

Human Thoughts and Quantum Mechanics (Predictive Study Through Mathematical Model)

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Abstract Human thoughts are in constant flux. Before being expressed in words, consciousness seems to exist in multiple states simultaneously, constantly evolving. When thoughts are verbalized, the dominant thought is extracted, resembling the superposition of states in quantum mechanics and their collapse upon observation. This suggests the possibility of representing the activity of the cerebrum using a mathematical model based on quantum mechanics. Brain cells could correspond to quantum bits (qubits) in quantum computation, and changes in the state of brain cells might be expressible through Schrödinger's wave equation. This paper proposes a mathematical model of the cerebrum based on such ideas. Introducing this mathematical model could deepen our understanding of cerebral diseases and potentially provide insights into methods for their treatment. For example, the mathematical model could give a method to estimate the effect of local damage on brain cells qualitatively and quantitatively, and the best selection and dosing of medicine. The potential danger of enabling external manipulation of the state of the cerebrum in undesirable directions should also be fully recognized.

Keywords Human Thoughts, Quantum Mechanics, State, Superposition, Collapse of State, Schrödinger Equation

1. Introduction

Human thoughts are in constant flux. Before being expressed in words, consciousness seems to exist in multiple states simultaneously, constantly evolving. When thoughts are verbalized, the dominant thought is extracted.

This property of human thoughts is very fascinating to the author. It resembles the superposition of states in quantum mechanics and their collapse upon observation [1,2].

To delve into this phenomenon, this paper first explores conversations with ChatGPT [3-6,7-12], a generative AI model, as a means of investigation. This is like using AI as a research assistant. By utilizing AI's wealth of knowledge, it becomes possible to conduct types of research previously impossible to the author. He also considers a mathematical model of brain activity inspired by associations with quantum mechanics.

Recent advancements in AI have led to its widespread applications, which are certainly welcome due to their societal benefits. However, one must question whether focusing solely on applications is sufficient. Just as understanding grammar enhances language use, comprehending the mathematical principles behind AI—and the brain's activities—should not be neglected.

2. Superposition States

2.1. The Curious Pictures

There is an image known as the Rubin vase (Rubin vase https://en.wikipedia.org/wiki/Rubin_vase). This image contains two perceptions: one of a vase and another of two faces facing each other. However, only one can be seen at any given time — either the vase or the faces. When one focuses on the vase's top edge, the vase is visible, and when the focus is on the bottom edge, the faces appear. This phenomenon can be loosely associated with the probability of focus being either at the top or the bottom. Additionally, if attention is drawn to the lighter color, the vase becomes visible, whereas focusing on the darker color reveals the faces.

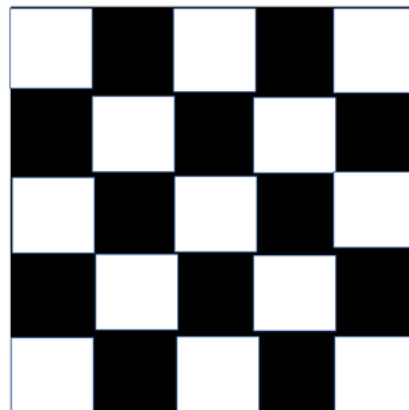


Figure 1. Illustration of white and black squares arranged in a grid

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There are other such images, and it is even possible to create your own. In Figure 1, white and black squares are arranged in a grid. When focusing on the white color, it appears as if white squares are arranged vertically, horizontally, or diagonally on a black background. Similarly, focusing on the black color yields multiple comparable interpretations.

In Figure 2, white convex lens shapes and black concave lens shapes are alternately arranged. When focusing on the white color, the white convex lens shapes appear aligned in a row. Conversely, focusing on the black color reveals the black concave lens shapes as aligned in a row.

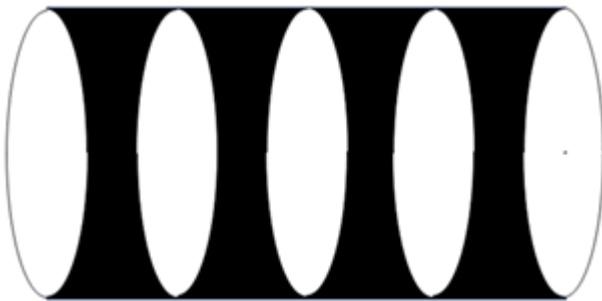


Figure 2. Illustration of alternating convex and concave lens shapes

These images contain multiple perceptions within a single sheet, where the way of observing the image determines how it is interpreted or what it signifies.

2.2. Drunk Driving and Election Results

Whether an individual involved in a traffic accident has been drinking cannot be determined merely by asking questions. However, analyzing their breath provides a definitive answer—an observation that resolves the uncertainty.

Similarly, the outcome of an election remains unknown until the votes are counted. This phenomenon, akin to others, illustrates how observation resolves ambiguity.

However, in this case, the result is not known until it is observed, but if observed a short time later, almost the same result is obtained, so it is simply a case of not knowing until it is observed, and it is not a quantum mechanical superposition.

2.3. Analogies between Human Thoughts and Quantum Mechanical Phenomena

A similar principle applies to human thoughts. Let us examine this idea through a conversation with ChatGPT. ChatGPT returns very interesting and affirmative viewpoints and suggestions. The details of the conversation are shown in Appendix A.

2.4. Interpretations and Applications

This perspective aligns with the 'quantum brain hypothesis', which posits that microstructures in the brain exhibit quantum behavior, forming the foundation of consciousness and creativity. Moreover, training in self-awareness and intentional thought selection may enhance mental flexibility

and freedom. Lastly, this model highlights the complexity of human behavior and decision-making, offering a new framework for understanding these phenomena.

In conversations with ChatGPT, as described above, interesting results were obtained. However, ChatGPT does not always provide the same response to identical questions. Each time, it offers different answers.

That said, these responses are not contradictory in meaning; rather, they reflect different perspectives. This phenomenon is quite common when humans ask questions of one another. In human interactions, both correct and incorrect answers may be given. In contrast, the range of variation in responses from a well-designed AI is not as wide.

2.5. The Scale of Brain Cells

In quantum mechanics, the states of atoms are superposed. This does not occur in classical mechanics. It is said that this phenomenon occurs in the world of quantum mechanics because the scale of the players, such as atoms and electrons, is small.

Now, let us consider the scale of a human brain cell. The volume of the brain is $10 \times 10^{-3} \text{ m}^3$, and the number of brain cells is 10 billion = 10^{10} , so the volume of a single cell is $1.0 \times 10^{-13} \text{ m}^3/\text{cell}$. From this, the scale of a single brain cell is $(1.0 \times 10^{-13})^{1/3} \text{ m}$ or 10^{-4} m .

Therefore, since the scale of a brain cell is more than 10,000 times that of an atom, it is unlikely to be a "superposition state" in quantum mechanics due to scale.

On the other hand, the pollen scale is 20 to 40 μm , which is close to the scale of brain cells. Let's consider the possibility of fluctuations in consciousness being due to the small scale. They may be due to fluctuations caused by Brownian motion rather than quantum mechanical fluctuations.

3. Brain Function

3.1. Everyday Experiences of Brain Activity

We experience various phenomena related to brain activity in our daily lives, such as:

1. Memory fades without refreshment.
2. Past experiences suddenly resurface in our minds.
3. Even if a memory can't be remembered immediately, it often revives if time is spent.
4. We dream.
5. The brain is constantly active.

3.2. A Hypothesis on Brain State Transitions

Neural networks are models that mimic brain function. Recently, generative AI has demonstrated incredible capabilities. Some speculate that AI may eventually match human intelligence, not only performing assigned tasks but also developing human-like cognitive abilities. This could lead to the so-called singularity, where AI surpasses human intelligence. From another perspective, neural networks—designed as imitations of certain brain functions—might

actually represent an accurate mathematical model of brain activity.

Based on this view, let us hypothesize that the brain functions as a kind of quantum computer and explore its mathematical underpinnings. Here, we introduce two assumptions inspired by quantum mechanics:

(1) Quantum Bits in Brain Cells

Brain cells are assumed to function as qubits, existing in superposition states of $|0\rangle$ and $|1\rangle$. That is, the state of a single cell can be written as:

$$|\psi\rangle = a|0\rangle + b|1\rangle,$$

where the probabilities of observing $|0\rangle$ and $|1\rangle$ are $|a|^2$ and $|b|^2$, respectively, with the constraint:

$$|a|^2 + |b|^2 = 1.$$

(2) Tensor Product of States

The overall state of all brain cells is expressed as the tensor product of individual cell states:

$$\begin{aligned} |\Psi\rangle &= |\psi_0\rangle \otimes |\psi_1\rangle \otimes \cdots \otimes |\psi_{N-1}\rangle \\ &= c_0 |00\cdots 00\rangle + c_1 |00\cdots 01\rangle + c_2 |00\cdots 10\rangle + \\ &\quad \cdots + c_{N-1} |11\cdots 11\rangle \\ &= c_0 |0\rangle + c_1 |1\rangle + c_2 |2\rangle + \cdots + c_{N-1} |N-1\rangle, \end{aligned}$$

where N represents the total number of brain cells.

(3) Time Evolution of Brain States [2]

The state of the brain evolves over time according to equations analogous to the Schrödinger:

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \hat{H} |\Psi\rangle,$$

where \hat{H} is the Hamiltonian operator. The coefficients in this equation represent interactions rather than Planck's constant and can incorporate terms for learning, forgetting, and other processes. Using an eigenfunction expansion of the interaction Hamiltonian:

$$|\Psi\rangle = \sum_{i=0}^{N-1} c_i |\Psi_i\rangle,$$

the equation can be expressed in matrix form as:

$$i\hbar \frac{d}{dt} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{N-1} \end{pmatrix} = \begin{pmatrix} H_{00} & H_{01} & \cdots & H_{0N-1} \\ H_{10} & H_{11} & \cdots & H_{1N-1} \\ \vdots & \vdots & \vdots & \vdots \\ H_{N-10} & H_{N-11} & \cdots & H_{N-1N-1} \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{N-1} \end{pmatrix},$$

The number of brain cells is estimated at 14 billion. However, what truly matters is the amount of information the brain can store—the number of possible patterns. The dimension of the eigenvalue problem equals the number of brain cells, making this the maximum number of eigenfunctions and thus the maximum capacity of information.

These eigenvalue problems also involve natural frequencies and eigenfunctions. For example, certain diseases or side

effects of medications can cause bodily tremors, which seem to be related to these natural oscillations of the brain.

When the brain is stimulated by the environment, transitions in consciousness occur. Such changes are hypothesized to result from variations in the interaction Hamiltonian, altering the brain's state.

A similar phenomenon occurs when a two-level atom is irradiated with a laser. The mathematical explanation of this quantum transition is provided in Appendix B.

In the case of the brain, experiences, learning, and sensory input likely induce changes in the interaction terms, leading to state transitions.

3.3. Insights from Laser Irradiation of Two-Level Atoms

Changes in brain states due to experiences or learning are autonomous and self-directed processes. However, understanding the brain's mathematical mechanisms may also reveal undesirable risks.

For example, a two-level atom's state can be altered by resonant laser irradiation. This suggests the possibility that targeted physical stimuli, such as sound waves resonant with specific brain regions, could induce changes in brain cell states, reflecting external intentions.

While this could potentially be used to treat diseases, it also raises concerns about the manipulation of human consciousness by others.

When asked, "Can physical stimuli alter brain cell states, as resonant laser light does for two-level atoms?" ChatGPT responded affirmatively.

ChatGPT's Answer:

"Yes, it is possible to alter brain cell states through physical stimulation. This concept has been widely studied in neuroscience."

Furthermore, it is known that applying electrical stimulation, light stimulation, or ultrasound stimulation to neurons can cause neural excitation, neural inhibition, or the strengthening or weakening of synaptic transmission. These methods are reportedly used in the treatment of neurological disorders.

The details of ChatGPT's responses are provided in Appendix C.

4. Conclusions

The author has always been interested by the sophisticated information processing and flashes of insight occurring within the human brain.

For example, how is it possible for humans to exchange and communicate complex and advanced information through language? This is truly a remarkable phenomenon.

On the other hand, human thoughts are always in motion. Even when we try to concentrate on a single item, maintaining concentration is challenging. This dynamic nature seems to be a defining feature of brain activity, and the author has attempted to examine its implications.

In conducting this study, the author engaged in dialogues with ChatGPT, an artificial intelligence model. From these

conversations, it may appear that the author might have uncovered the mathematical foundations underlying brain activity. The author proposed a mathematical model of the brain functions. However, even if this were the case, such understanding would likely only scratch the surface of the brain's overall functions.

Introducing this mathematical model could deepen our understanding of cerebral diseases. It could potentially provide insights into methods for their treatment. In the future, the mathematical model could, for example, give a method to estimate the effect of local damage on brain cells qualitatively and quantitatively, and the best selection and dosing of medicine.

However, constructing a mathematical model of brain function presents both favorable and potentially undesirable aspects. On the positive side, such models could aid in treating diseases. On the negative side, they may open the door to the intentional manipulation of human minds.

Appendix A: Conversation with ChatGPT on the Analogy between Human Thoughts and Quantum Mechanical Phenomena

The Question to ChatGPT:

Human thoughts does not seem deterministic but instead constantly fluctuates, resembling the superposition in quantum mechanics. In other words, consciousness does not exist as a single state but in multiple simultaneous states until expressed in words. This suggests that the human brain contains several coexisting thoughts that evolve over time. When verbalized, the dominant thought is selected. What are your thoughts on this?

ChatGPT's Answer:

This is a fascinating perspective. It is indeed possible to draw parallels between human consciousness and the superposition of states in quantum mechanics or the collapse of a wave function. This hypothesis has several key elements:

1. Simultaneous Existence of Multiple Thoughts

Human consciousness often contains contradictory thoughts and emotions simultaneously, akin to a 'superposition state' in quantum mechanics. For example, the coexistence of 'a motivated self' and 'a lazy self' often results in one state manifesting when action is taken.

2. Temporal Evolution and Fluctuations

Just as quantum states evolve, human thoughts and consciousness continuously change, influenced by external stimuli and internal cognitive processes.

3. The Act of Verbalization as 'Selection'

In quantum mechanics, observation collapses a wave function. Similarly, verbalization selects and stabilizes a specific thought among many, creating a concrete expression

while other possibilities fade into the background.

Interpretation and Possible Applications

Consciousness and the Quantum Brain Hypothesis

This perspective may be connected to the theory known as the "Quantum Brain Hypothesis." For example, it hypothesizes that the microstructure of the brain exhibits quantum behavior, which serves as the foundation for consciousness and creativity.

Training Self-Awareness

By cultivating the ability to recognize fluctuations in consciousness and intentionally choose one's mental state, individuals may develop a more free and flexible way of thinking.

Understanding the Multifaceted Nature of Humanity

Adopting this model clarifies why human behavior and decision-making cannot be explained solely through simple causal relationships. This is because multiple possibilities are constantly competing rather than being determined by a single thought process.

To deepen this perspective, integrating insights from philosophy, cognitive science, and psychology would be beneficial. Additionally, exploring the relationship between the framework of physics and consciousness may open entirely new horizons of understanding.

Appendix B: Manipulating the State of Two Level Atoms [2]

Consider the state of a two-level atom as depicted in **Figure A1**.

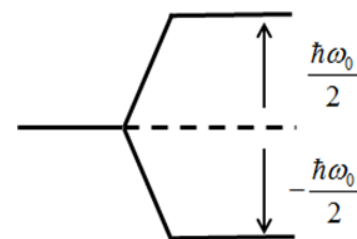


Figure A1. Energy Levels

Let the states of the two energy levels be denoted as $|0\rangle$ and $|1\rangle$. The state of the atom, $|\psi(t)\rangle$ at time t , evolves according to the Schrödinger equation:

$$i\hbar \frac{d|\psi(t)\rangle}{dt} = \hat{H}|\psi(t)\rangle,$$

where \hbar is the Planck's constant, and \hat{H} is the Hamiltonian operator. Expressing $|\psi\rangle$ in terms of its basis states:

$$|\psi(t)\rangle = a(t)|0\rangle + b(t)|1\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix},$$

the matrix representation of \hat{H} is given by:

$$\hat{H} = \begin{pmatrix} -\frac{\hbar\omega_0}{2} & 0 \\ 0 & \frac{\hbar\omega_0}{2} \end{pmatrix},$$

where $-\hbar\omega_0/2$ and $\hbar\omega_0/2$ are the energy levels of the two states.

When irradiating the atom with laser light of angular frequency ω , the interaction strength between the atom and the light is denoted by Ω . The Hamiltonian with the interaction included becomes:

$$\begin{aligned} \hat{H} &= \begin{pmatrix} -\frac{\hbar\omega_0}{2} & 0 \\ 0 & \frac{\hbar\omega_0}{2} \end{pmatrix} + \begin{pmatrix} 0 & -\frac{\hbar\Omega}{2}e^{i\omega t} \\ -\frac{\hbar\Omega}{2}e^{-i\omega t} & 0 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{\hbar\omega_0}{2} & -\frac{\hbar\Omega}{2}e^{i\omega t} \\ -\frac{\hbar\Omega}{2}e^{-i\omega t} & \frac{\hbar\omega_0}{2} \end{pmatrix}. \end{aligned}$$

The Schrödinger equation in matrix form is given by

$$i\hbar \begin{pmatrix} \frac{da(t)}{dt} \\ \frac{db(t)}{dt} \end{pmatrix} = \begin{pmatrix} -\frac{\hbar\omega_0}{2} & -\frac{\hbar\Omega}{2}e^{i\omega t} \\ -\frac{\hbar\Omega}{2}e^{-i\omega t} & \frac{\hbar\omega_0}{2} \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

The time-dependent exponential terms $e^{\pm i\omega t}$ complicate solving the equation directly. By performing a variable transformation, the equation can be simplified into a solvable form:

$$\begin{cases} a(t) = a'(t)e^{i\frac{\omega}{2}t} \\ b(t) = b'(t)e^{-i\frac{\omega}{2}t} \end{cases} \rightarrow \begin{cases} \frac{da'(t)}{dt} = \frac{i}{2}[(\omega_0 - \omega) + \Omega b'(t)] \\ \frac{db'(t)}{dt} = \frac{i}{2}[-(\omega_0 - \omega) + \Omega a'(t)] \end{cases}$$

When the laser's energy matches the energy gap, i.e., $\hbar\omega_0 = \hbar\omega$, the solution simplifies further:

$$\begin{aligned} \begin{cases} \frac{da'(t)}{dt} = i\frac{\Omega}{2}b'(t) \\ \frac{db'(t)}{dt} = i\frac{\Omega}{2}a'(t) \end{cases} &\rightarrow \begin{cases} \frac{d^2a'(t)}{dt^2} = -\frac{\Omega^2}{4}a'(t) \\ \frac{d^2b'(t)}{dt^2} = -\frac{\Omega^2}{4}b'(t) \end{cases} \\ &\rightarrow \begin{cases} a'(t) = a'(0)\cos\frac{\Omega}{2}t + ib'(0)\sin\frac{\Omega}{2}t \\ b'(t) = b'(0)\cos\frac{\Omega}{2}t + ia'(0)\sin\frac{\Omega}{2}t \end{cases} \end{aligned}$$

Recovering the original variables $a(t)$ and $b(t)$ from $a'(t)$ and $b'(t)$, the state of the atom $|\psi(t)\rangle$ is obtained as

$$\begin{aligned} |\psi(t)\rangle &= \left[a(0)\cos\frac{\Omega}{2}t + ib(0)\sin\frac{\Omega}{2}t \right] e^{-i\frac{\omega_0}{2}t} |0\rangle \\ &+ \left[b(0)\cos\frac{\Omega}{2}t + ia(0)\sin\frac{\Omega}{2}t \right] e^{i\frac{\omega_0}{2}t} |1\rangle. \end{aligned}$$

Ignoring the global exponential phase factor $\exp(\pm i\omega_0 t/2)$, the state of the atom can be expressed as:

$$\begin{aligned} |\psi(t)\rangle &= \left[a(0)\cos\frac{\Omega}{2}t + ib(0)\sin\frac{\Omega}{2}t \right] |0\rangle \\ &+ \left[b(0)\cos\frac{\Omega}{2}t + ia(0)\sin\frac{\Omega}{2}t \right] |1\rangle. \end{aligned}$$

This shows that by controlling the duration and phase of the laser irradiation, any desired superposition of $|0\rangle$ and $|1\rangle$ can be achieved.

Example: Initial State in the Ground Level

If the atom starts in the ground state $|0\rangle$, represented as $|\psi(0)\rangle = |0\rangle$, then the state of the atom at time t after the laser irradiation is applied becomes:

$$\begin{aligned} \text{State: } |\psi(t)\rangle &= \cos\left(\frac{\Omega}{2}t\right)|0\rangle + i\sin\left(\frac{\Omega}{2}t\right)|1\rangle \\ \rightarrow \text{Probability: } &\begin{cases} |a(t)|^2 = \left|\cos\left(\frac{\Omega}{2}t\right)\right|^2 \\ |b(t)|^2 = \left|\sin\left(\frac{\Omega}{2}t\right)\right|^2 \end{cases}. \end{aligned}$$

This oscillatory behavior is known as Rabi oscillation.

Appendix C: Manipulating Brain States Through Physical Stimulation

Question from the Author to ChatGPT:

"Can the states of brain cells be altered by physical stimulation, similar to how the states of two-level atoms are changed by resonant laser light irradiation?"

ChatGPT's Response:

"Yes, it is possible to alter the states of brain cells through physical stimulation. This concept is widely studied in neuroscience."

1. Physical Stimulation and Neural Activity

Physical stimulation, such as electrical, optical, or ultrasonic methods, can influence neural activity in the following ways:

- **Electrical Stimulation:** By applying electrical impulses directly to the brain via electrodes, neuronal firing (action potentials) can be triggered, activating or inhibiting specific neural circuits.
- **Optical Stimulation (Optogenetics):** Through genetic modification, neurons can be made sensitive to specific wavelengths of light. Shining light on these neurons allows selective activation or inhibition of neural activity.
- **Ultrasound Stimulation:** Non-invasive ultrasound can target specific brain regions and modulate neural activity, making it a promising method for research and therapeutic applications.

2. Examples of State Changes

Various changes in brain cell states can be achieved through physical stimulation, such as:

- **Neuron Excitation:** Triggering action potentials.
- **Neuron Inhibition:** Suppressing activity.
- **Synaptic Plasticity:** Strengthening or weakening synaptic connections.
- **Changes in Brain Waves:** Modifying patterns like alpha or gamma waves.

3. Applications of Brain State Manipulation

- **Treatment of Neurological Disorders:** Techniques like Deep Brain Stimulation (DBS) are used to treat Parkinson's disease, depression, and other conditions.
- **Learning and Memory Enhancement:** Stimulating the hippocampus can potentially improve memory formation and cognitive abilities.
- **Modulation of Consciousness:** Targeting specific brain regions (e.g., the thalamus or prefrontal cortex) may alter consciousness or cognitive states.

4. Cautionary Notes

Precise control is essential when manipulating brain cell states. Excessive or inappropriate stimulation can lead to brain damage or unintended adverse effects.

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