) \tag{12}$$

Which is mean, $R=R_1\cdot R_2$. So for two additive failure rates,$h_1(x)$ of $MOEU(\alpha,\theta)$and $h_2(x)$ of $MOEU(a,b)$, one can get the distribution of the system as. $f(t)=-\partial R/\partial t=R_1\cdot f_2+R_2\cdot f_1$, where,

$\frac{\partial R}{\partial t}=R_1\cdot\frac{\partial R_2}{\partial t}+R_2\cdot\frac{\partial R_1}{\partial t}=-R_1\cdot f_2-R_2\cdot f_1$, so,

$$f(t)=\frac{\alpha(\theta-t)}{(\alpha\theta+(1-\alpha)t)}\cdot\frac{ab}{(ab+(1-a)t)^2}+\frac{a(b-t)}{(ab+(1-a)t)}\cdot\frac{\alpha\theta}{(\alpha\theta+(1-\alpha)t)^2}$$
$$f(t)=\frac{a\alpha}{(\alpha\theta+(1-\alpha)t)(ab+(1-a)t)}\left\{\frac{b(\theta-t)}{(ab+(1-a)t)}+\frac{\theta(b-t)}{(\alpha\theta+(1-\alpha)t)}\right\} \tag{13}$$

According to the same argument, if we have for two additive failure rates $h_1(x)$ of $MOEU(\alpha,\theta)$ and $h_3(x)$ of (a,θ), then, one can get the distribution of the system as.

$$f(t)=\frac{\alpha(\theta-t)}{(\alpha\theta+(1-\alpha)t)}\cdot\frac{a\theta}{(a\theta+(1-a)t)^2}+\frac{a(\theta-t)}{(a\theta+(1-a)t)}\cdot\frac{\alpha\theta}{(\alpha\theta+(1-\alpha)t)^2}$$
$$=\frac{\alpha\theta a(\theta-t)}{(\alpha\theta+(1-\alpha)t)(a\theta+(1-a)t)}\left\{\frac{1}{(a\theta+(1-a)t)}+\frac{1}{(\alpha\theta+(1-\alpha)t)}\right\} \tag{14}$$

4. MOEU-Uniform Additive Failure Rate Model

Here we choice $MOEU(\alpha,\theta)$ for $h_1(x)$ and $u(0,\theta)$ for $h_2(x)$. So, Since $f_2(x)=1/\theta$, $0<x<\theta$, then $F_2(x)=x/\theta$, $R_2(x)=(\theta-x)/\theta$, and $h_2(x)=1/(\theta-x)$.

We have,

$$\int_0^t[h_1(x)+h_2(x)]dx=\int_0^t\frac{\theta}{(\theta-x)(\theta\alpha+(1-\alpha)x)}dx+\int_0^t\frac{1}{(\theta-x)}dx$$

$$since \int_0^t\frac{1}{(\theta-x)}dx=[-\ln|(\theta-x)|]_0^t$$
$$=-\ln|(\theta-t)|+\ln|\theta|$$
$$=\ln\left|\frac{\theta}{(\theta-t)}\right|,$$

then we get,

$$\int_0^t[h_1(x)+h_2(x)]dx=\ln\left|\frac{(\alpha\theta+(1-\alpha)t)}{\alpha(\theta-t)}\right|+\ln\left|\frac{\theta}{(\theta-t)}\right|=\ln\left|\frac{\theta(\theta\alpha+(1-\alpha)t)}{\alpha(\theta-t)^2}\right|$$

So, by (3), one can get the system Reliability as,

$$R=e^{-\int_0^t h(x)dx}\implies R=e^{-\ln\left|\frac{\theta(\theta\alpha+(1-\alpha)t)}{\alpha(\theta-t)^2}\right|}=\left(\frac{\alpha(\theta-t)^2}{\theta(\theta\alpha+(1-\alpha)t)}\right) \tag{15}$$

For two additive failure rates, $h_1(x)$ of $MOEU(\alpha,\theta)$ and $h_2(x)$ of $(0,\theta)$, then, one can get the distribution of the system as.

$$f(t)=-\frac{\partial R}{\partial t}=-\frac{\theta(\theta\alpha+(1-\alpha)t)(-2\alpha(\theta-t))-\alpha(\theta-t)^2\theta(1-\alpha)}{\theta^2(\theta\alpha+(1-\alpha)t)^2}$$
$$=\frac{\alpha(t-\theta)\{\alpha(t-\theta)-(t+\theta)\}}{\theta(\theta\alpha+(1-\alpha)t)^2},\quad 0<t<\theta, \tag{16}$$

5. MOEU-Truncated Exponential Additive Failure Rate Model

The probability density function of truncated exponential distribution from the right can be derived as,

$$f^*(x) = \frac{f(x)}{F(b)-F(a)} = \frac{\lambda e^{-\lambda x}}{1-e^{-\lambda\theta}} \ \ when\ \ a = 0\ and\ \ b = \theta\ ,$$

so the cumulative distribution is

$$F^*(x) = \int_0^x \frac{\lambda e^{-\lambda t}}{1-e^{-\lambda\theta}} dt = \frac{-1}{1-e^{-\lambda\theta}}\left[e^{-\lambda t}\right]_0^x = \frac{-e^{-\lambda x} + e^{-\lambda(0)}}{1-e^{-\lambda\theta}} \Longrightarrow F^*(x) = \frac{1-e^{-\lambda x}}{1-e^{-\lambda\theta}}.$$

and then the reliability function is

$$R^*(x) = 1 - F^*(x) = 1 - \frac{1-e^{-\lambda x}}{1-e^{-\lambda\theta}}\ ,\ \ 0 < x < \theta\ .$$

$$= \frac{1-e^{-\lambda\theta} - 1 + e^{-\lambda x}}{1-e^{-\lambda\theta}} \Longrightarrow R^*(x) = \frac{e^{-\lambda x} - e^{-\lambda\theta}}{1-e^{-\lambda\theta}}\ , so\ the\ hazard\ function\ will\ be,$$

$$h^*(x) = \frac{f^*(x)}{R^*(x)} = \frac{\lambda e^{-\lambda x}/(1-e^{-\lambda\theta})}{(e^{-\lambda x}-e^{-\lambda\theta})/(1-e^{-\lambda\theta})} = \frac{\lambda}{1-e^{-\lambda(\theta-x)}}. \quad (17)$$

Here we choice $MOEU(\alpha,\theta)$ for $h_1(x)$ and truncated exponential from the right for $h_2(x)$.
Now, since

$$\int_0^t \frac{\lambda}{1-e^{-\lambda(\theta-x)}} dx = \int_0^t \frac{\lambda e^{-\lambda x}}{(e^{-\lambda x} - e^{-\lambda\theta})} dx$$

$$= \left[-\ln\left|(e^{-\lambda x} - e^{-\lambda\theta})\right|\right]_0^t$$

$$= -\ln\left|(e^{-\lambda t} - e^{-\lambda\theta})\right| + \ln\left|(1 - e^{-\lambda\theta})\right|$$

$$= \ln\left|\frac{1-e^{-\lambda\theta}}{(e^{-\lambda t} - e^{-\lambda\theta})}\right|$$

So, we can write

$$\int_0^t h(x)dx = \int_0^t [h_1(x) + h_2(x)]dx = \int_0^t [h_1(x) + h_2(x)]dx = \int_0^t \frac{\theta}{(\theta - x)(\theta\alpha + (1-\alpha)x)} dx + \int_0^t \frac{\lambda}{1-e^{-\lambda(\theta-x)}} dx$$

We get $\ln\left|\frac{(\alpha\theta+(1-\alpha)t)}{\alpha(\theta-t)}\right| + \ln\left|\frac{1-e^{-\lambda\theta}}{(e^{-\lambda t}-e^{-\lambda\theta})}\right| = \ln\left|\frac{(1-e^{-\lambda\theta})(\theta\alpha+(1-\alpha)t)}{\alpha(e^{-\lambda t}-e^{-\lambda\theta})(\theta-t)}\right|$

And then the reliability function of the system can be written by (3) as,

$$R = e^{-\int_0^t h(x)dx} = e^{-\ln\left|\frac{(1-e^{-\lambda\theta})(\alpha\theta+(1-\alpha)t)}{\alpha(\theta-t)(e^{-\lambda t}-e^{-\lambda\theta})}\right|}$$

$$= \left|\frac{\alpha(\theta-t)(e^{-\lambda t} - e^{-\lambda\theta})}{(1-e^{-\lambda\theta})(\alpha\theta + (1-\alpha)t)}\right|$$

$$= \frac{\alpha}{1-e^{-\lambda\theta}}\left\{\frac{(\theta-t)(e^{-\lambda t}-e^{-\lambda\theta})}{(\theta\alpha+(1-\alpha)t)}\right\} \quad (18)$$

It follows that, for two additive failure rates $h_1(x)$ of $MOEU(\alpha,\theta)$ and $h_2(x)$ of truncated exponential(λ) at, then one can get the distribution of the system as follows,

$$f(t) = -\grave{R} = \frac{\alpha}{1-e^{-\lambda\theta}}\left\{\frac{(\alpha\theta+(1-\alpha)t)\{\lambda e^{-\lambda t}(\theta-t)+\left(e^{-\lambda t}-e^{-\lambda\theta}\right)\}+(\theta-t)(1-\alpha)\left(e^{-\lambda t}-e^{-\lambda\theta}\right)}{(\alpha\theta+(1-\alpha)t)^2}\right\}, 0 < t < \theta\,. \quad (19)$$

Where, $\grave{R} = \frac{\alpha}{1-e^{-\lambda\theta}}\left\{\frac{(\alpha\theta+(1-\alpha)t)\{-\lambda(\theta-t)e^{-\lambda t}-(e^{-\lambda t}-e^{-\lambda\theta})\}-(\theta-t)(1-\alpha)(e^{-\lambda t}-e^{-\lambda\theta})}{(\alpha\theta+(1-\alpha)t)^2}\right\}$

6. MOEU-Truncated Weibull Additive Failure Rate Model

The pdf of the truncated Weibull from the right at θ can be derived as

$$f^*(x)=\frac{f(x)}{F(\theta)-F(0)}=\frac{\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-\left(\frac{x}{\lambda}\right)^k}}{1-e^{-\left(\frac{\theta}{\lambda}\right)^k}}\ ,\ 0<x<\theta\,.$$

so, the distribution function can be defined as

$$F^*(x)=\int_0^x\frac{\frac{k}{\lambda}(\frac{t}{\lambda})^{k-1}e^{-\left(\frac{t}{\lambda}\right)^k}}{1-e^{-\left(\frac{\theta}{\lambda}\right)^k}}dt=\frac{-1}{1-e^{-\left(\frac{\theta}{\lambda}\right)^k}}\left[e^{-\left(\frac{t}{\lambda}\right)^k}\right]_0^x=\frac{1-e^{-\left(\frac{x}{\lambda}\right)^k}}{1-e^{-\left(\frac{\theta}{\lambda}\right)^k}}\ ,$$

and then the reliability function will be,

$$R^*(x)=1-F^*(x)=1-\frac{1-e^{-\left(\frac{x}{\lambda}\right)^k}}{1-e^{-\left(\frac{\theta}{\lambda}\right)^k}}$$

$$=\frac{1-e^{-\left(\frac{\theta}{\lambda}\right)^k}-1+e^{-\left(\frac{x}{\lambda}\right)^k}}{1-e^{-\left(\frac{\theta}{\lambda}\right)^k}}$$

$$=\frac{e^{-\left(\frac{x}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}}{1-e^{-\left(\frac{\theta}{\lambda}\right)^k}},$$

so the failure rate function will be,

$$h^*(x)=\frac{f^*(x)}{R^*(x)}=\frac{\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-\left(\frac{x}{\lambda}\right)^k}\Big/(1-e^{-\left(\frac{\theta}{\lambda}\right)^k})}{e^{-\left(\frac{x}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}\Big/(1-e^{-\left(\frac{\theta}{\lambda}\right)^k})}=\frac{\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}}{1-e^{-(\frac{\theta-x}{\lambda})^k}} \tag{20}$$

So if we choice $MOEU(\alpha,\theta)$ for $h_1(x)$ and truncated Weibull(λ,κ) from the right at θ for $h_2(x)$

$$then\ \int_0^t h(x)dx=\int_0^t[h_1(x)+h_2(x)]dx=\int_0^t\frac{\theta}{(\theta-x)(\theta\alpha+(1-\alpha)x)}dx+\int_0^t\frac{\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}}{1-e^{-(\frac{\theta-x}{\lambda})^k}}dx$$

$$since\int_0^t\frac{\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}}{1-e^{-\left(\frac{\theta-x}{\lambda}\right)^k}}dx=\int_0^t\frac{\frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-(\frac{x}{\lambda})^k}}{e^{-\left(\frac{x}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}}dx$$

$$=\left[-\ln\left|e^{-\left(\frac{x}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}\right|\right]_0^t$$

$$=-\ln\left|e^{-\left(\frac{t}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}\right|+\ln\left|1-e^{-(\frac{\theta}{\lambda})^k}\right|$$

$$=\ln\left|\frac{1-e^{-(\frac{\theta}{\lambda})^k}}{e^{-\left(\frac{t}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}}\right|$$

Then $\int_0^t h(x)dx=\ln\left|\frac{(\alpha\theta+(1-\alpha)t)}{\alpha(\theta-t)}\right|+\ln\left|\frac{1-e^{-(\frac{\theta}{\lambda})^k}}{e^{-\left(\frac{t}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}}\right|$

$$=\ln\left|\frac{(\alpha\theta+(1-\alpha)t)(1-e^{-\left(\frac{\theta}{\lambda}\right)^k})}{\alpha(\theta-t)(e^{-\left(\frac{t}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k})}\right|,$$

so by (3) the reliability function of the system is,

$$R = e^{-\int_0^t h(x)dx} = \left(\frac{\alpha(\theta-t)(e^{-\left(\frac{t}{\lambda}\right)^k} - e^{-\left(\frac{\theta}{\lambda}\right)^k})}{(\alpha\theta+(1-\alpha)t)(1-e^{-\left(\frac{\theta}{\lambda}\right)^k})}\right) \tag{21}$$

for two additive failure rates, $h_1(x)$ of $MOEU(\alpha,\theta)$ and $h_2(x)$ of truncated weibull(λ,κ) from the right at θ, then, one can get the distribution of the system as,

$$f(t) = -\frac{\partial R}{\partial t} = \frac{\alpha}{(1-e^{-\left(\frac{\theta}{\lambda}\right)^k})}\left\{\frac{(\alpha\theta+(1-\alpha)t)\left[(\theta-t)\frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1}e^{-\left(\frac{t}{\lambda}\right)^k}+\left(e^{-\left(\frac{t}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}\right)\right]+(\theta-t)(1-\alpha)\left(e^{-\left(\frac{t}{\lambda}\right)^k}-e^{-\left(\frac{\theta}{\lambda}\right)^k}\right)}{(\alpha\theta+(1-\alpha)t)^2}\right\} \tag{22}$$

7. MOEU-Truncated Frechet Additive Failure Rate Model

The pdf of truncated Frechet from the right at θ can be derived as, $f^*(x) = \frac{f(x)}{F(\theta)-F(0)} = \frac{ab\,x^{-(b+1)}e^{-ax^{-b}}}{e^{-a\theta^{-b}}}$, $0 < x < \theta$. so the distribution function can be derived as,

$$F^*(x) = \int_0^x \frac{ab\,t^{-(b+1)}e^{-at^{-b}}}{e^{-a\theta^{-b}}}\,dt = \frac{1}{e^{-a\theta^{-b}}}\left[e^{-at^{-b}}\right]_0^x = \frac{e^{-ax^{-b}}-1}{e^{-a\theta^{-b}}},$$

and then the reliability function as, $R^*(x) = 1 - F^*(x) = \frac{e^{-a\theta^{-b}}-e^{-ax^{-b}}+1}{e^{-a\theta^{-b}}}$

So the hazard function will be

$$h^*(x) = \frac{f^*(x)}{R^*(x)} = \frac{(ab\,x^{-(b+1)}e^{-ax^{-b}})/e^{-a\theta^{-b}}}{(e^{-a\theta^{-b}}-e^{-ax^{-b}}+1)/e^{-a\theta^{-b}}} = \frac{(ab\,x^{-(b+1)}e^{-ax^{-b}})}{(e^{-a\theta^{-b}}-e^{-ax^{-b}}+1)} \tag{23}$$

Now, if we choice $h_1(x)$ of $MOEU(\alpha,\theta)$ and $h_2(x)$ of truncated Frechet$(a,b)$from the right at,

$$then \int_0^t h(x)dx = \int_0^t [h_1(x)+h_2(x)]dx = \int_0^t \frac{\theta}{(\theta-x)(\theta\alpha+(1-\alpha)x)}dx + \int_0^t \frac{(abx^{-(b+1)}e^{-ax^{-b}})}{(e^{-a\theta^{-b}}-e^{-ax^{-b}}+1)}dx$$

$$since \int_0^t \frac{abx^{-(b+1)}e^{-ax^{-b}}}{e^{-a\theta^{-b}}-e^{-ax^{-b}}+1}dx = \left[-\ln\left|e^{-a\theta^{-b}}-e^{-ax^{-b}}+1\right|\right]_0^t$$

$$= -\ln\left|e^{-a\theta^{-b}}-e^{-at^{-b}}+1\right| + \ln\left|e^{-a\theta^{-b}}-1+1\right|$$

$$= \ln\left|\frac{e^{-a\theta^{-b}}}{e^{-a\theta^{-b}}-e^{-at^{-b}}+1}\right|$$

Then $\int_0^t h(x)dx = \ln\left|\frac{(\alpha\theta+(1-\alpha)t)}{\alpha(\theta-t)}\right| + \ln\left|\frac{e^{-a\theta^{-b}}}{e^{-a\theta^{-b}}-e^{-at^{-b}}+1}\right|$

$$= \ln\left|\frac{(\alpha\theta+(1-\alpha)t)e^{-a\theta^{-b}}}{\alpha(\theta-t)(e^{-a\theta^{-b}}-e^{-at^{-b}}+1)}\right|$$, so the reliability function of the system is,

$$R = e^{-\int_0^t h(x)dx} = e^{-\ln\left|\frac{(\alpha\theta+(1-\alpha)t)e^{-a\theta^{-b}}}{\alpha(\theta-t)(e^{-a\theta^{-b}}-e^{-at^{-b}}+1)}\right|}$$

$$= \left(\frac{\alpha(\theta-t)(e^{-a\theta^{-b}}-e^{-at^{-b}}+1)}{(\alpha\theta+(1-\alpha)t)e^{-a\theta^{-b}}}\right) \tag{24}$$

For two additive failure rates $h_1(x)$ of $MOEU(\alpha,\theta)$ and $h_2(x)$ of truncated Frechet(a,b), then, one can get the probability distribution of the system as,

$$f(t) = \frac{-\partial R}{\partial t} = \frac{\alpha}{e^{-a\theta^{-b}}}\left\{\frac{(\alpha\theta+(1-\alpha)t)\left[(\theta-t)ab\,t^{-(b+1)}e^{-at^{-b}}+(e^{-a\theta^{-b}}-e^{-at^{-b}}+1)\right]+(\theta-t)(1-\alpha)(e^{-a\theta^{-b}}-e^{-at^{-b}}+1)}{(\alpha\theta+(1-\alpha)t)^2}\right\} \tag{25}$$

8. MOEU-Truncated Rayleigh Additive Failure Rate Model

The pdf of truncated Rayleigh from the right at θ can be derived as,

$$f^*(x) = \frac{f(x)}{F(\theta) - F(0)} = \frac{\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}}{1 - e^{-\frac{\theta^2}{2\sigma^2}}}, \quad 0 < x < \theta .$$

so the distribution function is,

$$F^*(x) = \int_0^x \frac{\frac{t}{\sigma^2} e^{-\frac{t^2}{2\sigma^2}}}{1 - e^{-\frac{\theta^2}{2\sigma^2}}} dt = \frac{1}{1 - e^{-\frac{\theta^2}{2\sigma^2}}} \left[-e^{-\frac{t^2}{2\sigma^2}} \right]_0^x = \frac{1 - e^{-\frac{x^2}{2\sigma^2}}}{1 - e^{-\frac{\theta^2}{2\sigma^2}}},$$

and the reliability function is,

$$R^*(x) = 1 - F^*(x) = 1 - \frac{1 - e^{-\frac{x^2}{2\sigma^2}}}{1 - e^{-\frac{\theta^2}{2\sigma^2}}} = \frac{e^{-\frac{x^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}}}{1 - e^{-\frac{\theta^2}{2\sigma^2}}}$$

So, the hazard function will be

$$h^*(x) = \frac{f^*(x)}{R^*(x)} = \frac{\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} / 1 - e^{-\frac{\theta^2}{2\sigma^2}}}{e^{-\frac{x^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}} / 1 - e^{-\frac{\theta^2}{2\sigma^2}}} = \frac{x/\sigma^2}{1 - e^{-\frac{(\theta^2 - x^2)}{2\sigma^2}}}$$

Now, if we choice $MOEU(\alpha, \theta)$ for $h_1(x)$ and truncated Rayleigh(σ) from the right at θ for $h_2(x)$ then, $\int_0^t h(x)dx = \int_0^t [h_1(x) + h_2(x)]dx = \int_0^t \frac{\theta}{(\theta - x)(\alpha\theta + (1-\alpha)x)} dx + \int_0^t \frac{x/\sigma^2}{1 - e^{-\frac{(\theta^2 - x^2)}{2\sigma^2}}} dx$

$$since \int_0^t \frac{x/\sigma^2}{1 - e^{-\frac{\theta^2 - x^2}{2\sigma^2}}} dx = \int_0^t \frac{\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}}{e^{-\frac{x^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}}} dx$$

$$= \left\{ -\ln \left| e^{-\frac{x^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}} \right| \right\}_0^t = -\ln \left| e^{-\frac{t^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}} \right| + \ln \left| 1 - e^{-\frac{\theta^2}{2\sigma^2}} \right|$$

$$= \ln \left| \frac{1 - e^{-\frac{\theta^2}{2\sigma^2}}}{e^{-\frac{t^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}}} \right|,$$

$$then \int_0^t h(x)dx = \ln \left| \frac{(\alpha\theta + (1-\alpha)t)}{\alpha(\theta - t)} \right| + \ln \left| \frac{1 - e^{-\frac{\theta^2}{2\sigma^2}}}{e^{-\frac{t^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}}} \right|$$

$$= \ln \left| \frac{(\alpha\theta + (1-\alpha)t)(1 - e^{-\frac{\theta^2}{2\sigma^2}})}{\alpha(\theta - t)(e^{-\frac{t^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}})} \right|,$$

so the reliability function of the system is ,

$$R = e^{-\int_0^t h(x)dx} = e^{-\ln \left| \frac{(\alpha\theta + (1-\alpha)t)(1 - e^{-\frac{\theta^2}{2\sigma^2}})}{\alpha(\theta - t)(e^{-\frac{t^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}})} \right|} = \left(\frac{\alpha(\theta - t)(e^{-\frac{t^2}{2\sigma^2}} - e^{-\frac{\theta^2}{2\sigma^2}})}{(\alpha\theta + (1-\alpha)t)(1 - e^{-\frac{\theta^2}{2\sigma^2}})} \right), \quad (26)$$

For two additive failure rates, $h_1(x)$ of $MOEU(\alpha, \theta)$ and $h_2(x)$ of truncated Rayleigh(σ) from the right at θ then, one can get the distribution of the system as,

$$f(t)=\frac{-\partial R}{\partial t}=\frac{\alpha}{(1-e^{-\frac{\theta^2}{2\sigma^2}})}\left\{\frac{(\alpha\theta+(1-\alpha)t)\left[(\theta-t)\frac{t}{\sigma^2}e^{-\frac{t^2}{2\sigma^2}}+(e^{-\frac{t^2}{2\sigma^2}}-e^{-\frac{\theta^2}{2\sigma^2}})\right]+(\theta-t)(1-\alpha)(e^{-\frac{t^2}{2\sigma^2}}-e^{-\frac{\theta^2}{2\sigma^2}})}{(\alpha\theta+(1-\alpha)t)^2}\right\} \quad (27)$$

9. MOEU- Doubly Truncated Cauchy Additive Failure Rate Model

The pdf of doublytruncated Cauchy from the right at θ and from the left at zero can be derived as,

$$f^*(x)=\frac{f(x)}{F(\theta)-F(0)}=\frac{b/\{\pi(b^2+(x-a)^2\}}{\frac{1}{2}+\frac{1}{\pi}\tan^{-1}(\frac{\theta-a}{b})-\frac{1}{2}-\frac{1}{\pi}\tan^{-1}(\frac{-a}{b})},$$

$$=\frac{b/\{\pi(b^2+(x-a)^2\}}{\frac{1}{\pi}\tan^{-1}(\frac{\theta-a}{b})-\frac{1}{\pi}\tan^{-1}(\frac{-a}{b})},0<x<\theta.$$

So the distribution function is

$$F^*(x)=\int_0^x\frac{b/\{\pi(b^2+(t-a)^2\}}{\frac{1}{\pi}\tan^{-1}(\frac{\theta-a}{b})-\frac{1}{\pi}\tan^{-1}(\frac{-a}{b})}dt$$

$$=\frac{\left\{\tan^{-1}(\frac{t-a}{b})\right\}_0^x}{\frac{1}{\pi}\tan^{-1}(\frac{\theta-a}{b})-\frac{1}{\pi}\tan^{-1}(\frac{-a}{b})}=\frac{\tan^{-1}(\frac{x-a}{b})-\tan^{-1}(\frac{-a}{b})}{\tan^{-1}(\frac{\theta-a}{b})-\tan^{-1}(\frac{-a}{b})},$$

And the reliability function is

$$R^*(x)=1-F^*(x)=\frac{\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{x-a}{b}\right)}{\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{-a}{b}\right)},$$

and then the hazard function will be,

$$h^*(x)=\frac{f^*(x)}{R^*(x)}=\frac{b/\{b^2+(x-a)^2\}}{\tan^{-1}(\frac{\theta-a}{b})-\tan^{-1}(\frac{x-a}{b})}, \quad (28)$$

Now, if we choice $MOEU(\alpha,\theta)$ for $h_1(x)$ and doublytruncated Cauchy(a,b) from the right at θ and from the left at zero for $h_2(x)$ then,

$$\int_0^t h(x)dx=\int_0^t[h_1(x)+h_2(x)]dx=\int_0^t\frac{\theta}{(\theta-x)(\alpha\theta+(1-\alpha)x)}dx+\int_0^t\frac{b/\{b^2+(x-a)^2\}}{\tan^{-1}(\frac{\theta-a}{b})-\tan^{-1}(\frac{x-a}{b})}dx$$

$$\textit{since}\int_0^t\frac{b/\{b^2+(x-a)^2\}}{\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{x-a}{b}\right)}dx=\left\{-\ln\left|\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{x-a}{b}\right)\right|\right\}_0^t$$

$$=-\ln\left|\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{t-a}{b}\right)\right|+\ln\left|\tan^{-1}(\frac{\theta-a}{b})-\tan^{-1}(\frac{-a}{b})\right|$$

$$=\ln\left|\frac{\tan^{-1}(\frac{\theta-a}{b})-\tan^{-1}(\frac{-a}{b})}{\tan^{-1}(\frac{\theta-a}{b})-\tan^{-1}(\frac{t-a}{b})}\right|$$

Then,

$$\int_0^t h(x)dx=\ln\left|\frac{(\alpha\theta+(1-\alpha)t)}{\alpha(\theta-t)}\right|+\ln\left|\frac{\tan^{-1}(\frac{\theta-a}{b})-\tan^{-1}(\frac{-a}{b})}{\tan^{-1}(\frac{\theta-a}{b})-\tan^{-1}(\frac{t-a}{b})}\right|,$$

$$=\ln\left|\frac{(\alpha\theta+(1-\alpha)t)(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{-a}{b}\right))}{\alpha(\theta-t)(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{t-a}{b}\right))}\right| \quad (29)$$

So, the reliability of the system can be written as

$$R = e^{-\ln\left|\frac{(\alpha\theta+(1-\alpha)t)(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{-a}{b}\right))}{\alpha(\theta-t)(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{t-a}{b}\right))}\right|} = \frac{\alpha(\theta-t)(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{t-a}{b}\right))}{(\alpha\theta+(1-\alpha)t)(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{-a}{b}\right))} \quad (30)$$

So, for two additive failure rates, $h_1(x)$ of $MOEU(\alpha,\theta)$ and $h_2(x)$ of doublytruncated Cauchy(a,b) from the right at θ and from the left at zero, one can get the distribution of the system as

$$f(t) = \frac{-\partial R}{\partial t}$$

$$= \frac{\alpha}{(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{-a}{b}\right))}\left\{\frac{(\alpha\theta+(1-\alpha)t)\left[\frac{b(\theta-t)}{b^2+(t-a)^2}+(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{t-a}{b}\right))\right]+(\theta-t)(1-\alpha)(\tan^{-1}\left(\frac{\theta-a}{b}\right)-\tan^{-1}\left(\frac{t-a}{b}\right))}{(\alpha\theta+(1-\alpha)t)^2}\right\} \quad (31)$$

10. MOEU-Doublytruncated Gumbel Additive Failure Rate Model

The pdf of doublytruncated Gumbel from the right, at θ and from the left at zero, can be derived as,

$$f^*(x) = \frac{f(x)}{F(\theta)-F(0)} = \frac{\frac{1}{b}e^{-\left(\left(\frac{x-a}{b}\right)+e^{-\left(\frac{x-a}{b}\right)}\right)}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{-a}{b}\right)}}}, \qquad 0 < x < \theta.$$

So the distribution function is

$$F^*(x) = \int_0^x \frac{\frac{1}{b}e^{-\left(\left(\frac{t-a}{b}\right)+e^{-\left(\frac{t-a}{b}\right)}\right)}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{-a}{b}\right)}}}dt = \frac{1}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{-a}{b}\right)}}}\left\{e^{-e^{-\left(\frac{t-a}{b}\right)}}\right\}_0^x = \frac{e^{-e^{-\left(\frac{x-a}{b}\right)}}-e^{-e^{-\left(\frac{-a}{b}\right)}}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{-a}{b}\right)}}}$$

And the reliability function is

$$R^*(x) = 1-F^*(x) = \frac{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{x-a}{b}\right)}}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{-a}{b}\right)}}},$$

and then the hazard function will be

$$h^*(x) = \frac{f^*(x)}{R^*(x)} = \frac{\frac{1}{b}e^{-\left(\left(\frac{x-a}{b}\right)+e^{-\left(\frac{x-a}{b}\right)}\right)}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{x-a}{b}\right)}}} \quad (32)$$

Now, if we choice $MOEU(\alpha,\theta)$ for $h_1(x)$ and doublytruncated Gumbel(a,b) from the right at θ and from the left at zero for $h_2(x)$ then,

$$\int_0^t h(x)dx = \int_0^t [h_1(x)+h_2(x)]dx = \int_0^t \frac{\theta}{(\theta-x)(\alpha\theta+(1-\alpha)x)}dx + \int_0^t \frac{\frac{1}{b}e^{-\left(\left(\frac{x-a}{b}\right)+e^{-\left(\frac{x-a}{b}\right)}\right)}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{x-a}{b}\right)}}}dx,$$

$$since \int_0^t \frac{\frac{1}{b}e^{-\left(\left(\frac{x-a}{b}\right)+e^{-\left(\frac{x-a}{b}\right)}\right)}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{x-a}{b}\right)}}}dx = \left\{-\ln\left|e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{x-a}{b}\right)}}\right|\right\}_0^t$$

$$= -\ln\left|e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{t-a}{b}\right)}}\right| + \ln\left|e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{-a}{b}\right)}}\right|$$

$$= \ln\left|\frac{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{-a}{b}\right)}}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}}-e^{-e^{-\left(\frac{t-a}{b}\right)}}}\right|$$

$$then \int_0^t h(x)dx = \ln\left|\frac{(\theta\alpha+(1-\alpha)t)}{\alpha(\theta-t)}\right| + \ln\left|\frac{e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{-a}{b}\right)}}}{e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{t-a}{b}\right)}}}\right|$$

$$= \ln\left|\frac{(\theta\alpha+(1-\alpha)t)(e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{-a}{b}\right)}})}{\alpha(\theta-t)(e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{t-a}{b}\right)}})}\right| \quad (33)$$

So the reliability function of the system can be written as

$$R = e^{-\ln\left|\frac{(\theta\alpha+(1-\alpha)t)(e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{-a}{b}\right)}})}{\alpha(\theta-t)(e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{t-a}{b}\right)}})}\right|}, = \frac{\alpha(\theta-t)(e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{t-a}{b}\right)}})}{(\alpha\theta+(1-\alpha)t)(e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{-a}{b}\right)}})}, \quad (34)$$

For two additive failure rates, $h_1(x)$ of $MOEU(\alpha,\theta)$ and $h_2(x)$ of doublytruncated Gmbel(a,b) from the right at θ and from the left at zero, one can get the distribution of the system as.

$$f(t) = \frac{-\partial R}{\partial t}$$

$$= \frac{\alpha}{\left(e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{-a}{b}\right)}}\right)} \left\{\frac{(\alpha\theta+(1-\alpha)t)\left[-(\theta-t)\frac{1}{b}e^{-\left(\frac{t-a}{b}\right)}e^{-e^{-\left(\frac{t-a}{b}\right)}} + (e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{t-a}{b}\right)}})\right] + (\theta-t)(1-\alpha)(e^{-e^{-\left(\frac{\theta-a}{b}\right)}} - e^{-e^{-\left(\frac{t-a}{b}\right)}})}{(\alpha\theta+(1-\alpha)t)^2}\right\} \quad (35)$$

11. Summary and Conclusions

In spite of the great importance of the uniform distribution uses, but unfortunately the form of the distribution and its properties reduced the distribution applications, especially in real life. This issue has made us think to construct other distributions based on the uniform distribution, So that the new distributions have flexible forms and properties to represent a lot of other applications.

A combination of (Marshall-Olkin Extended Uniform distribution) MOEU(α,θ) model and every one of some probability models are developed on lines of the well known linear failure rate model .We derive here the additive failure rate model of MOEU(α,θ) and every one of MOEU(a,b), MOEU(a,θ), uniform(θ), truncated exponential (λ,θ), truncated Weibull (λ,k,θ), truncated Frechet (a,b,θ), truncated Rayleigh(σ^2,θ), truncated Cauchy (a,b,θ) and truncated Gumbel(a,b,θ) distributions.

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