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Re‒clarifying and Corroborating the Physical Meaning of Planck's Constant *h* being Integral of One Cycle *EM* Wave Energy per Frequency *ν* More Rigorously and Completely

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Abstract This paper aims to re-clarify, corroborate and supplement the conclusion of physical meaning of Planck's constant being integral of one cycle *EM* wave energy per frequency *ν* through more rigorously and accurately calculating and analyzing of various respective theoretical and practical approaches and viewpoints.

Keywords Planck's constant, Quanta energy, *hν*, Quantum mechanics, Black body radiation, Ether, Bulk modulus, Physical meaning of Planck's constant

1. Introduction

The previous paper of author [9] preliminarily reached a conclusion that the physical meaning of Planck's constant is integral of one cycle *EM* wave energy per frequency *ν*. It is believable that the conclusion is reasonable correct. However, considering some derivation and calculation seems not very perfect and may not be very rigorous, this paper would like to provide complement derivation and calculation together with previous results to build a complete perfect theory and conclusion.

To corroborate and re-clarify accurately why physical meaning of Planck's constant *h* is integral of one cycle *EM* wave energy per frequency *ν*, some key points are worth noting.

1. The quanta energy $E = h\nu$ of *EM* wave being produced or being absorbed by electrons is the measurable smallest energy unity and the only direct or indirect detection tool is electron (inside the sensor.)
2. The energy of *EM* wave produced by motion of electron or point source are not consist of quanta energies before detected; only the energy absorbed or detected by an electron is quanta energy *hν*. Critically, because when motion of an electron generates *EM* wave, multi-electrons located at various directions and distances can sense or receive the quanta wave energy *hν* separately and independently.
3. The decisive and critical measure to calculate energy of a periodical signal or wave is to set the domain (boundary) or (lower - upper) limit of the integral to one wavelength λ or one period *T*.

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1.1. Analyzing and Discussing Some Crucial Points in Precedent Derivations of Physical Meaning of Planck's Constant

1.1.1. Radiation of *EM* Wave from a Point Source (an Electron) before Detected is not Quanta Energy, and is not n$h\nu$

It is said that the *EM* wave emitted from a point source is spherical wave. Radius of a spherical wave varies in a sinusoidal function. The wave propagates outward from the point source at the wave propagation speed.

The energy in a spherical wave propagates equally in all directions; no one direction is preferred over any other. When the source emits power, at any distance *r* from the source must be distributed over a spherical surface of area $4\pi r^2$. Hence, the wave intensity at a distance *r* from the point source is $I = \frac{P_{EM}}{A} = \frac{P_{EM}}{4\pi r^2}$. For example, the intensities at distances (for examples) r_1 and r_5 are $I_1 = \frac{P_{EM}}{4\pi {r_1}^2}$ and $I_5 = \frac{P_{EM}}{4\pi {r_5}^2}$, the ratio of intensities on these two spherical surfaces is $I = \frac{I_1}{I_5} = \frac{r_5^2}{r_1^2}$.

When one electron emits *EM* wave, various electrons (inside receiving antennas or sensor) at any direction and at any distance away from the point source can receive the signal of *EM* wave, although the intensities at different distances (different radius) and different angles are various.

According to axioms of spherical wave, the emitted power density, at any distance *r* away from the point source must be distributed over a spherical surface of area $4\pi r^2$, however, in practices, electrons (sensors) either being located at light year away or located meters away can receive the *EM* wave as well.

Every electron inside a sensor which is randomly located, will generate a closed tiny circle subtend a solid angular centered at the point source of the spherical wave. They have same patch of areas with different distance (*r*) from the point source, different areas subtend different solid angles, thus densities of energy or (densities of photons) distribution they experienced must be different, and as farer as lower. As shown in **Fig.1.1**.

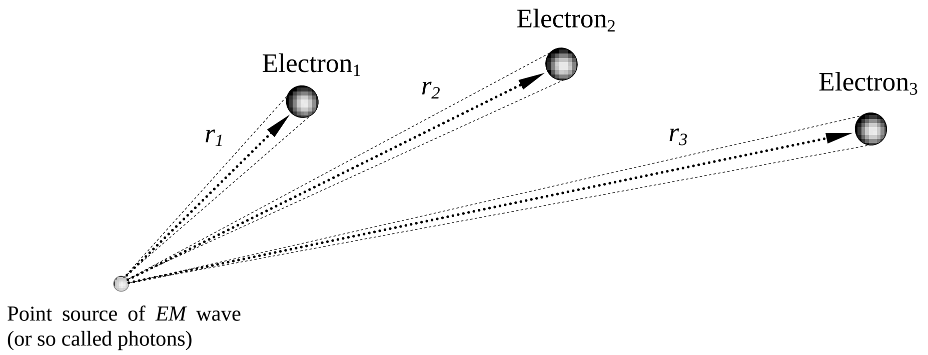

Figure 1.1

The radiation from a transmitting antenna can be received by various receiving antennas located in different directions and distances. As shown in **Fig.1.2**.

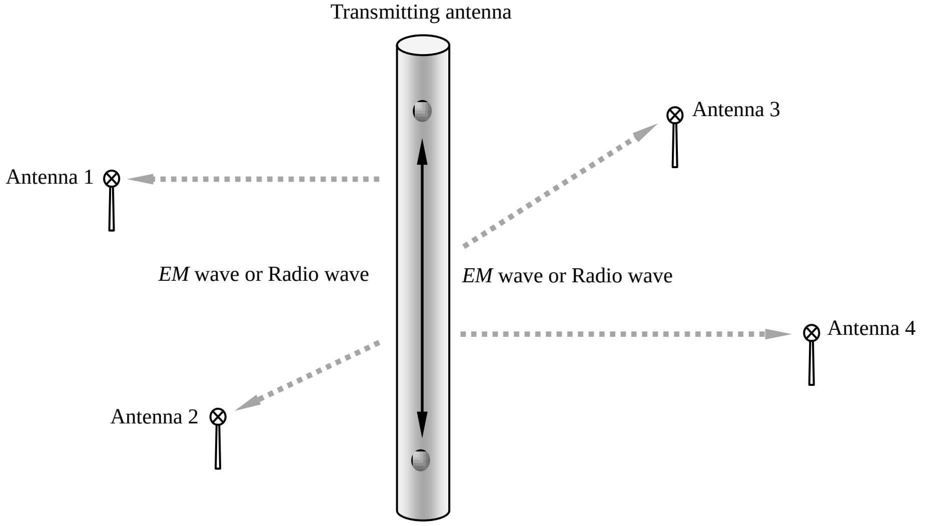

Figure 1.2. Movements of electron inside a transmitting antenna produces *EM* wave, electrons insider various receiving antenna can detect quanta energy independently and separately

If *EM* wave consist of quanta energies or photons, how can their energy densities at cosmic space light years away keep as same high as meters away?

Max Planck comes up with a formula for the spectral energy density of blackbody radiation $u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/kT}-1}$, the formula does not manifest physical meaning of Planck's constant *h*, theoretically and practically; one cannot quantitatively control the radiation energy of *EM* wave of a point source blocked by a closed surface to scale of single quanta, therefore one cannot detect *EM* energy of a monochromatic frequency emitted from a point source blocked by a closed surface to the scale of exact one quanta.

These above mentioned phenomena verify that the *EM* wave energy before received by electrons is not ready quanta energy (photon). Before detected, *EM* wave energy is not existed in the form of quanta but globally distributed field of *EM* wave. Therefore, the existing of quanta energy is not depend upon the theoretical energy density of the *EM* wave but upon the energy absorbed by an electron or say an electron can absorb only quanta *EM* wave energy.

Quanta energy reflects the response characteristics of electron (inside the sensor or detector) to *EM* wave, not the intrinsic property of *EM* wave itself.

If one can measure or receive the *EM* wave by other sensor or particle (except electron) and verifies it being also quanta energy ($h\nu$), then one can say that the *EM* wave originally is consist of quanta energies, or the *EM* wave energy before absorbed or detected by electron is consist of quanta energies.

1.1.2. We can Conclude the Decisive Points as Following

So far as we can only make sure is that the *EM* wave energy packet absorbed or detected by electron is quanta energy.

Both *EM* waves and light waves are produced by oscillating or back and forth movements, i.e., acceleration – deceleration motions of electrons and technologies we can rely on to measure the *EM* wave or light waves and X-rays directly or indirectly is no alternative choice but electrons.

The *EM* radiation emits from a point source can be received by multi electrons located at various angles and distance.

Thus, photoelectric effect measured quanta energy reflects only the response characteristic of electron corresponding to light wave or *EM* wave, not the original intrinsic properties of *EM* wave.

Therefore, when one calculates the quanta energy, one should consider the quantity of energy around a single detecting electron, in other words, it is most reasonable to consider integral of *EM* energy packet over one wave cycle which is absorbed by an electron.

1.1.3. The Determination of Boundary or Limits of Upper and Lower of Integral of *EM* Wave Energy

It is worth noting that, as shown in **Fig.1.3**, the tracks of one *EM* wave cycle left no energy behind the current cycle, only the current instant cycle has energy, when calculating the integral of *EM* wave energy respect to time.

As shown in **Fig.1.4**, the cycles following the current cycle are either the other wave cycles (other quanta energy) or tracks of same cycle with empty energy, when calculating the integral of *EM* wave energy respect to distance.

Therefore when one calculate the integral of a periodic signal, it is meaningless to set the domain or limit of integration as non-integer periods, and it is even more ignorant and absurd to set upper limit of the integral to infinite or more than one wavelength λ or more than one period *T*.

Since the signal is repeated periodically, the boundary of the periodic signal must be bounded by the wavelength. The total energy of one wave cycle is equal to the integral of energy over one wave cycle. Therefore, it is properly right and reasonable to set lower-upper limit of integral to one wavelength λ or one period *T*.

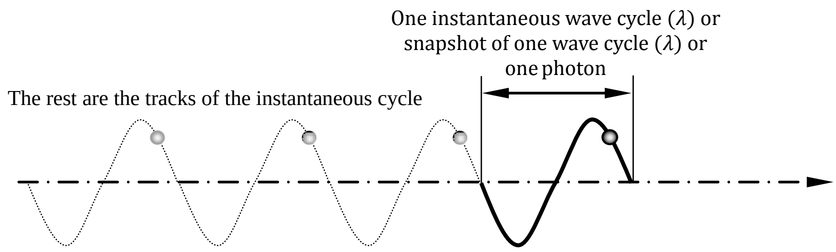

Figure 1.3

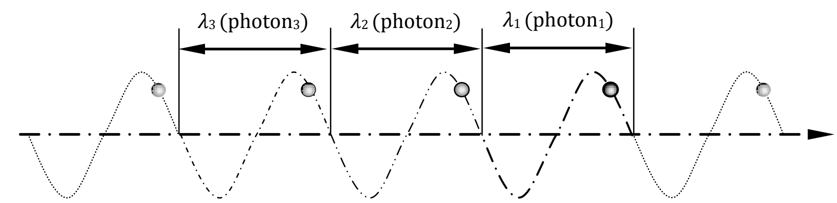

Figure 1.4

1.1.4. Discussions of Some Precedent Approaches and Viewpoints in Deriving Physical Meaning of Planck's Constant *h*

Some researchers have published their approaches (e.g. Brucholz [7], Chang [8], Kwiat [9] and Liu [10]) in their articles and essays, trying to derive physical meaning of Planck's constant. It is helpful to have some evaluation, research, analysis and discussion on these approaches by the way.

1.1.4.1. Bruchholz [7]

Bruchholz asserts in an essay that a photon must have a geometric boundary, the integration of its energy density over a bounded volume must have $E = h\nu$.

$$A_{y=}A_y(\omega \cdot (t - x))$$

The field strengths respectively flow densities become

$$E_y = \frac{\partial A_y}{\partial t} = \omega A'_y\big(\omega \cdot (t - x)\big) \text{ and}$$

$$B_z = -\frac{\partial A_y}{\partial x} = \omega A_y^{'}(\omega \cdot (t-x))$$

$$\iiint \frac{\varepsilon_0}{2}\left(E_y^2 + B_z^2\right) dy\, dz = \omega^2 \varepsilon_0 \iiint {A_y^{'}}^2 (\omega(t-x))\, dy\, dz$$

Bruchholz emphasized, this volume integral would be impossible without the boundary, because the linear solution, being alone, is not physically meaningful for the infinite extension.

Discussion

The assertion of photon having boundary sounds right.

However, it is inefficient to set up the domain or range of integration as a volume integral rather than a line integral.

And Bruchholz did not try to determine the "boundary" or limit of integral in the essay.

1.1.4.2. Chang [8]

In the article, Chang treats the photon as a wave packet which is made up of an oscillating electro-magnetic field as a wave packet of electro-magnetic radiation and directly calculating the total energy and momentum contained within the wave packet.

Then examine whether E is proportional to the oscillating frequency ν. If yes, the proportional constant will be identified as the Planck's constant h.

The integration of the energy density described in equation below over the entire volume of the wave packet: U

$$\langle U \rangle = \iiint\limits_V U(x,y,z)\, dx\, dy\, dz = \iiint\limits_V (\varepsilon\omega^2 A_0^2)\, dx\, dy\, dz$$

$$= \varepsilon\omega^2 \int_0^{\infty}\int_0^{2\pi} A_T^2(r,\theta) r\, d\theta\, dr \int_{-\infty}^{\infty} A_L^2(z-ct)\, dz$$

Discussion

The train of thought of treating the photon as a wave packet and calculating the total energy within the wave packet deems correct.

However, the $h\nu$ or quanta energy is connected or proportional to a monochromatic frequency ν which should be treated as constant, thus it may not be necessary to analyze frequency distribution.

It is inefficient to set up the domain or limit of integration as a volume integral rather than a line integral and setting the upper limit of integral to infinite rather than one wavelength may not be proper.

1.1.4.3. Kwiat [9]

In the approach of the paper, coupled string like real wave functions is assumed, therefore Planck constant h is interpreted as result of exchange interactions between two coupled strings.

Discussion

Schrödinger equation $-i\hbar\frac{\partial}{\partial t}\psi(x,t) = \mathcal{H}\psi(x,t)$ is a wave equation which is not derived from any classical mechanics model. Kwiat assumes $\frac{1}{k_s}\frac{\partial y}{\partial x} = -\frac{1}{2}\hbar$, seems equivalent to trying set up a classical mechanical wave model similar to spring wave mechanical model with tension or sound wave mechanical model with bulk modulus.

However, the assumption seems absence of concrete mechanical ground.

The "string theory" seems an idea out of expediency, temporary substitute of medium ether, may not be the result of comprehensive considerations.

The problems are:

Why only two strings interacts each other rather than multi strings are interacting roundly?

Any so called string, its two ends must be fixed then can the interaction between two strings be coupled, thus, how are the ends of string fixed?

By this way, it may be better to try to find the "bulk modulus" of the continuous uniform distributed medium ether (or modern ether) rather than consider "tension" of discrete "strings".

(Modern) ether theory could comprehensively contain string theory. To seek physical meaning of Planck's constant consider more on the quanta energy original emitting and receiving objects (electrons producing *EM* wave and response to *EM* wave when detecting *EM* wave) may be more efficient.

1.1.4.4. Liu [10]

The wave energy expressions in the approach are inspired by expression of Poynting vector.

According to the principle of wave energy flow density vector $\boldsymbol{S} = \frac{\boldsymbol{1}}{\boldsymbol{\mu_0}}\boldsymbol{E} \times \boldsymbol{B}$, to find the flow rate density with respect to time t

$$U_E(x,t) = \frac{\partial E(x,t)}{\partial t} \quad \text{and} \quad U_B(x,t) = \frac{\partial B(x,t)}{\partial t}$$

Similar to the Poynting vector, the derivative of energy flow density respect to time represents flow rate density through a unit surface area perpendicular to the direction of wave propagation, is expressed as

$$U_S = \frac{U_E \times U_B}{\mu_0}$$

$$= \omega^2 \frac{E_{max}\, sin(kx-\omega t) B_{max}\, sin(kx-\omega t)}{\mu_0}$$

$$E_\lambda = \int_0^\lambda U_S dx = \int_0^\lambda \omega^2 \frac{E_{max}\sin kx B_{max}\sin kx}{\mu_0} dx$$

$$= 2\pi^2 c \frac{E_{max}\, B_{max}}{\mu_0}\nu$$

Discussion

The result and conclusion should be not wrong, however, the coming up expressions and equations are needed to be checked whether are compatible with the conventional electromagnetic wave expression and theory. For this reason, this paper aims to improve the derivation and calculation more rigorously and accurately to conform to the Maxwell's equation and formula of *EM* wave.

1.2. More Accurate and Rigorous Calculation of the Integral of *EM* Wave Energy over One Wavelength Based on Maxwell Equations

1.2.1. The *E* Wave and the *B* Wave should have a Phase Difference $\boldsymbol{\phi = 90^o}$ **or** $\boldsymbol{\frac{\pi}{2}}$

Examining prevailing description of *EM* wave formulas, the amplitudes of *E* wave field and *B* wave field are reaching the crests and troughs without phase difference. Thus the instantaneous sum of the *E* energy and *M* energy is not a constant.

At the nodes, the sum of total energy is zero, where is the energy of the wave stored to restore the amplitudes of the oscillator of *EM* wave? This obviously violates the energy conservation law. As shown in **Fig.2.1.a**. Therefore, regarding the law of energy conservation, the *EM* wave should be depicted as shown as **Fig.2.1.b**, where point of the crest of *E* wave falls in the node of *B* wave, whilst the point of crest of *B* wave falls in the node of *E* wave. I.e. the *E* wave and the *B* wave should have a phase difference $\phi = 90^o$ or $\frac{\pi}{2}$. Hence, any instantaneous value of the sum of potential energies is a constant.

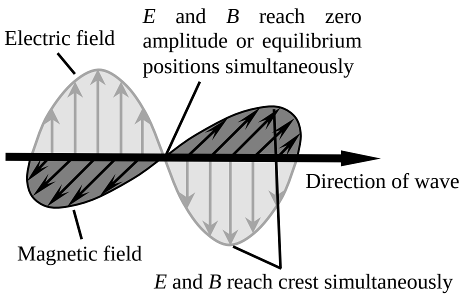

Fig.2.1.a

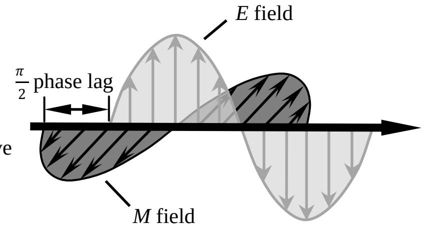

Fig.2.1.b

Figure 2.1

However, the mainstream of physics community may not agree with the above measure the moment. For the sake of a complementary consideration, we take the two assertions separately into account when we calculating the *EM* wave energy.

1.2.2. General Measure

In *EM* wave, both the electric potential energy density and magnetic potential energy density are proportional to ω^2 and transforming each other.

The key point is that if *EM* wave is described by the general solution in the *x* axis direction,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

The monochromatic (constant frequency) harmonic waves in the *x* direction, namely

$$y = Acos\omega\left(t - \frac{x}{v}\right) \quad \text{then} \quad \frac{\partial y}{\partial t} = -\omega Asin\omega(t - \frac{x}{v})$$

The $\frac{\partial y}{\partial t}$ could be considered as equivalent velocity in *y* direction or velocity of amplitude, then the equivalent kinetic energy density of wave must be proportional to second power of $\frac{\partial y}{\partial t}$ i.e.,

$$K \quad \propto \quad (\frac{\partial y}{\partial t})^2 \quad \propto \quad \omega^2 A^2 sin^2\omega(t - \frac{x}{v})$$

The equivalent kinetic energy must be equal to potential energy, therefore

$$U \quad \propto \quad (\frac{\partial y}{\partial t})^2 \quad \propto \quad \omega^2 A^2 sin^2\omega(t - \frac{x}{v})$$

1.2.3. Deriving the Formula of Calculation of the *EM* Wave Energy over One Cycle Considering the *E* and *M* without Phase Lag

According to **Fig.2.1a** depicted (the case 1), considering the wave propagates along the x-axis, then the energy density of an electro-magnetic field is known to be

$$U = \frac{1}{2}\left[\varepsilon_0 \boldsymbol{E}^2 + \frac{1}{\mu_0}\boldsymbol{B}^2\right] \qquad (2.1)$$

Where $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$ are the dielectric permittivity and magnetic permeability of the vacuum, $\boldsymbol{E}$ and $\boldsymbol{B}$ are electric field and magnetic induction, respectively. According to the Maxwell's theory, $\boldsymbol{E}$ and $\boldsymbol{B}$ can be derived from the scalar potential Φ and the vector potential $\boldsymbol{A}$:

$$\begin{cases} \boldsymbol{B} = \nabla \times \boldsymbol{A} \\ \boldsymbol{E} = -\nabla\boldsymbol{\Phi} - \frac{\partial \boldsymbol{A}}{\partial t} \end{cases} \qquad (2.2)$$

Consider the x-axis direction propagation, (assume being the direction of an electron of the detecting sensor), since $\boldsymbol{\Phi = 0}$ then

$$\boldsymbol{B} = \frac{\partial A_y}{\partial x} \quad \text{and} \quad \boldsymbol{E} = \frac{\partial A_y}{\partial t}$$

The energy packet density is then described in following equation U,

$$U = \frac{1}{2}\left[\varepsilon_0 \boldsymbol{E}^2 + \frac{1}{\mu_0}\boldsymbol{B}^2\right] = \frac{1}{2}\left[\varepsilon_0\left(\frac{\partial A_y}{\partial t}\right)^2 + \frac{1}{\mu_0}\left(\frac{\partial A_y}{\partial x}\right)^2\right] =$$

$$\frac{1}{2}\left[\varepsilon_0(\omega A_0)^2 + \frac{1}{\mu_0}(kA_0)^2\right] = \varepsilon_0\omega^2 A_0^2 cos^2(kx - \omega t) \qquad (2.3)$$

We can calculate the integral of the total energy over one wavelength by integrating (2.3).

If we take a snapshot of the wave at time then the energy density in a wavelength λ is the integral of the expression respect with *x* over one wavelength λ:

$$E_\lambda = \int_0^\lambda U dx = \int_0^\lambda \varepsilon_0 \omega^2 A_0^2 cos^2(kx)dx$$
$$= \varepsilon_0 \omega^2 A_0^2 \int_0^\lambda cos^2 kx dx = \varepsilon_0 \omega^2 A_0^2 \frac{1}{2}\lambda \qquad (2.4)$$

Substituting $\omega = 2\pi f$ into (2.4) obtains,

$$E_\lambda = \frac{1}{2}\varepsilon_0 (2\pi f)^2 A_0^2 \lambda = \frac{1}{2}(2\pi)^2 \varepsilon_0 A_0^2 f^2 \lambda \qquad (2.5)$$

Substituting $f\lambda = c$ into (2.5) we have the expression of energy in one wavelength as

$$E_\lambda = 2\pi^2 c\varepsilon_0 A_0^2 f \quad \text{or} \quad E_\lambda = 2\pi^2 c\varepsilon_0 A_0^2 \nu \qquad (2.6)$$

1.2.4. Deriving the Formula of Calculation of the *EM* Wave Energy over One Cycle Considering the *E* and *M* has $\mathbf{90^o}$ **or** $\frac{\pi}{2}$ Lag

According to **Fig.2.1b** depicted, (case2 the energy conservation law is obeyed), the *E* and *M* has $\frac{\pi}{2}$ or 90° phase lag, the energy density of an electro-magnetic field should be

$$U = \frac{1}{2}[\varepsilon_0 (\omega A_0)^2 sin^2(kx - \omega t)$$
$$+ \frac{1}{\mu}(kA_0)^2 cos^2(kx - \omega t)\Big] = \frac{1}{2}\varepsilon_0 \omega^2 A_0^2 \qquad (2.7)$$

The integral of energy over one cycle is

$$E_\lambda = \int_0^\lambda U dx = \frac{1}{2}\varepsilon_0 \omega^2 {A_0}^2 \int_0^\lambda dx = \frac{1}{2}\varepsilon_0 \omega^2 {A_0}^2 \lambda \quad (2.8)$$

Substituting the equations $\omega = 2\pi f$ and $f\lambda = c$ into (2.8), we obtain the expression of energy over one wavelength is

$$E_\lambda = \frac{1}{2}\varepsilon_0 (2\pi)^2 {A_0}^2 f^2 \lambda = 2\pi^2 \varepsilon_0 c {A_0}^2 f$$
$$\text{or} \quad E_\lambda = 2\pi^2 \varepsilon_0 c {A_0}^2 \nu \qquad (2.9)$$

Comparing Eq.(2.9) with Eq.(2.6), the result is same.

1.2.5. Estimation or a Hypothesis of the Amplitude of *EM* Wave

Regarding both of transmitting and receiving of the *EM* wave are using by electrons; the maxim amplitude A_0 of *EM* wave could be equal to or proportional to the radius ($R_e = D_e/2$) of electron. (There were some preliminary discussions of this hypothesis in [6].)

An explanatory illustrational diagram is shown as **Fig.2.2**.

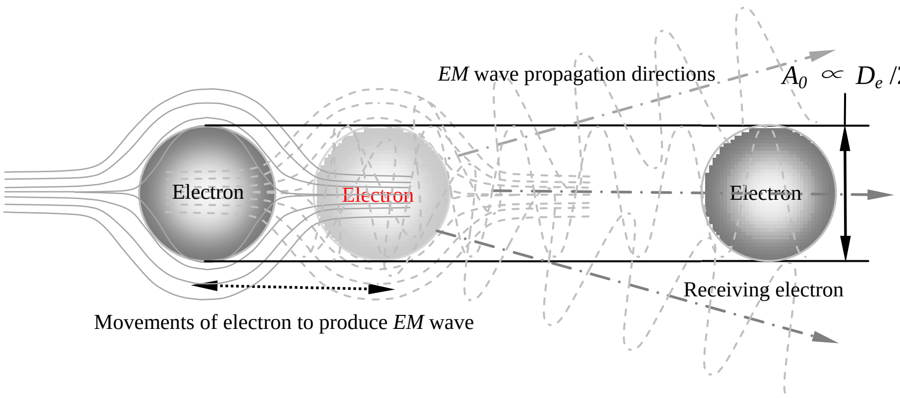

Figure 2.2. Amplitude A_0 is equal to or proportional to radius of electron ($D_e/2$)

1.3. Rigorous Logical Deductions And New Discovery of a General Law about Wave Energy Re-Clarify and Corroborate the Planck's Constant Being the Integral of One cycle *EM* Wave Energy Per Frequency ν

1.3.1. There is a Novel Uncovered General Law of One Cycle Wave Energy

It is worth noting that there is a previously un-clarified or covered general principle or law about wave energy that the integral of energy over one cycle of any wave is proportional to a constant h_x and its frequency *f*.

Normally, the linear distributed potential energy density *U* of a sinusoidal wave is proportional to second power of ω *y* ($\omega^2 y^2$), where ω is angular frequency, *y* is the amplitude.

If potential energy density *U* (or power) of wave is proportional to ω^2, then at least mathematically, the integral of wave energy over one cycle or one wavelength λ is definitely proportional to a constant and frequency *f* or *ν*.

This novel uncovered general law is not exception for electromagnetic waves and not be noticed by researchers previously.

1.3.2. Reviewing the Integral of Energy over One Cycle of Sound Wave with Respect to Frequency

"To evaluate the kinetic energy of this volume of air, we need to know its speed. We have

$$K_\lambda = \int dK = \int_0^\lambda \frac{1}{2}\rho A(\omega s_{max})^2 \sin^2 kx dx$$
$$= \frac{1}{2}\rho A(\omega s_{max})^2 \int_0^\lambda \sin^2 kx dx$$
$$= \frac{1}{2}\rho A(\omega s_{max})^2 (\frac{1}{2}\lambda) = \frac{1}{4}\rho A(\omega s_{max})^2 \lambda$$

" [1]

We know that the angular frequency is $\omega = 2\pi f$ and velocity is $v = f\lambda$. Insert the equations $\omega = 2\pi f$ and velocity $v = f\lambda$ into the energy expression of one wavelength K_λ obtains

$$K_\lambda = \frac{1}{4}\rho A(\omega s_{max})^2\lambda = \frac{1}{4}\rho A v(2\pi s_{max})^2 f$$

Let $\frac{1}{4}\rho A v(2\pi s_{max})^2 = h_{SoundWave}$; the density of medium, A and v remain unchanged, the coefficient $h_{SoundWave}$ is a constant, therefore the total energy in one wavelength of sound wave is $K_\lambda = h_{SoundWave}\, f = h_{SoundWave}\, \nu$, i.e., proportional to a constant and frequency.

1.3.3. Reviewing the Integral of Energy over One Cycle of String Wave with Respect to Frequency

"To obtain the total potential energy in one wavelength, we integrate this expression over all the string segments in one wavelength:

$$U_\lambda = \int dU = \int_0^\lambda \frac{1}{2}\mu\omega^2 A^2 \sin^2 kx dx = \frac{1}{4}\mu\omega^2 A^2\lambda$$

$$K_\lambda = \int dK = \frac{1}{4}\mu\omega^2 A^2\lambda$$

The total energy in one wavelength of the wave is the sum of the potential and kinetic energies:

$$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2}\mu\omega^2 A^2\lambda$$

" [1]

We know that the angular frequency is $\omega = 2\pi f$ and the velocity is $v = f\lambda$. Insert the equations $\omega = 2\pi f$ and $v = f\lambda$ into the one wavelength E_λ obtains

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2\lambda = \frac{1}{2}\mu v(2\pi)^2 A^2 f$$

Express the $\frac{1}{2}\mu v(2\pi)^2 A^2$ as h_{string} , the magnitudes of μ, v and A remain unchanged, the coefficient h_{string} is constant, then the total energy in one wavelength of string wave is $E_\lambda = h_{string}\, f$ or $E_\lambda = h_{string}\, \nu$, i.e., proportional to a constant and frequency.

1.3.4. The Novel Uncovered General Law of One Cycle Wave Energy is Even Actually Applicable to Simple harmonic Motion e.g. the Block-Spring System

As we know (as shown in **Fig. 3.1**), the displacement of a block-spring system is

$$x = x_{max}\, cos(\omega t) = Acos(\omega t).$$

The maxim displacement x_{max} actually is the amplitude A, and $\omega^2 = \frac{k}{m}$, $\omega = 2\pi f$.

The equivalent wavelength λ actually is equal to $2x_{max}$ ($2A$), and the average velocity is $v_{average} = f\lambda = f2x_{max}$.

As we well know prevailingly, the total energy of block-spring system is

$$E = \frac{1}{2}k{x_{max}}^2 = \frac{1}{2}kA^2 \quad (3.1)$$

Substituting $A = x_{max}$, $\frac{k}{m} = \omega^2$ $v_{average} = f2x_{max}$ and $\omega = 2\pi f$ into (3.1) obtains

$$\frac{1}{2}k{x_{max}}^2 = \frac{1}{2}m\omega^2{x_{max}}^2 = \frac{1}{2}m(2\pi f)^2 x_{max}\frac{\lambda}{2}$$
$$= \pi^2 m\frac{1}{2}v_{average}\, x_{max}\, f = \frac{1}{2}\pi^2 m v_{avareg}\, Af = h_{bs} f$$

Since the coefficient $h_{bs} = \frac{1}{2}\pi^2 m v_{avareg}\, A$ is a constant, therefore, the total energy of a block-spring system is also proportional to a constant h_{bs} and frequency f.

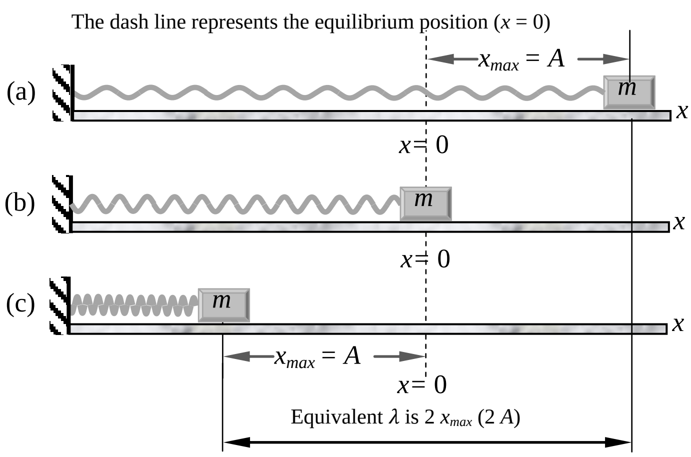

Figure 3.1

1.3.5. Reviewing and Analyzing the Integral of Energy over One Cycle of *EM* Wave with Respect to Frequency ν

Review the equation (2.9) derived in Section (2.3), we have found the total *EM* wave energy over one wavelength:

$$E_\lambda = \frac{1}{2}\varepsilon_0(2\pi)^2{A_0}^2 f^2\lambda = 2\pi^2\varepsilon_0 c{A_0}^2 f$$

or $$E_\lambda = 2\pi^2\varepsilon_0 c{A_0}^2\nu \quad (3.9)$$

Where c as being light speed is constant; A_0 as being max amplitudes is constant; coefficient $\boldsymbol{\varepsilon}$ as being permittivity is constant. Then the coefficient $2\pi^2\varepsilon c{A_0}^2$ is constant.

Notice that the quanta energy ($h\nu$) is Planck' constant h multiplying by the wave frequency ν ($E = h\nu$), the integral of *EM* wave energy over one wavelength is also a constant $2\pi^2 \varepsilon c {A_0}^2$ multiply by the wave frequency ν. Therefore the Planck's constant h must be equal to the coefficient $2\pi^2 \varepsilon c {A_0}^2$.

1.4. Analysis Manifests that the Planck's Constant *h* is Decisively Proportional to "Bulk Modulus and Density" ($\boldsymbol{\varepsilon_0}$ Combine with $\boldsymbol{\mu_0}$) of Vacuum or Modern ether

1.4.1. Relation of $\boldsymbol{h_{SoundWave}}$ and Bulk Modulus of Sound Wave Medium Air

The energy expression of one wavelength K_λ of sound wave is

$$K_\lambda = \frac{1}{4}\rho A(\omega s_{max})^2\lambda = \frac{1}{4}\rho A v(2\pi s_{max})^2 f \qquad (4.1)$$

Where v is speed $v = \sqrt{\frac{B}{\rho}}$, substituting $v = \sqrt{\frac{B}{\rho}}$ into (4.1), obtains

$$K_\lambda = \frac{1}{4}\rho A v(2\pi s_{max})^2 f = \frac{1}{4}\rho A\sqrt{\frac{B}{\rho}}(2\pi s_{max})^2 f$$

$$= \frac{1}{4}A\sqrt{\rho B}(2\pi)^2 {s_{max}}^2 f \qquad (4.2)$$

$$h_{SoundWave} = \frac{1}{4}A\sqrt{\rho B}(2\pi)^2 {s_{max}}^2 \qquad (4.3)$$

1.4.2. Relation of $\boldsymbol{h_{String}}$ and Bulk Modulus (Tension *T*) of String Wave Medium

The total energy in one wavelength of string wave is

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2\lambda = \frac{1}{2}\mu v(2\pi)^2 A^2 f \qquad (4.4)$$

Where v is speed: $v = \sqrt{\frac{T}{\mu}}$, T is tension of string, substituting $v = \sqrt{\frac{T}{\mu}}$ into (4.4), obtains

$$E_\lambda = \frac{1}{2}\mu v(2\pi)^2 A^2 f = \frac{1}{2}\mu\sqrt{\frac{T}{\mu}}(2\pi)^2 A^2 f$$

$$= \frac{1}{2}\sqrt{\mu T}(2\pi)^2 A^2 f \qquad (4.5)$$

$$h_{stringe} = \frac{1}{2}\sqrt{\mu T}(2\pi)^2 A^2 \qquad (4.6)$$

1.4.3. Relation of $\boldsymbol{h}$ and Bulk Modulus and Density of Vacuum or Ether

The total *EM* wave energy over one wavelength is:

$$E_\lambda = \frac{1}{2}\varepsilon_0(2\pi)^2 {A_0}^2 f^2\lambda = 2\pi^2\varepsilon_0 c {A_0}^2 f$$

$$\text{or} \quad E_\lambda = 2\pi^2\varepsilon_0 c {A_0}^2 \nu \qquad (4.7)$$

Where c is speed $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, substituting $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ into (4.7), obtains

$$E_\lambda = 2\pi^2\varepsilon_0\frac{1}{\sqrt{\mu_0\varepsilon_0}}{A_0}^2\nu = 2\pi^2\sqrt{\frac{\varepsilon_0}{\mu_0}}{A_0}^2\nu \qquad (4.8)$$

Therefore Planck's constant h is

$$h = 2\pi^2\sqrt{\frac{\varepsilon_0}{\mu_0}}{A_0}^2 \qquad (4.9)$$

1.4.4. Further Derivation and Analysis

We know that $v = \sqrt{\frac{B}{\rho}}$ for sound wave, $v = \sqrt{\frac{T}{\mu}}$ for string wave and $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ for *EM* wave.

The ρ and B is density and bulk modulus of medium of sound wave in below expression,

$$h_{SoundWave} = \frac{1}{4}A\sqrt{\rho B}(2\pi)^2 {s_{max}}^2 \qquad (4.10)$$

The μ and T is linear density and tension of string wave in below expression,

$$h_{stringe} = \frac{1}{2}\sqrt{\mu T}(2\pi)^2 A^2 \qquad (4.11)$$

The $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$ are the dielectric permittivity and magnetic permeability of the vacuum in below expression,

$$h = 2\pi^2\sqrt{\frac{\varepsilon_0}{\mu_0}}{A_0}^2 \qquad (4.12)$$

Make a comparison:

We see that, the $\boldsymbol{\varepsilon_0}$ combine with $\boldsymbol{\mu_0}$ is playing corresponding roles similar to the linear density $\boldsymbol{\mu}$ of string wave and density $\boldsymbol{\rho}$ of medium of sound wave, tension $\boldsymbol{T}$ of string wave and bulk modulus $\boldsymbol{B}$ of medium of sound wave.

One could say that the $\boldsymbol{\varepsilon_0}$ combine with $\boldsymbol{\mu_0}$ are density and bulk modulus of vacuum or free space if one insists that ether does not exist.

(Factually ether is existed, because if light or *EM* wave "medium is wave itself", then the changes of amplitude of wave is following up with and synchronized with the wave propagation speed c.

This synchronous actions of relationship result in amplitude of wave not being actually manifested changes at a particular stationary position x of a receiving antenna.

Then the amplitude will be only function of x, the expression will be $S = S_{max}\cos(kx)$.

Thus any static sensor or antenna fixed on a particular position x, there is no AC signal can be perceived. Therefore photon is not existed, and medium ether must be existed! There are more details in reference [5] and [6].)

So as a matter of fact, alternatively, we can frankly say that the $\boldsymbol{\varepsilon_0}$ combine with $\boldsymbol{\mu_0}$ are density and bulk modulus of (modern) ether if we objectively recognize that modern ether actually and factually exists.

Whatever, Planck's constant h is mainly proportional to $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$ determined density and bulk modulus of "modern ether" or "vacuum" or "free space". (There are some preliminary discussions of the deduction in [5].)

1.5. Try to Calculate the Value of Planck's Constant According to above Derived Physical Meaning and Deduced Conclusive Formula

In section 4.4, based on the result of integral of *EM* wave energy over one wavelength, we further derived the equation (4.12) for calculation of Planck's constant:

$$h = 2\pi^2\sqrt{\frac{\varepsilon_0}{\mu_0}}{A_0}^2 \qquad (4.12)$$

As we well know the values of these parameters in the equation:

$\boldsymbol{\varepsilon}_0$ = 8.854 2 × 10^{-12} $C^2/N/m^2$ and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.

And we have a hypothesis in Section 2.5 of this paper that the maxim amplitude A_0 of *EM* wave could be equal to or proportional to the radius of electron R_e.

We try to use a most realistic approach to estimate the radius of electron R_e, to take the ratio of proton/electron mass, thus, divides the proton's radius by the cube root of this number. This ratio would set the electron's radius at approximately 12 times smaller than a proton: at $R_e \cong 9.1 \text{x} 10^{-17}$ m.

Thus, substituting all available values of $\boldsymbol{\varepsilon}_0$, μ_0 and R_e into (4.12), obtains

$$h_{\text{calc}} = 2\pi^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} {A_0}^2$$

$$= 2 \times (3.1416)^2 \sqrt{\frac{8.854\ 2 \times 10^{-12}}{4\pi \times 10^{-7}}} (9.1 \times 10^{-17})^2$$

$$= 4.34 \times 10^{-34} \text{ J.s}$$

Taking the new theoretical calculated value of Planck's constant $h_{\text{calc}} = 4.34 \times 10^{-34}$ J.s to compare with the value from NIST : $h = 6.626 \times 10^{-34}$ J.s, they are of the same order of magnitude.

Considering the radius of electron is an approximate value, the derivations of this paper are scientifically correct.

2. Conclusions

Based on the previous rigorous analysis, deductions and calculations in this article, **we can generally and clearly conclude that:**

1. The integral of energy over one cycle of any waves is proportional to a constant h_x determined by the wave parameters and is proportional to the wave frequency *f* or *v*, i.e.,

$$\boldsymbol{K_\lambda} \text{ or } \boldsymbol{E_\lambda} \text{ or } \boldsymbol{K_T} \equiv \boldsymbol{hf} \text{ or } \boldsymbol{h\nu}.$$

However, for different waves the value of the constant $\boldsymbol{h_x}$ is distinctively different and for *EM* wave, the constant $\boldsymbol{h_x}$ had been called Planck's constant $\boldsymbol{h}$ beforehand.

2. The Planck's constant *h* actually and factually is the integral of energy over one wavelength of *EM* wave, i.e., $E_\lambda = 2\pi^2\varepsilon_0 c {A_0}^2 \nu$ per frequency *ν*.
3. Accordingly, the Planck's constant $h = 2\pi^2\varepsilon_0 c {A_0}^2$.
4. The maxim amplitude A_0 could be equal to or proportional to radius of electron R_e.

Planck's constant *h* is mainly proportional to ε_0 and μ_0 determined density and bulk modulus of "modern ether".

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