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Designs of One-Element Refracting System for Gaussian and Annular-Gaussian Beams Transformations

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Abstract In this paper, the design of one-lens (instead of two-lens) refracting system is proposed to provide particular laser irradiance profiles. Specifically, the one-lens surfaces are derived for three types of laser beam transformations: (i) Gaussian to Annular-uniform; (ii) Annular-Gaussian to Annular-uniform; and (iii) Annular-Gaussian to Gaussian. It will be demonstrated that the one-element designs outperform the two-element ones (for the same system parameters previously reported) for these beams regarding the system length. In particular, the system length is reduced by almost 39% for both the Gaussian and Annular-Gaussian to Annular-uniform beam transformation for the same starting refraction angle. For the beam transformation of Annular-Gaussian to Gaussian beam transformation, this system length is reduced by almost 34% with slightly larger initial refraction angle.

Keywords Beam Shaping, Refracting system, One-lens design

1. Introduction

Gaussian or near-Gaussian laser beams are heavily used in many applications in which the laser beam is being focused to a small spot [1]. However, in many other industrial, military, medical applications laser beams with uniform intensity profiles are needed to illuminate evenly an entire processed area such as in coherent optical image processing, optical pattern recognition, Fourier transform-based correlation, and materials processing tasks [2-8]. In this regard, beam shaping techniques were proposed to do the conversion of Gaussian and non-Gaussian laser beams to uniform and other beams profiles. Refractive optical system technique is among many successful and efficient methods that can achieve this conversion [9-14]. This technique relies on geometric optics for designing laser beam shapers. The designed optical element functions as a field mapping of the input beam distribution to provide a desired output beam profile. Over the years, single-element and two-element refracting systems have been proposed to achieve beam shapers [15-20].

In this work, one-element refracting beam shaping systems are designed instead of two-element systems that were recently reported in [21]. The mathematical expressions of the input and output lens surfaces are derived for the following beams transformations: (i) Gaussian to Annular-uniform; (ii) Annular-Gaussian to Annular-uniform; and (iii) Annular-Gaussian to Gaussian. It will be demonstrated that the one-element designs achieve smaller system length, compared to the two-element refracting systems previously reported for the same system parameters.

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2. Design Considerations

As stated in [9], two main conditions govern the refracting optical system for beam shaping:

1. The geometrical optics intensity law, i.e., the ratio of the input beam power to the output beam power must equal to a constant.
2. The optical path length, i.e., all input rays that enter and leave the lens must travel the same optical path length.

In addition, one can add a third condition which is related to geometrical optics rays tracing, i.e.:

3. All input rays that enter and leave the lens must be parallel to each other's.

A geometrical configuration of half of the proposed axially-symmetric refracting system is shown in Figure 1 for Gaussian to annular-uniform beam transformation. Note that with some minor modifications, the set-up in Figure 1 can also be used for other beam shapers.

The lens is made from glass (index of refraction n = 1.5172). D is the distance between the input and the output surfaces. A ray enters the lens with an angle θ_{ii} and then is refracted by an angle θ_{ri}. This ray reaches the second lens surface with an angle θ_{ro} and then it is refracted by an angle

θ_{io} and exits parallel to the first incident ray. In Figure 1, $y_i(r_i)$ and $y_o(r_o)$ represent the mathematical expressions as functions of the radial distances r_i and r_o of the input and output surfaces, respectively.

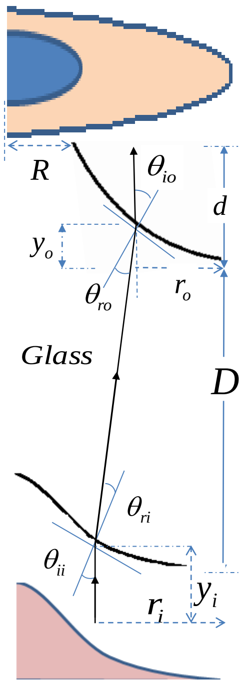


Figure 1. The geometric set-up of the one-element refraction system beam transformation

The above first condition implies that the ratio between the two cross-sectional areas of the two beams is set to a constant:

$$\frac{Input\ Intensity}{Output\ Intensity} = \frac{Input\ cross-sectional\ area}{Output\ cross-sectional\ area} = constant = k^2 \quad (1)$$

The second condition is translated into this equation:

$$y_i + n\sqrt{(r_i - r_o)^2 + (D - y_i + y_o)^2} + d - y_o = f \quad (2)$$

where f is a constant. Adding (-D) term to both sides of Eq. (2):

$$-(D - y_i + y_o) + n\sqrt{(r_i - r_o)^2 + (D - y_i + y_o)^2} = f - d - D$$

From the geometry of the set-up in Figure 1, one can deduce that:

$$tan(\theta_{ii} - \theta_{ri}) = tan(\theta_{io} - \theta_{ro}) = (r_i - r_o)/(D - y_i + y_o) \quad (3)$$

Let $f' = f\text{-}d\text{-}D$ and use Eq. (3), we obtain:

$$f' = (r_i - r_o)\left[\frac{n - cos(\theta_{ii} - \theta_{ri})}{sin(\theta_{ii} - \theta_{ri})}\right] \quad (4)$$

Further, the input and the output ray's parallelism dictate that both the input and the output surfaces slope must be equal to each other, i.e.:

$$dy_i/dr_i = dy_o/dr_o = tan\theta_{ii} = tan\theta_{io} \quad (5)$$

Furthermore, by applying Snell's law at the input lens ($\sin\theta_{ii} = n\sin\theta_{ri}$) and using trigonometric identities, one can obtain from Eq. (4) and Eq. (5) the following important relationship:

$$dy_i/dr_i = dy_o/dr_o = n\{(f'/(r_i - r_o))^2 - n^2 + 1\}^{-1/2} \quad (6)$$

It is worth mentioning that since the quantity under the square root must be kept positive, then there would be a minimum value for f' and consequently a minimum value for the length of the system D for any specific beam parameters.

Eq. (6) is the main equation that will be used to obtain the lens' input and output surfaces. The design of the refracting lens for beam shaping starts with defining the relationship between the radial distances r_i and r_o from the cross-sectional areas of the laser beams using Eq. (1). Next for a specific value of system length (f' or D), one can determine the numerical values for the surfaces slope dy_i/dr_i and dy_o/dr_o from Eq. (6). Finally, by integration the surfaces slopes, we can obtain the numerical values of the surfaces $y_i(r_i)$ and $y_o(r_o)$. Practical issues regarding the fabrication of the lens implies the following preferable characteristics:

i. The system length is preferred to be as small as possible.
ii. The surface slope (the refracting angles) is preferred to be small.
iii. The radii of curvature $\rho(r)$ of the surfaces need to be large.

$$\rho(r) = \frac{[1 + (dy/dr)^2]^{3/2}}{d^2y/dr^2} \quad (7)$$

3. Laser Beam Transformations

The above-mentioned design procedure will be applied in this section to transform three types of beams.

3.1. Gaussian to Annular-Uniform Beam Transformation

As shown in Figure 1, an input Gaussian beam will be redistributed by the designed lens surfaces to form an output annular-uniform beam. The law of intensities implies that the ratio k^2 between the two cross-sectional areas is given by:

$$k^2 = \frac{2I_o \int_0^{r_i} re^{-r^2/w_o^2}\, dr}{I_o(r_o^2 - R^2)} \quad (8)$$

where r_o and R are the outer and inner radii of the output annular beam, and r_i is the radius of the input Gaussian beam. Eq. (8) will be solved to obtain:

$$r_o = \left\{w_o^2(1 - e^{-r_i^2/w_o^2})/k^2 + R^2\right\}^1 \quad (9a)$$

$$r_i = w_o\{-ln(1 - k^2(r_o^2 - R^2)/w_o^2)\}^{1/2} \quad (9b)$$

These radii can be used along with Eq. (6) to design the lens refracting system transformation. In order to compare our proposed one-lens design to the two-lens design in [21], the same beams parameters are used, namely R = 0.5 cm;

$k = 1$; $w_o = 2$; as well as the same starting refracting angle $\theta_{ri} = 20.77^o$. To determine this refracting angle, the system length D, the initial surface slope dy/dr, and the initial radii of curvature are plotted for different values of f' as shown in Figure 2. As indicated in Figure 2a, a system length D = 2.397 cm which corresponds to f' = 1.318 generates $\theta_{ri} = 20.77^o$. Therefore, the system length D of the one-lens design is reduced by almost 39% compared to the value D = 3.928 cm in [21].

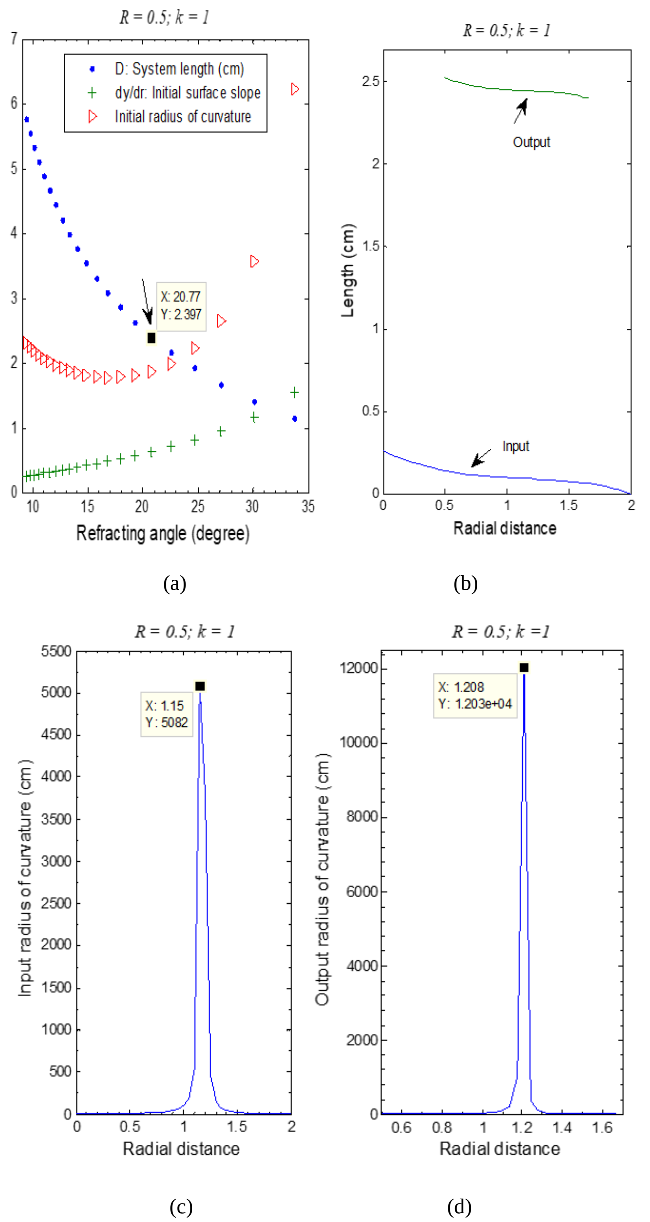


Figure 2. Gaussian to Annular-uniform design: (a) effect of the refracting angle on the system length; (b) the input and the output surfaces $y_i(r_i)$ and $y_o(r_o)$; (c) and (d) the radii of curvature of the surfaces

Now, by substituting f'= 1.318 in Eq. (6) and using Eqs. (5) and (9), we can obtain the numerical values for the surface slopes dy_i/dr_i and dy_o/dr_o. The mathematical expressions for these slopes are achieved using a polynomial curve-fitting routine to yield:

$$\frac{dy_i}{dr_i} = (0.0221)r_i^4 - (0.152)r_i^3 + (0.0493)r_i^2 + (0.3693)r_i - 0.3312 \tag{10a}$$

$$\frac{dy_o}{dr_o} = -(0.757)r_o^4 + (3.0288)r_o^3 - (4.9594)r_o^2 + (4.0512)r_o - 1.4178 \tag{10b}$$

Integrating the above expressions to provide approximate equations for the lens' surfaces as:

$$y_i(r_i) = (4.42x10^{-3})r_i^5 - (0.038)r_i^4 + (0.01643)r_i^3 + (0.1846)r_i^2 - (0.3312)r_i + 0.259 \tag{11a}$$

$$y_o(r_o) = -(0.1514)r_o^5 + (0.7572)r_o^4 - (1.6531)r_o^3 + (2.0256r_o^2 - (1.4178)r_o + 0.4943 \tag{11b}$$

These equations are plotted in Figure 2b. The radii of curvature of the input and output surfaces of the lens are shown in Figure 2c and Figure 2d, respectively, where the radius of curvature for input surface changes from a minimum 4.039 cm to reach a maximum value of 5082 cm; while the radius of curvature for output surface changes from a minimum of 0.695 cm to reach a maximum value of 12030 cm. Note that these radii of curvatures of this design are smaller than the reported ones in [21]. These radii values can be improved at the expense of having longer system lengths D and larger initial refracting angles θ_{ri}.

3.2. Annular-Gaussian to Annular-Uniform Beam Transformation

The refracting one-lens design process can be applied to transform annular-Gaussian to annular-uniform beams. In this case, the cross-sectional areas ratio for the two beams is given by:

$$k^2 = \frac{2I_o \int_{R_g}^{r_i} r[1 - R_o e^{-2r^2/w_o^2}]e^{-2r^2/w^2} dr}{I_o(r_o^2 - R_a^2)} \tag{12}$$

where R_a and r_o are the inner and the outer radii of the output annular-uniform beam and I_o is the peak input beam intensity. For the input annular-Gaussian beam, R_g and r_i are the inner and the outer radii; w_o is the annular-Gaussian beam radius; $w^2 = (M^2 - 1)w_o^2$ is the beam spot size in the large Fresnel number limit; is the magnification; R_o is the reflectivity of the central mirror [1].

The output radius r_o is obtained from Eq. (12) as:

$$r_o = \left\{ k^{-2} \left[\frac{\frac{R_0}{2}}{(1/w_0^2 + 1/w^2)} [e^{-2r_i^2(1/w_0^2+1/w^2)} - e^{-2R_g^2(1/w_0^2+1/w^2)}] - w^2(e^{-2r_i^2/w^2} - e^{-2R_g^2/w^2})/2 \right] + R_a^2 \right\}^{1/2} \tag{13}$$

For and $w = w_o$, Eq. (13) is simplified to:

$$r_o = \left\{ \left(\frac{w_0}{k}\right)^2 \left[R_0 \left(e^{\frac{-4r_i^2}{w_0^2}} - e^{\frac{-4R_g^2}{w_0^2}} \right)/4 - \left(e^{-2r_i^2/w_0^2} - e^{-2R_g^2/w_0^2} \right)/2 \right] + R_a^2 \right\}^{1/2} \tag{14}$$

The two-lens design of [21] uses the beam parameters R_o = 0.9; R_a= 0.5 cm; R_g = 1 cm; k = 1; w_o = 2; and the starting refracting angle θ_{ri} = 20.77°. We have used the same parameters in our one-lens design. Figure 3a shows the system length D, the initial surface slope dy/dr, and the initial radii of curvature versus the starting refracting. It is shown in the figure that the system length D = 2.397 cm corresponds to f'= 1.318 and θ_{ri} = 20.77°. Thus, again the system length is reduced by almost 39% compared to length D = 3.928 cm in [21] while the radii of curvatures are expected to be lower as it will be demonstrated later.

Now, the numerical values for the surface slopes dy_i/dr_i and dy_o/dr_o can be obtained by substituting f'= 1.318 in Eq. (6) and using Eqs. (5) and (9). Using these numerical values, we can get the mathematical expressions for these slopes as:

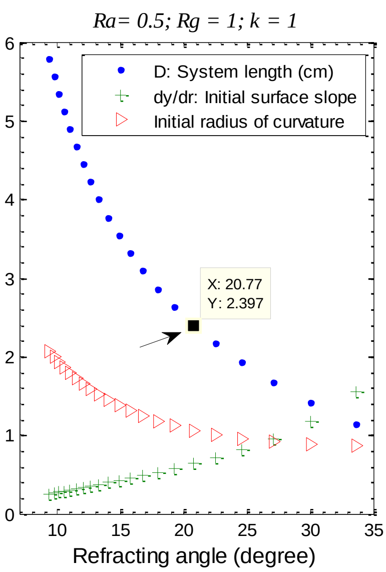
$$\frac{dy_i}{dr_i} = (8.0616)r_i^4 - (43.7218)r_i^3 + (88.8551)r_i^2 - (79.0662)r_i + 26.5428 \tag{15a}$$

$$\frac{dy_o}{dr_o} = (377.7578)r_o^4 - (1.0138x10^3)r_o^3 + (1.0138x10^3)r_o^2 - (445.5489)r_o + 73.1431 \tag{15b}$$

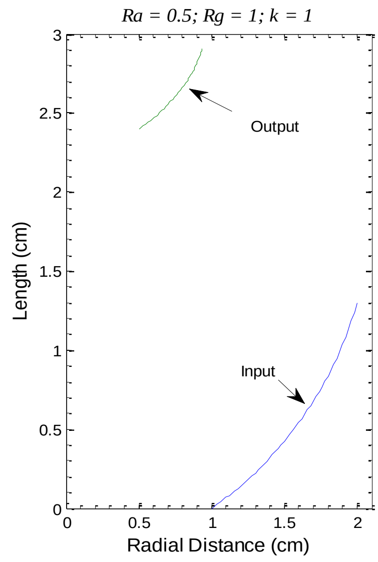
Integrating and using appropriate initial conditions, one can obtain the approximate equations for lens surfaces as:

$$y_i(r_i) = (1.6123)r_i^5 - (10.9304)r_i^4 + (29.6184)r_i^3 - (39.5331)r_i^2 + (26.5428)r_i - 7.3099 \tag{16a}$$

$$y_o(r_o) = (75.5515)r_o^5 - (0.2534x10^3)r_o^4 + (0.3379x10^3)r_o^3 - (222.7744)r_o^2 + (73.1431)r_o - 9.6402 \tag{16a}$$



(a)



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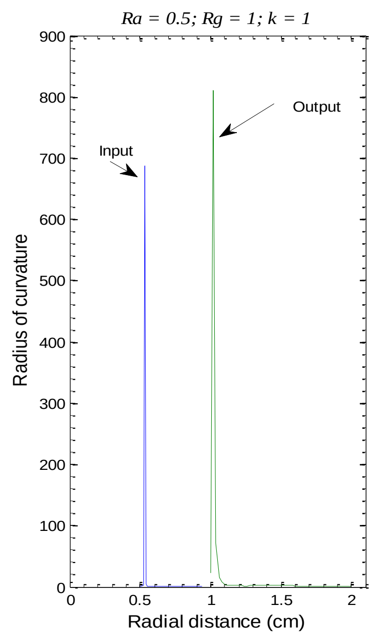


(c)

Figure 3. Annular-Gaussian to Annular-uniform design: (a) choosing the system length for a specific refracting angle; (b) the input and the output surfaces; and (c) the input and the output radii of curvatures

Figure 3b and 3c shows the surfaces and the radii of curvature of this one-lens refracting design where the radius of curvature for input surface changes from a minimum value of 0.3515 cm reaching a value of 810.9 cm; and the radius of curvature for output surface changes from a minimum value of 0.0404 cm reaching a value of 687.8 cm.

3.3. Annular-Gaussian to Gaussian Beam Transformation

For this type of beam shaping, the cross-sectional areas ratio for the two beams is written as:

$$k^2 = \frac{2I_o \int_{R_a}^{r_i} r[1-R_o e^{-2r^2/w_{oa}^2}]e^{-2r^2/w^2}\,dr}{I_o w_{og}^2 (1-e^{-r_o^2/w_{og}^2})} \qquad (17)$$

where w_{oa} is the input annular Gaussian beam radius; r_i and R_a are the outer and the inner radii of the input annular-Gaussian beam; w_{og} is the radius of the output Gaussian beam; I_o is the peak input beam intensity; $w^2 = (M^2-1)w_o^2$ is the beam spot size in the large Fresnel number limit; is the magnification; R_o is the reflectivity of the central mirror [1]. Note that $w = w_{oa}$ when $M = \sqrt{2}$. From Eq. (17) we can obtain the output radius of the Gaussian beam as:

$$r_o = w_{og}\sqrt{-\ln\left[1+\frac{w_{oa}^2}{2k^2 w_{og}^2}(e^{-2r_i^2/w_{oa}^2} - e^{-2R_a^2/w_{oa}^2}) - \frac{R_o w_{oa}^2}{4k^2 w_{og}^2}(e^{-4r_i^2/w_{oa}^2} - e^{-4R_a^2/w_{oa}^2})\right]} \qquad (18)$$

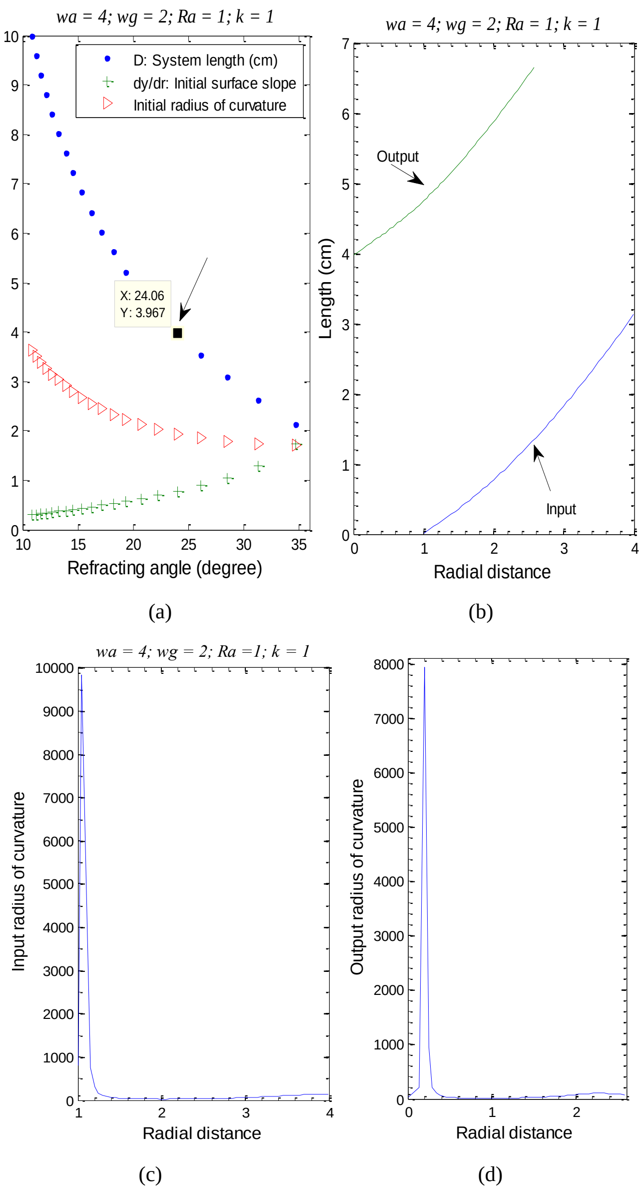


Figure 4. Annular-Gaussian to Gaussian design for the same D used in [21], but with a smaller θ_{ri}: (a) choosing the system length for a specific initial refracting angle; (b) the input and the output surfaces; and (c) the input and the output radius of curvature of the surfaces

It is worth mentioning that a design with system length D = 2.397 cm which is similar to the previous two designs is not possible due to the fact that the quantity under the square root in Eq. (6) must be kept positive. Thus, this leads to having a minimum value for f' and D for any specific design beam parameters. In this subsection, we provide a design for transforming beams with R_a = 1, w_{oa} = 4, w_{og} = 2, k = 1, R_o = 0.9, which are the same parameters used in [21]. It is found that the minimum value for f' is 1.624 which corresponds to system length D = 2.594 cm and a larger refracting angle of θ_{ri} = 31.62°. Note that this value of D is still smaller than the one reported in [21]. In Figure 4, we illustrate a designed system that has D = 3.967 cm, θ_{ri} = 24.06°, and f' = 2.24. This design has almost the same length D = 3.928 as in [21], but with a smaller θ_{ri}. Again, using these values in the design equations, we can obtain the numerical values for the surface's slopes dy_i/dr_i and dy_o/dr_o as well as the mathematical expressions for the lens surfaces:

$$\frac{dy_i}{dr_i} = (0.0174)r_i^4 - (0.2084r_i^3 + (0.8762)r_i^2 - (1.2447)r_i + 1.2696 \qquad (19a)$$

$$\frac{dy_o}{dr_o} = (0.0598)r_o^4 - (0.3927)r_o^3 + (0.8352)r_o^2 - (0.3032)r_o + 0.7376 \qquad (19b)$$

$$y_i(r_i) = (3.48\text{x}10^{-3})r_i^5 - (0.01521)r_i^4 + (0.2921)r_i^3 - (0.6223)r_i^2 + (1.2696)r_i \text{ - } 0.8906 \qquad (20a)$$

$$y_o(r_o) = (0.1196)r_o^5 - (0.0982)r_o^4 + (0.2784)r_o^3 - (0.1516)r_o^2 + (0.7376)r_o \qquad (20b)$$

For this design, the radius of curvature for input surface changes from a minimum value of 24.47 cm and reaches a maximum value of 9840 cm. Further, the radius of curvature for output surface changes from a minimum value of 13.68 cm and reaches a maximum value of 7931 cm.

As shown in Figure 4a, the system length can be reduced at the expense of a larger starting angle of refraction. For instance, Figure 5 shows a design for f' = 1.792, D = 2.987

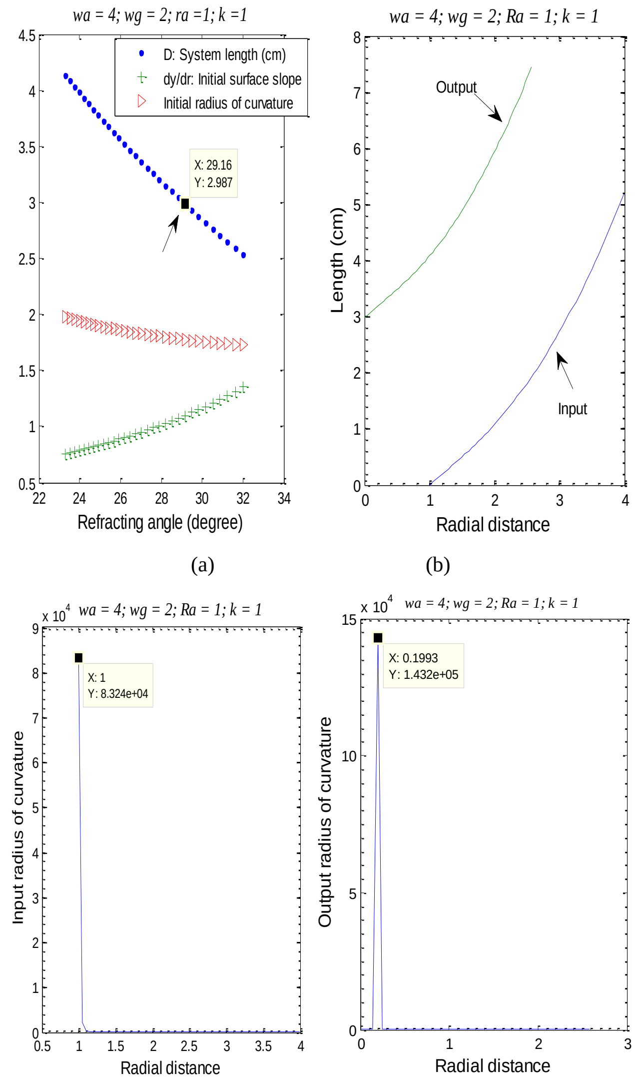
cm, θ_{ri} = 29.16°, R_a= 1 cm; k = 1, w_a = 4, w_g = 2. The surface slopes and the lens surfaces expressions are found to be:

$$\frac{dy_i}{dr_i} = (0.0066)r_i^4 - (0.1139)r_i^3 + (0.7223)r_i^2 - (1.1349)r_i + 1.4816 \tag{21a}$$

$$\frac{dy_o}{dr_o} = (0.0447)r_o^4 - (0.3698)r_o^3 + (1.095)r_o^2 - (0.3981)r_o + 1.0056 \tag{21b}$$

$$y(r_i) = (1.32x10^{-3})r_i^5 - (0.0285)r_i^4 + (0.2408)r_i^3 - (0.5674)r_i^2 + (1.4816)r_i \text{ - } 1.1278 \tag{22a}$$

$$y(r_o) = (8.94x10^{-3})r_o^5 - (0.09245)r_o^4 + (0.365)r_o^3 - (0.1991)r_o^2 + (01.0056)r_o \tag{22b}$$



(c)

Figure 5. An alternative design for Annular-Gaussian to Gaussian transformation with D = 2.987 cm, θ_{ri} = 29.16°: (a) choosing the system length for a specific initial refracting angle; (b) the input and the output surfaces; and (c) the input and the output radius of curvature of the surfaces

The radius of curvature of the input surface ranges from a minimum value of 11.7 cm to a maximum value of 83249 cm; while the radius of curvature of the output surface ranges from a minimum value of 6.65 cm to a maximum value of 143200 cm.

4. Conclusions

One-element lens designs for transforming laser beam profiles are considered in this work using optical refracting system. The lens-design is applied to three types of beam transformations, (i) Gaussian to Annular-uniform; (ii) Annular-Gaussian to Annular-uniform; and (iii) Annular-Gaussian to Gaussian. The mathematical expressions for the aspheric surfaces are derived for the three types of laser beams. In addition, few important parameters such as the length of the set-up, the radii of curvature of the surfaces, and the initial refracting angle for annular beam were discussed. These designs are compared to the two-element refracting systems previously reported in [21]. It was demonstrated that the one-element designs outperform the two-element ones regarding the system length D and the initial refracting angles for annular beams at the price of having smaller values for the surfaces radii of curvatures.

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