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Commercial Sex on University Campuses: An Epidemiological Approach

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Abstract The study employs an epidemiological modelling approach to explain the transmission dynamics of commercial sex work within university campuses. It utilizes a deterministic model that categorizes individuals into Abstained, Social Sex Worker, and Habitual Commercial Sex Worker (ASH) compartments to explore their transmission dynamics. The basic reproduction number, stability analysis, commercial sex-free and endemic equilibrium solutions, and simulations were performed. From the analytical solution, both the commercial sex-free and endemic equilibria were observed to be globally asymptotically stable. The basic reproduction number was below the minimum threshold, and the commercial sex-free equilibrium was locally asymptotically stable when $\Re_0 < 1$. Numerical simulations indicated that the abstained commercial sex population begins to decline as the social and habitual commercial sex populations begin to increase and remain asymptotic to the horizontal axis.

Keywords Stability, Epidemiological, Equilibrium, Commercial Sex, University

1. Introduction

Commercial sex work is the exchange of sexual services or performances for money or goods. 'The oldest profession in the world' is a common phrase used to describe commercial sex work since this practice has been recorded throughout human history and dates back to ancient societies [1] Throughout history, there has been controversy surrounding the exchange of sex for material goods or benefits. Thus, commercial sex encompasses various forms of sex work conducted with a commercial aspect, where individuals provide sexual services in exchange for compensation [2]. This compensation can take many forms, including cash payments, gifts, goods, or other services. Although this behaviour is frowned upon in many countries around the world, some countries have either legalized or are in the process of legalizing the act. Despite its illegality, commercial sex work is increasingly prevalent among young individuals in various forms, particularly in bustling commercial hubs [3].

There is a growing trend that commercial sex work is gaining ground in higher-learning institutions [4]. Studies have revealed that college students are involved in commercial sex work activities either within campus premises or by sharing their contact information at various bars, motels, and guest houses, enabling potential clients to reach out to them [5]. Commercial sex work has transcended brothels and streets, permeating every facet of society, including university campuses. Efforts by government and university authorities to address the issue through legal and institutional frameworks have, rather than mitigating the problem, exacerbated it [6]. Commercial sex work occurring on a university campus is generally not something that universities support. Universities have codes of conduct and policies that prohibit illegal activities, including prostitution on their campuses [7]. These policies are in place to ensure the safety and well-being of students and staff. Many universities provide support services for students, including counselling, health services, and resources for sexual health and well-being. These services are intended to ensure the well-being and safety of students and often include information about safe and consensual sexual practices.

Commercial sex work is not limited to a specific gender; both males and females engage in this activity. However, it is more common among females than males. For the purpose of this study, both male and female individuals engaging in sexual acts would be considered.

Over the years, different authors have used various approaches to address commercial sex work [8-10]. For instance, an Analysis of Commercial Sex Work in Nigerian universities was studied by [11] considering a cursory review of the history of Nigerian tertiary institutions indicating that campus commercial sex work is on the ascendancy. [12] applied a nested concurrent mixed-method approach to

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investigate the nature and practice of commercial sex work in the Cape Coast Metropolis of Ghana. [13] used an exploratory mixed-method design to conduct a study on commercial sex work in selected public universities in Ghana to inform policy and programme decisions for safer sex at the universities. [14] applied a long-standing survey to explore the impact of question-wording on attitudes toward trading sexual services. Prostitution on university campuses was also considered by [15] using exploratory data analysis. Furthermore, quantitative frameworks have been developed to understand the dynamics of commercial sex work taking place on university campuses. However, mathematical models have not been used extensively to understand the dynamics of commercial sex work in this context. To address this gap, the study therefore seeks to employ an epidemiological modeling approach to examine the dynamics of commercial sex work on university campuses.

2. Model Formulation

A mathematical model designed to describe the diverse transitions experienced by individuals within the university campus sex categories is proposed in this section. The model assumes homogeneous mixing among the university student population, which is divided into three distinct compartments: Abstained (A), Social (S), and Habitual Commercial sex worker compartments. The Abstained Commercial sex worker compartment represents students who are not involved in the commercial sex work trade, the Social Commercial sex worker compartment includes those who occasionally engage in sex work, and the Habitual Commercial sex worker compartment consists of students who have embraced sex work as a daily activity. The category of students in this compartment are a threat to themselves and also to any individual around them. The diagram of the transitions among the three compartments is indicated in Figure 1.

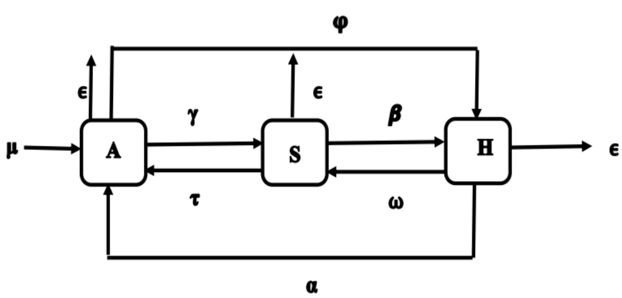

Figure 1. Compartmental Diagram of the Relationship between the three Compartments

Table 1. Variables Used and their meaning

r	Description
A(t)	Abstained Commercial Sex Worker
S(t)	Social Commercial Sex Worker
H(t)	Habitual Commercial Sex Worker

Table 2. Model Parameters and Definitions

Variable	Description
γ	Transmission rate of Abstained commercial sex worker to Social commercial sex worker
τ	Recovery rate of Social commercial sex worker to Abstained commercial sex worker
β	Transmission rate of Social commercial sex worker to Habitual commercial sex worker
ω	Recovery rate of Habitual commercial sex worker to Social commercial sex worker
φ	Transmission rate of Abstained commercial sex worker to Habitual commercial sex worker
α	Recovery rate of Habitual commercial sex worker to Abstained commercial sex worker
μ	Recruitment rate
ϵ	Departure rate

From Figure 1, transitions within these three compartments exhibit specific interaction patterns. When an individual in a lower state moves to a higher state, interactions are assumed to be involved. For instance, an Abstained (A) commercial sex worker must interact with either a Social (S) or Habitual (H) commercial sex worker before transitioning to one of these compartments. This implies that interactions occur when an Abstained individual moves to the Social or habitual commercial sex worker compartments. Additionally, interactions also occur when an individual transitions from the social sex worker compartment to the habitual sex work compartment. Conversely, there are no interactions when an individual transitions from a higher state to a lower state. Therefore, there is no interaction when an individual moves from the social commercial sex work compartment to the Abstained compartment, nor when an individual in the Habitual compartment moves to the Abstained or Social Compartments. Similarly, no interactions are involved when an individual moves from the occasional sex worker category to the habitual sex worker category. These transitions to a lower state are assumed to be based on the individual's recovery. The conversion from Abstained Commercial Sex Worker (A) to Social Commercial Sex Worker (S) and Habitual Commercial Sex Worker (H), as well as from Social Commercial Sex Worker to Habitual Commercial Sex Worker, is modelled using the interactions $\gamma AS, \varphi AH$ and βSH respectively. Individuals in the population move from the Abstained Commercial Sex Worker compartment to the Social Commercial Sex Worker compartment, from Abstained Commercial Sex Worker to Habitual Commercial Sex Worker, and from Social Commercial Sex Worker to Habitual Commercial Sex Worker at rates γ, φ and β respectively. These three parameters are referred to as the mean transmission rates. The recovery rate of the model is given by τ, ω and α. τ is the rate of recovery by which a Social Commercial Sex worker revert back to Abstained Commercial Sex worker. ω is the recovery rate by which a Habitual Commercial Sex worker revert back to a Social

Commercial Sex Worker whilst α is the recovery rate by which Habitual Commercial Sex Worker revert back to Abstained Commercial Sex Worker. The variables and parameters used in the study are described in Tables 1 and 2 respectively. μ is the recruitment rate, which is defined as the rate by which new individuals enter the Abstained population whereas $\in$ is departure rate, that is the rate at which individuals leaves the various compartments. It is assumed that the recruitment rate is equal to the departure rate.

2.1. Model Equations

From the diagram in Figure 1, the resulting system of nonlinear differential equation is given by;

$$\frac{dA}{dt} = \mu - \gamma AS + \tau S - \varphi AH + \alpha H - \in A \quad (1)$$

$$\frac{dS}{dt} = -\epsilon S - \tau S + \gamma AS + \omega H - \beta SH \quad (2)$$

$$\frac{dH}{dt} = \varphi AH - \epsilon H - \alpha H - \omega H + \beta SH \quad (3)$$

Subject to initial conditions:

$$A(0) = A_o, S(0) = S_o, H(0) = H_o \quad (4)$$

and the total population $N = A + S + H$ then

$$\frac{dN}{dt} = \frac{dA}{dt} + \frac{dS}{dt} + \frac{dH}{dt} \quad (5)$$

Therefore

$$\frac{dN}{dt} = \mu - \in (A + S + H) = \mu - \in N \quad (6)$$

3. Positivity of Solution

According to [16] positivity of solution is very essential in biological epidemiological modelling involving the human population since some solutions cannot be negative and as such from equations (1)-(3)

$$\left\{(A, S, H) \to R_+^3 \mid A_o > 0, S_o > 0, H_o > 0\right\}$$

Proof

From equation (1),

$$\frac{dA}{dt} = \mu - \gamma AS + \tau S - \varphi AH + \alpha H - \in A$$

$$\frac{dA}{dt} = \mu - \tau S + \alpha H - (\gamma S + \varphi H + \in) A \quad (7)$$

Accordingly,

$$\frac{dA}{dt} \geq -(\gamma S + \varphi H + \in) A \quad (8)$$

$$\int \frac{dA}{A} \geq -\int (\gamma S + \varphi H + \in) dt \quad (9)$$

Let $q_1(t) = \int (\gamma S + \varphi H + \in) dt$ and c be a constant, thus

$$\ln A(t) \geq -q_1(t) + c \quad (10)$$

$$A(t) \geq e^{-q_1(t) + c} \quad (11)$$

$$A(t) \geq e^{-q_1(t)} . e^{c} \quad (12)$$

Let $Q_1 = e^c$ in equation (10) therefore,

$$A(t) \geq Q_1 e^{-q_1(t)} \quad (13)$$

At $t = 0$, $A(t) = A_0$ and $A_0 > 0$ by theorem 1 and thus $Q_1 = e^c \geq 0$

Hence,

$$A(t) \geq A_0 e^{-q_1(t)} \geq 0, \forall t > 0$$

Also, considering equation (2),

$$\frac{dS}{dt} = - \in S - \tau S + \gamma AS + \omega H - \beta SH$$

$$\frac{dS}{dt} = \omega H - (\in + \tau - \gamma A - \beta H) S \quad (14)$$

Let

$$\frac{dS}{dt} \geq -(\in + \tau - \gamma A - \beta H) S \quad (15)$$

$$\int \frac{dS}{S} \geq -\int (\in + \tau - \gamma A - \beta H) dt \quad (16)$$

Let $q_2(t) = \int (\in + \tau - \gamma A - \beta H) dt$ and c be a constant then

$$\ln S(t) \geq -q_2(t) + c \quad (17)$$

This also implies that,

$$S(t) \geq e^{-q_2(t) + c} \quad (18)$$

$$S(t) \geq e^{-q_2(t)} . e^{c} \quad (19)$$

Let $Q_2 = e^c$ in equation (1), thus,

$$S(t) \geq Q_2 e^{-q_2(t)} \quad (20)$$

At $t = 0$, $S(t) = Q_2 = S_0$ and $S_0 > 0$ by theorem 1 and consequently,

$$S(t) \geq S_0 e^{-q_2(t)} \geq 0, \forall t > 0 \quad (21)$$

Applying the same procedure to equation (3) gives

$$H(t) \geq H_0 e^{-q_3(t)} \geq 0, \forall t > 0 \quad (22)$$

This completes the proof.

4. Equilibrium Solutions

The commercial sex-free and endemic equilibrium is obtained from the developed model. A commercial sex-free equilibrium is a state solution to the model in which the studied population remains in the absence of individuals involved in commercial sex on university campuses. An Endemic Equilibrium (EE) point is defined as a positive steady-state solution when individuals involved in commercial sex persist in the studied population.

4.1. Commercial Sex-Free Equilibrium

The developed model in equations (1)-(3) has a steady state in the form $\left(A^*, S^*, H^*\right)$. For a campus commercial sex-free equilibrium, which is characterized by the absence of individuals who are not involved in the commercial sex trade on campus, we let $\frac{dA}{dt} = \frac{dS}{dt} = \frac{dH}{dt} = 0$ in equations (1)-(3) resulting in the commercial sex-free equilibrium;

$$E_0 = \left(A^*, S^*, H^*\right) = \left(\frac{\mu}{\in}, 0, 0\right) \tag{23}$$

4.2. Secondary Commercial Sex-Free Equilibrium

There also exists a second equilibrium point for equation (1) when habitual commercial sex workers are absent on a university campus. Suppose the social commercial sex worker is not considered to be a problem; then this equilibrium point is considered as the second commercial sex-free equilibrium point, which is expressed as:

$$E_1 = \left(A^*, S^*, H^*\right) = \left(\frac{\in + \tau}{\gamma}, \frac{-\in^2 - \in \tau + \gamma\mu}{\in \gamma}, 0\right) \tag{24}$$

4.3. Endemic Equilibrium

Suppose that at a given point in time, all three states; the abstained group, the occasional sex worker, and the habitual sex worker coexist within the population. Since commercial sex work persists on some university campuses, the endemic equilibrium points $E_2 = \left(A^*, S^*, H^*\right)$ for these three states is expressed as;

$$A^* = -\frac{1}{\varphi}\left(\beta\sqrt{C} - \alpha - \omega - \in\right), \quad S^* = \sqrt{C} \quad \text{and}$$

$$H^* = \frac{1}{\varphi}\left(\in \beta\sqrt{C} - \in \varphi\sqrt{C} - \alpha \in + \mu\varphi - \omega \in - \in^2\right) \tag{25}$$

Where

$$C = \begin{pmatrix} \sqrt{\beta^2 \in + \beta \in \gamma - \beta \in \varphi} + (-\alpha\beta \in - \alpha\beta\gamma - \beta \in^2 - 2\beta \in \omega \\ + \beta\mu\varphi - \in^2 \gamma + \in^2 \varphi - \in \gamma\omega + \in \omega\varphi + \in \varphi\tau) + \\ \alpha \in \omega + \in^2 \omega + \in \omega^2 - \mu\omega\varphi \end{pmatrix}$$

4.4. Basic Reproduction Number

The basic reproduction number, denoted by R_0, is a crucial parameter that predicts whether an infection will spread throughout a population. To obtain this, the next-generation matrix technique is employed [17]. When an infected individual is introduced into a susceptible population, the total number of secondary infections caused by this individual during the epidemic is referred to as the basic reproduction number.

The basic reproduction number is derived using the next-generation matrix.

Let $f = \begin{pmatrix} \gamma AS \\ 0 \end{pmatrix}$ and

$$\upsilon = \begin{pmatrix} -\beta SH + \omega H - \tau S - \in S \\ \varphi AH + \beta SH - \alpha H - \omega H - \in H \end{pmatrix}$$

Partially differentiating f and υ with respect to S and H gives

$$\partial f_{S,H} = \begin{pmatrix} \gamma A & 0 \\ 0 & 0 \end{pmatrix} \tag{26}$$

$$\partial \upsilon_{S,H} = \begin{pmatrix} -H\beta - \tau - \in & -S\beta + \omega \\ H\beta & \varphi A + S\beta - \alpha - \omega - \in \end{pmatrix} \tag{27}$$

Substituting the commercial sex free equilibrium into Equations (25) and (26) result in the Jacobian matrix of f and υ as follows:

$$F = \begin{pmatrix} \frac{\gamma\mu}{\in} & 0 \\ 0 & 0 \end{pmatrix} \tag{28}$$

and

$$V = \begin{pmatrix} -\tau - \in & \omega \\ 0 & \frac{\mu\varphi}{\in} - \alpha - \omega - \in \end{pmatrix} \tag{29}$$

Calculating the inverse of V in equation (29)

$$V^{-1} = \begin{pmatrix} \frac{1}{\in + \tau} & \frac{\omega \in}{(\in + \tau)\left(\mu\varphi - \alpha \in - \omega \in - \in^2\right)} \\ 0 & \frac{\in}{\mu\varphi - \alpha \in - \omega \in - \in^2} \end{pmatrix} \tag{30}$$

Therefore, the next generation matrix, G is given by;

$$G = FV^{-1} = \begin{pmatrix} \frac{\gamma\mu}{\in(\in + \tau)} & \frac{\gamma\mu\omega}{(\in + \tau)\left(\mu\varphi - \alpha \in - \omega \in - \in^2\right)} \\ 0 & 0 \end{pmatrix} \tag{31}$$

Evaluating the eigenvalues of G yields;

$$\lambda_1 = 0 \quad \text{and} \quad \lambda_2 = -\frac{\gamma\mu}{\in(\in + \tau)} \tag{32}$$

Therefore, the basic reproduction number $\Re_0 = \rho\left(FV^{-1}\right)$, where ρ is the spectral radius of the next generation matrix $\left(FV^{-1}\right)$ is represented as:

$$R_0 = \frac{\gamma\mu}{\in\left(\in+\tau\right)} \tag{33}$$

4.5. Second Basic Reproduction number

The second reproduction number $\Re_s$ is defined as the average number of secondary cases generated by a typical Habitual Sex worker in an abstained campus environment. The secondary cases are cases of new habitual sex workers recruited from the abstained class. To derive the secondary basic reproduction number,

Let

$$f = \begin{pmatrix} \gamma AS \\ 0 \end{pmatrix} \text{ and } \upsilon = \begin{pmatrix} -\beta SH + \omega H - \tau S - \in S \\ \varphi AH + \beta SH - \alpha H - \omega H - \in H \end{pmatrix}$$

Partially differentiating f and υ with respect to S and H gives

$$\partial f_{S,H} = \begin{pmatrix} \gamma A & 0 \\ 0 & 0 \end{pmatrix} \tag{34}$$

$$\partial \upsilon_{S,H} = \begin{pmatrix} -H\beta - \tau - \in & -S\beta + \omega \\ H\beta & \varphi A + S\beta - \alpha - \omega - \in \end{pmatrix} \tag{35}$$

Substituting the Secondary commercial sex free equilibrium into Equations (34) and (35) yields the Jacobian matrix of f and υ at the secondary disease-free equation;

$$F = \begin{pmatrix} -\tau - \in & 0 \\ 0 & 0 \end{pmatrix} \tag{36}$$

$$V = \begin{pmatrix} -\tau - \in & \omega - \dfrac{(-\in^2 - \in\tau + \gamma\mu)\beta}{\in\gamma} \\ 0 & -\dfrac{\left(\in+\tau\right)\varphi}{\gamma} + \dfrac{(-\in^2 - \in\tau + \gamma\mu)\beta}{\in\gamma} - \left(\alpha + \omega + \in\right) \end{pmatrix} \tag{37}$$

The inverse of V is calculated and given by

$$V^{-1} = \begin{pmatrix} \dfrac{1}{-\left(\in+\tau\right)} & \dfrac{-\omega\gamma\in - \beta\in^2 - \beta\in\tau + \beta\gamma\mu}{-\left(\in+\tau\right)\left(-\alpha\in\gamma - \beta\in^2 - \beta\in\tau + \beta\gamma\mu - \omega\gamma\in - \in^2\gamma - \varphi\in^2 - \varphi\in\tau\right)} \\ 0 & \dfrac{\in\gamma}{\left(-\alpha\in\gamma - \beta\in^2 - \beta\in\tau + \beta\gamma\mu - \omega\gamma\in - \in^2\gamma - \varphi\in^2 - \varphi\in\tau\right)} \end{pmatrix} \tag{38}$$

Therefore, the next generation matrix, G is given as

$$G = FV^{-1} = \begin{pmatrix} 1 & \dfrac{-\beta\in^2 - \beta\in\tau + \beta\gamma\mu - \omega\gamma\in}{\left(-\alpha\in\gamma - \beta\in^2 - \beta\in\tau + \beta\gamma\mu - \omega\gamma\in - \in^2\gamma - \varphi\in^2 - \varphi\in\tau\right)} \\ 0 & 0 \end{pmatrix} \tag{39}$$

Evaluating the eigenvalues of G in equation (39) gives

$$\lambda_1 = 0 \text{ and } \lambda_2 = 1.$$

Therefore, the basic reproduction number $\Re_s$, which is the dominant eigenvalue of G becomes $\Re_s = 1$.

5. Stability Analysis

The equilibrium points of a system can be classified as stable, unstable, or asymptotically stable according to the nature of the eigenvalues of the coefficient matrix of the system of ordinary differential equations or the Jacobian matrix of the nonlinear systems of ordinary differential equations about such a point.

5.1. Local Stability of the Commercial Sex-Free Equilibrium

Theorem 1

The Commercial Sex-free Equilibrium (CSFE) is Locally Asymptotically Stable if $\Re_0 < 1$ and unstable if $\Re_0 > 1$

Proof

Let the Jacobian matrix of for equations (1)-(3) be given by

$$J = \begin{pmatrix} -H\varphi - S\gamma - \in & -A\gamma + \tau & -\varphi A + \alpha \\ S\gamma & A\gamma - H\beta - \in - \tau & -\beta S + \omega \\ H\varphi & H\beta & \varphi A + \beta S - \in - \alpha - \omega \end{pmatrix} \tag{40}$$

The Jacobian matrix J evaluated at the commercial sex free equilibrium $\varepsilon_0 = \left(A^*, S^*, H^*\right) == \left(\dfrac{\mu}{\in}, 0, 0\right)$ is given by:

$$J = \begin{pmatrix} -\epsilon & -\dfrac{\mu\gamma}{\in} + \tau & -\dfrac{\mu\varphi}{\in} + \alpha \\ 0 & \dfrac{\mu\gamma}{\in} - \in - \tau & \omega \\ 0 & 0 & \dfrac{\mu\varphi}{\in} - \alpha - \omega - \in \end{pmatrix} \tag{41}$$

The eigenvalues of the Jacobian matrix J, is given as

$$\lambda = \begin{pmatrix} -\in \\ -\dfrac{\alpha \in +\omega \in + \in^2 - \mu\varphi}{\in} \\ \dfrac{-\in^2 - \in \tau + \gamma\mu}{\in} \end{pmatrix} \tag{42}$$

Which implies that

$\lambda_1 = -\in$, $\lambda_2 = -\dfrac{\alpha \in +\omega \in + \in^2 - \mu\varphi}{\in}$ and

$\lambda_3 = -\dfrac{\in^2 + \in \tau - \gamma\mu}{\epsilon}$.

Here, $\lambda_3 = -\dfrac{\tau \in + \in^2 - \gamma\mu}{\in} < 0$ can further be simplified as $-\left(\in \tau + \in^2\right) + \gamma\mu < 0$ which implies $\gamma\mu < \left(\varepsilon\tau + \varepsilon^2\right)$.

Thus, $\dfrac{\gamma\mu}{\left(\varepsilon\tau + \varepsilon^2\right)} < 1$, which can be written as; $\dfrac{\gamma\mu}{\varepsilon\left(\tau + \varepsilon\right)} < 1$

But $\Re_0 = \dfrac{\gamma\mu}{\varepsilon\left(\tau + \varepsilon\right)}$

This implies $\Re_0 < 1$, indicating that all the eigenvalues are negative. Thus, commercial sex free equilibrium DFE is locally asymptotically stable.

5.2. Global Stability of Commercial Sex Free Equilibrium

Theorem 2

Suppose that $\Re_0 < 1$, then the DFE point is globally asymptotically stable [19].

Proof: To prove global stability of Commercial sex free equilibrium, the following Lyapunov function must be constructed:

$$G\left(A,S,H\right) = \frac{1}{2}((A - A^0) + (S - S^0) + (H - H^0))^2 \tag{43}$$

Differentiating with respect to time gives;

$$\dot{G}\left(A,S,H\right) = ((A - A^0) + (S - S^0) + (H - H^0))(\dot{A} + \dot{S} + \dot{H})$$

But $(\dot{A} + \dot{S} + \dot{H}) = \mu - \varepsilon N$

Then

$$\dot{G}\left(A,S,H\right) = ((A - A^0) + (S - S^0) + (H - H^0))\left(\mu - \varepsilon N\right) \tag{44}$$

Which can further be simplified as;

$$\dot{G}\left(A,S,H\right) = -((A - A^0) + (S - S^0) + (H - H^0))\left(\varepsilon N - \mu\right) \tag{45}$$

$\dot{G}\left(A,S,H\right) \le 0$ if and only if $\varepsilon N > \mu$. Thus, by the invariance principle of Lasalle, DFE is globally asymptotically stable.

5.3. Global Stability of Endemic Equilibrium

Theorem 3

If $\Re_0 > 1$, then the model system has a unique endemic equilibrium point (EE) point which is globally asymptotically stable in invariant positivity region [20].

Using the approach of Ojo and Akinpelu (2017), the Lyapunov function is defined as:

$$V(t) = (A - A^* \ln A) + (S - S^* \ln S) + (H - H^* \ln H) \tag{46}$$

Differentiating with respect to time gives;

$$\dot{V}(t) = \left(1 - \frac{A^*}{A}\right)\dot{A} + \left(1 - \frac{S^*}{S}\right)\dot{S} + \left(1 - \frac{H^*}{H}\right)\dot{H} \tag{47}$$

By substituting equation of the system gives;

$$\begin{aligned} \dot{V}(t) = {} & \left(1 - \frac{A^*}{A}\right)\left(\mu - \gamma AS + \tau S - \varphi AH + \alpha H - \varepsilon A\right) + \\ & \left(1 - \frac{S^*}{S}\right)\left(-\varepsilon S - \tau S + \gamma AS + \omega H - \beta SH\right) + \\ & \left(1 - \frac{H^*}{H}\right)\left(\varphi AH - \varepsilon H - \alpha H - \omega H + \beta SH\right) \end{aligned} \tag{48}$$

Further simplification gives;

$$\begin{aligned} \dot{V}(t) = {} & \mu\left(1 - \frac{A^*}{A}\right) + \gamma AS\left(\frac{A^*}{A} - \frac{S^*}{S}\right) + \tau S\left(\frac{S^*}{S} - \frac{A^*}{A}\right) + \\ & \varphi AH\left(\frac{A^*}{A} - \frac{H^*}{H}\right) + \alpha H\left(\frac{H^*}{H} - \frac{A^*}{A}\right) - \varepsilon A\left(1 - \frac{A^*}{A}\right) - \\ & \varepsilon S\left(1 - \frac{S^*}{S}\right) + \omega H\left(\frac{H^*}{H} - \frac{S^*}{S}\right) + \beta SH\left(\frac{S^*}{S} - \frac{H^*}{H}\right) - \\ & \varepsilon H\left(1 - \frac{H^*}{H}\right) \end{aligned} \tag{49}$$

Which can again simplify further as;

$$\begin{aligned} \dot{V}(t) = {} & -\mu\left(1 - \frac{A}{A^*}\right) - \gamma AS\left(\frac{S^*}{S} - \frac{A^*}{A}\right) - \tau S\left(\frac{A^*}{A} - \frac{S^*}{S}\right) - \\ & \varphi AH\left(\frac{H^*}{H} - \frac{A^*}{A}\right) - \alpha H\left(\frac{A^*}{A} - \frac{H^*}{H}\right) - \varepsilon A\left(1 - \frac{A^*}{A}\right) - \\ & \varepsilon S\left(1 - \frac{S^*}{S}\right) - \omega H\left(\frac{S^*}{S} - \frac{H^*}{H}\right) - \beta SH\left(\frac{H^*}{H} - \frac{S^*}{S}\right) - \\ & \varepsilon H\left(1 - \frac{H^*}{H}\right) \end{aligned} \tag{50}$$

$$\Rightarrow \dot{V}(t) \le 0 \tag{51}$$

Thus, $\dot{V}(t) \le 0$ for $\Re_0 > 1$. Thus, by the invariance principle of Lasalle [21], EE is globally asymptotically stable.

6. Numerical Simulations

In this section, the numerical simulations of the Commercial sex model are presented using estimated parameter values. These parameter values are indicated in Table 3 to explain the dynamics of commercial sex on university campuses. All simulations in this section were performed using MATLAB software.

Table 3. Model Parameters and Definitions

Variable	Value	Source
γ	0.50	Estimated
τ	0.10	Estimated
β	1.70	Estimated
ω	1.08	Estimated
φ	1.5	Estimated
α	0.0005	Estimated
μ	0.1	Estimated
ϵ	0.1	Estimated

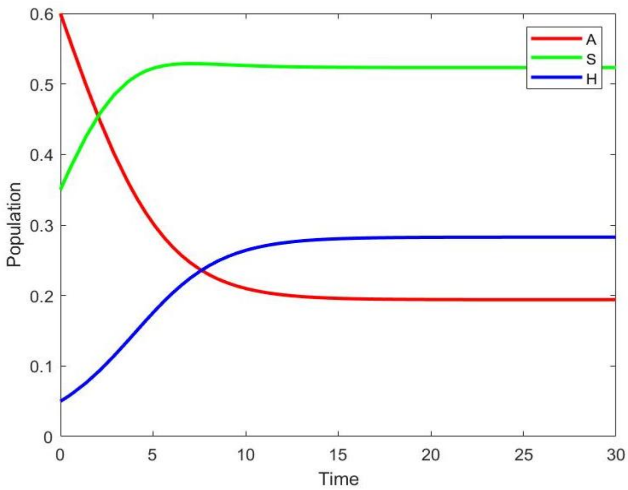

Figure 2. A Simulation of the three Commercial Sex Compartments

With the introduction of a commercial sex worker into the campus population, it can be observed that the Abstained Commercial Sex population begins to decline, resulting in an increase in both the Social Commercial Sex population and the Habitual Commercial Sex population during the initial stage. Over time, the Abstained Commercial Sex population, Social Commercial Sex population, and Habitual Commercial Sex population all become asymptotic to the horizontal axis.

Simulations for the individual compartments are displayed in the following graphs.

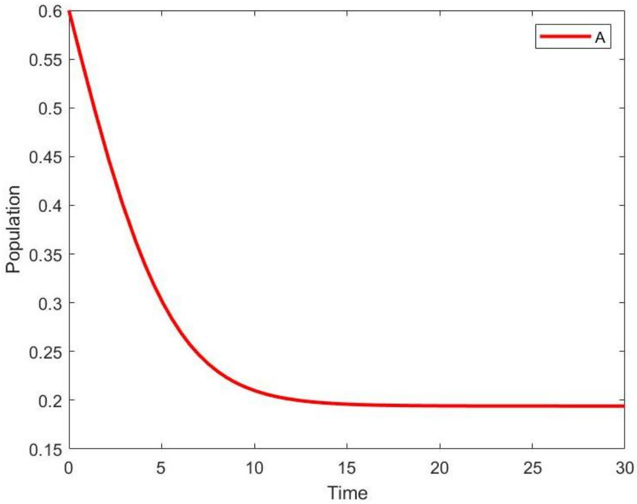

Figure 3. A Simulation of the Abstained Commercial Sex Compartment

From Figure 3, it is evident that no student in the university population has immunity to commercial sex; hence, the Abstained Commercial Sex population equals the total university population. When individuals involved in the commercial sex trade are introduced into the population, the population begins to decline gradually and becomes asymptotic to the horizontal axis over time. This results in a corresponding rise in the Social Commercial Sex population. The reason is that, with the introduction of individuals involved in the commercial sex trade into the university population, there is an interaction between the various compartments. This interaction leads to a progression from the Abstained Commercial Sex compartment, thereby inducing the behavior in other individuals. Consequently, the behavior begins to spread through the university population, causing the Abstained Commercial Sex population to decline and remain asymptotic, as it is not completely eradicated from the population.

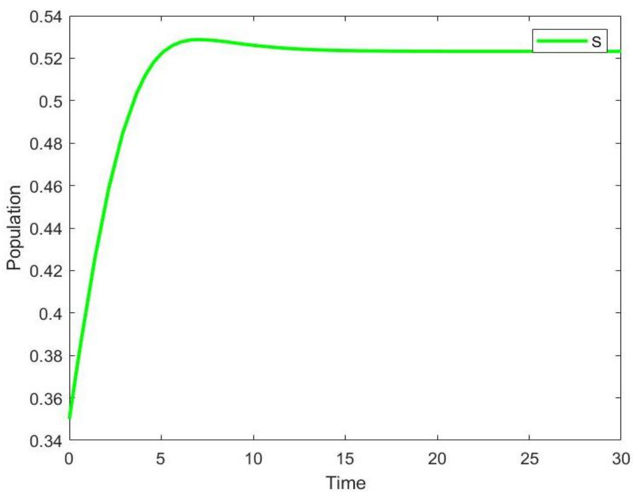

Figure 4. A Simulation of the Social Commercial Sex Compartment

From Figure 4, it was observed that initially, few individuals are involved in the commercial sex trade. However, over time, the behavior begins to spread through the university population, putting more people at risk. Consequently, the population of social commercial sex workers rises to its peak, declines steadily for a short period, and then remains asymptotic to the horizontal axis. The graph also demonstrates that the transmission rate significantly impacts the spread of the commercial sex trade within the university population. If the transmission rate is high, the rate at which people migrate from the Abstained Commercial Sex group to the Social Commercial Sex group will also be high.

Furthermore, Figure 5 shows a gradual rise in the Habitual Commercial Sex population from an initial value of 0.05 to its peak value of around 0.3 of the total population about ten days after the introduction of individuals involved in commercial sex into the university population. It then continues to remain constant and asymptotic to the horizontal axis since there is no permanent immunity to the commercial sex trade unless serious initiatives are introduced by various stakeholders to reduce the rate at which students are getting involved in the trade to the lowest minimum.

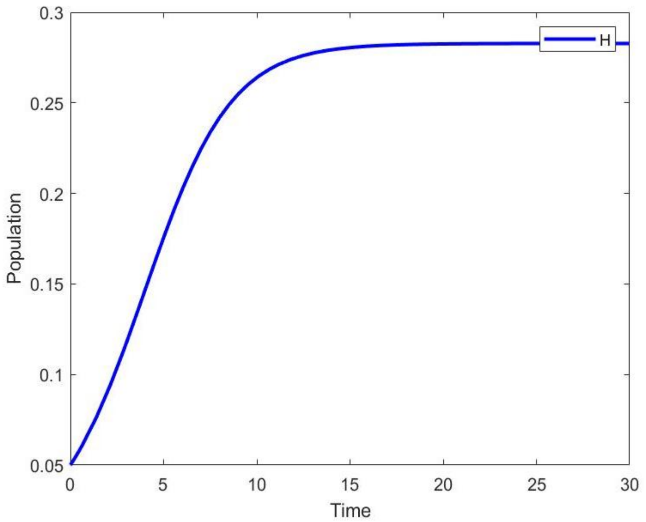

Figure 5. A Simulation of the Habitual Commercial Sex Compartment

7. Conclusions

The study applied mathematical models to model and analyze the transmission dynamics of commercial sex on university campuses. The basic reproductive number determined was less than the maximum threshold, implying that the commercial sex trade on university campuses will not become epidemic under current conditions. Additionally, the secondary disease-free equilibrium indicated that, on average, each student involved in commercial sex causes exactly one new case by inducing other students into the act. Both the disease-free and endemic equilibrium states were found to be globally asymptotically stable. This stability is independent of initial conditions, ensuring that the commercial sex trade will not fluctuate wildly or lead to unexpected outbreaks once the stable state is achieved. The findings suggest that targeted interventions can stabilize the commercial sex trade dynamics and prevent its spread. From the numerical simulations, it was observed that commercial sex can spread rapidly in a geographical area when the number of individuals involved is higher. The transmission dynamics of the commercial sex by this study are similar to the results in the study by [13].

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