

Unsteady MHD Free Convective Chemically Reacting Flow over a Heated Vertical Plate with Heat Source, Thermal Radiation and Oscillating Wall Temperature, Concentration and Suction Effects

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Abstract The problem of transient MHD free convective chemically reacting fluid flow past a hot vertical porous plate with the attendant effects of thermal radiation, heat source and varying wall temperature, concentration and suction is investigated. The governing non-linear and coupled partial differential equations are non-dimensionalized, and linearized using the oscillating perturbation series expansion solutions. The resulting equations are solved and the expressions for the temperature, concentration, velocity, Nusselt number, Sherwood number and force on the plate wall are obtained. The flow characteristics are quantified and presented graphically. The results, amidst others, show that increase in the Hartmann number increases the velocity and skin friction; increase in the convection force increases the velocity and skin friction. Furthermore, it is seen that increase in the Raleigh number increases the temperature, velocity and skin friction but decreases the rate of heat transfer; increase in the heat source parameter increases the temperature, velocity and skin friction but decreases the rate of heat transfer; increase in the chemical reaction rate increases the concentration, Sherwood number, and velocity and skin friction. These results are bench-marked with the results in some existing literatures and they are in agreement.

Keywords Chemical reaction, Free convection, Heat source, Oscillating boundaries, Thermal radiation

1. Introduction

Convective flows involving MHD chemical reactions over porous plates have relevance in, amongst others, engineering, power generating systems and geophysics.

In many natural and engineering processes, there are different levels of interactions which influence the flow variables. Some are highly interactive, some moderately or low, and others non-interactive. In the highly interactive flow cases, the simultaneous heat and mass transfer effects exist, and they are seen as thermo-diffusion and diffusion-thermo effects. A number of research works involving the different levels of interactions exist. In some the effects of magnetic field are played down, and in others they are considered. With the magnetic field neglected, for the highly interactive flow over vertical plates, [1,2] studied

natural convective heat and mass transfer effects on the unsteady flow with cross-diffusive effects over an accelerating semi-infinite vertical porous plate; [3] considered the mixed convective flow with double diffusive effects over a vertical wavy plate; Also, for the moderate or less interactive flow over vertical plates but with the magnetic field effects neglected, [4] studied the unsteady natural convective flow over a porous plate in the presence of time-dependent temperature, concentration and suction; [5] investigated the transient mixed convective flow over a porous vertical plate in the presence of periodic suction; [6] considered the radiation effects in a mixed convective flow over an isothermal vertical porous plate; [7] studied the unsteady mixed convective flow over a vertical porous plate with heat and mass transfer, periodic suction and oscillatory free stream effects.

Magnetic field plays very important roles in the dynamics of fluids. Upon this, a great number of researches have been carried out in this domain of study. For the highly interactive flow, [8] examined the viscous effects on a steady MHD free convective flow through a vertical porous plate; [9,10]

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Published online at <http://journal.sapub.org/ajfd>

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investigated the effects of simultaneous heat and mass transfer, thermal radiation and Hall currents on the unsteady free convective flow over a vertical porous plate; [11] studied the effects of chemical reaction, convection, thermal radiation, cross-diffusion and magnetic field on the transient flow over a porous vertical plate. With the effects of magnetic field in view, for the moderate or less interactive flow, [12-22] studied the free convective flow over porous plates in the presence of suction, thermal radiation, chemical reaction rate and viscous dissipation. *Importantly*, for a partial in-depth review of literature, [23] studied the unsteady natural convective flow over a porous vertical plate using numerical approach, and noticed that the velocity is increased by the convective currents and Darcy number but is decreased by the magnetic field parameter, Prandtl and Schmidt numbers. [24] investigated the effect of chemical reaction rate in a transient free convective flow over a vertical plate in the presence of oscillating temperature and variable suction, and found that the magnetic field parameter increases the skin friction; the Prandtl number decreases the velocity, temperature, skin friction and Nusselt number; the Schmidt number decreases the velocity, concentration, skin friction and Sherwood number; the convective currents increase the skin friction; chemical reaction rate decreases the velocity. [25] examined the unsteady mixed convective flow over a porous plate using similarity transformation and a numerical approach, and saw that the velocity is increased by the convective currents but it is decreased by the suction and magnetic field parameters; the heat source source/sink parameter increases the temperature and skin friction.

Gundagani et al., [23] studied the MHD free convective flow past a vertical porous plate using the finite element numerical approach. In their work, the effects of heat source, thermal radiation and chemical reaction were neglected. Upon this, we are motivated to improve on the same model by investigating amongst others, the effects of the afore-mentioned parameters using the oscillating perturbation series solutions analysis.

This paper is organized as follows: section 2 is the methodology, and section 3 is the conclusion.

2. Methodology

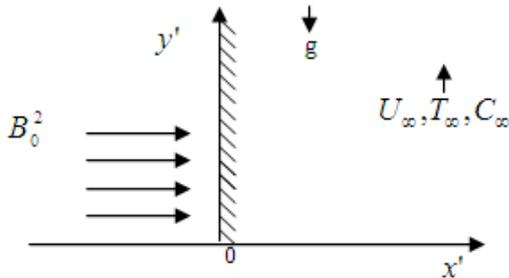


Figure 1. A schematic model of a moving vertical plate in a flowing fluid

The unsteady MHD natural convective chemically reacting flow over a vertical porous plate with heat source,

thermal radiation, oscillating wall temperature, concentration and suction effects is considered. We assumed the y' - axis is along the plate; $y'=0$ is the edge of the plate, and is taken to be the origin; the x' -axis is normal to the plate; the flow region is between $y'=0$ and $y'=\infty$; the fluid is viscous, incompressible, Newtonian, optically transparent, magnetically susceptible (due to the influence of the Earth or applied magnetic field) and non-highly chemically interactive; the plate is porous, heated, coated with some chemical substance, and vertically accelerating; the flow is naturally convective; the pressure is along the x -axis and is equal to the hydrostatic pressure such that $p_o(x')=0$; the wall temperature and concentration and suction are oscillating. The hotness of the plate creates room for radiation and promotes heat exchange in the fluid system. The porous nature of the plate allows the fluid to permeate it; the chemical substance on the plate, which is of a higher concentration than that of the fluid is to enhance chemical reaction and suction. The magnetic field is constant and normal to the plate. Upon the one-dimensional mathematical theory for transient flows, the only independent variables are y' and t' . If (u',v') are the velocity components in the (x',y') coordinates of the Cartesian system, where u' is the axial velocity and v' is the normal velocity and suction, T and C are the fluid temperature and concentration (quantity of material being transported), respectively; T_∞ and C_∞ are the fluid temperature and concentration at equilibrium; T_w and C_w are the temperature and concentration at which the plate is maintained. Now, under the Boussinesq approximations, the governing flow equations are:

$$\frac{dv'}{dy'} = 0 \quad (1)$$

$$\rho \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = \mu \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma_e B_o^2}{\mu \mu_m} - \frac{\mu}{\kappa} \right) u' + \rho g \beta_t (T' - T_\infty) + \rho g \beta_c (C' - C_\infty) \quad (2)$$

$$\rho C_p \left(\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right) = k \frac{\partial^2 T'}{\partial y'^2} + Q(T' - T_\infty) - \frac{\partial q_y'}{\partial y'} \quad (3)$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - k_r^2 (C' - C_\infty) \quad (4)$$

with the boundary condition

$$t \leq 0 : u' = 0, T' = T_\infty, C' = C_{\infty*} \text{ for all } y' \quad (5)$$

$$t > 0 : u' = u_p = 0, T' = T_w (1 + e^{i\omega' t'}),$$

$$C' = C_w (1 + e^{i\omega' t'}) \text{ at } y' = 0 \quad (6)$$

$$u' = 0, T' \rightarrow T_\infty, C' \rightarrow C_\infty \text{ at } y' \rightarrow \infty \quad (7)$$

where β_t and β_c are the volumetric expansion coefficient for temperature and concentration respectively;

ρ is the fluid density; u_p is the plate velocity; μ is the viscosity of the fluid; μ_m is the magnetic permeability of the fluid; \mathbf{g} the gravitational field vector acting in the reverse direction of the flow; κ is the permittivity of the porous medium; B_o^2 is the uniform magnetic field strength, σ_e is the electrical conductivity of the fluid; k is the thermal conductivity of the fluid; C_p is the specific heat capacity at constant pressure; D is the diffusion coefficient; k_r^2 is the chemical reaction term; ω' is the frequency of oscillation; Q is the heat source/sink; q'_y is the radiative heat flux; α is the optical depth of penetration, σ is the Boltzman constant,

Radiative heat transfer has great influence on high temperature regime and is comparable with convective heat transfer. The analysis of radiation is based the optically thin or thick limits. For an optically thin medium with low density and $\alpha \ll 1$, the radiative heat flux is given in equation (3) can be expressed in the spirit of [26]; as seen in [27] as

$$\frac{\partial q'_y}{\partial y'} = -4\alpha^2 \frac{\partial(T' - T_\infty)}{\partial y'} \tag{8}$$

where $\alpha^2 = \int_0^\infty \delta_1 \gamma \frac{\partial B}{\partial T'}$

and δ_1 is the radiation absorption coefficient, γ is the frequency of the radiation, B is the Planck's function.

Now, by equation (8), equation (3) becomes

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{Q}{\rho C_p} (T' - T_\infty) - \frac{4\alpha^2 T_\infty^3}{\rho C_p} \frac{\partial T'}{\partial y'} \tag{9}$$

More so, being oscillatory, the suction/injection can be prescribed as

$$v' = -v_o \left(1 + \varepsilon A e^{i\omega t'}\right) \tag{10}$$

where $\varepsilon < 1$, a small perturbation parameter, v_o is the uniform suction, A is a position constant, $\varepsilon A \ll 1$; (see [24], [27]). The negative sign in the suction means that the suction is towards the plate. And, by equation (1), suction is not a function of y' but t' .

We introduce the following dimensionless quantities:

$$u = \frac{u'}{v_o}, v = \frac{v'}{v_o}, y = \frac{t' v_o}{4\omega t'}, \omega = \frac{4\nu\omega'}{v_o^2}$$

$$\Theta = \frac{T' - T_\infty}{T_w - T_\infty}, \Phi = \frac{C' - C_\infty}{C_w - C_\infty}, M^2 = \frac{\sigma_e B_o^2 \nu}{\rho v_o^2},$$

$$\chi^2 = \frac{\nu^2}{\kappa v_o^2}, Pr = \frac{\nu}{k}, Sc = \frac{\nu}{D}, N^2 = \frac{\nu Q}{\rho C_p v_o^2}$$

$$Gr = \frac{g \beta_t (T_w - T_\infty)}{v_o^3}, Gc = \frac{g \beta_c (C_w - C_\infty)}{v_o^3},$$

$$Ra^2 = \frac{4\alpha^2 (T_w - T_\infty)}{\rho C_p v_o^2 k} \tag{11}$$

where M^2 is the Hartmann number, N^2 is the heat source parameter, χ^2 is the porosity parameter, Gr is the Grashof number due to temperature gradient, Gc is the Grashof number due concentration gradient, Pr is the Prandtl number, Sc is the Schmidt number, Ra^2 is the Raleigh number, Θ is dimensionless temperature, Φ is the dimensionless concentration, ω is the frequency of oscillation, into the governing equations.

By equations (10) and (11), equations (2), (4), (9), (6) and (7) become

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - M^2 u - Gr\Theta - Gc\Phi \tag{12}$$

$$\frac{\partial \Theta}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right) \frac{\partial \Theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \Theta}{\partial y^2} + \lambda^2 \Theta \tag{13}$$

$$\frac{\partial \Phi}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right) \frac{\partial \Phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \Phi}{\partial y^2} - \delta^2 \Phi \tag{14}$$

with the boundary conditions:

$$u = 0, \Theta = 1 + \varepsilon e^{i\omega t}, \Phi = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \tag{15}$$

$$u = 0, \Theta = 0, \Phi = 0 \text{ at } y \rightarrow \infty \tag{16}$$

An examination of equations (12) – (14) shows that they are non-linear and coupled. To linearize and make them tractable, we seek for oscillating perturbation series expansion solutions of the form:

$$u(y, t) = u_o(y) + \varepsilon u_1(y) e^{i\omega t} + \dots \tag{17}$$

$$\Theta(y, t) = \Theta_o(y) + \varepsilon \Theta_1(y) e^{i\omega t} + \dots \tag{18}$$

$$\Phi(y, t) = \Phi_o(y) + \varepsilon \Phi_1(y) e^{i\omega t} + \dots \tag{19}$$

Substituting equations (17) – (19) appropriately into equations (12) – (16), we have:

for the zeroth order

$$\frac{\partial u_o}{\partial y^2} + \frac{\partial u_o}{\partial y} - M_1^2 u_o = -Gr\Theta_o - Gc\Phi_o \tag{20}$$

$$\frac{1}{Pr} \frac{\partial^2 \Theta_o}{\partial y^2} + \frac{\partial \Theta_o}{\partial y} + \lambda^2 \Theta_o = 0 \tag{21}$$

$$\frac{1}{Sc} \frac{\partial^2 \Phi_o}{\partial y^2} + \frac{\partial \Phi_o}{\partial y} - \delta^2 \Phi_o = 0 \tag{22}$$

where $M_1^2 = M^2 + \chi^2$; $\lambda^2 = N^2 + Ra^2 + \frac{i\omega}{4}$

with the boundary conditions:

$$u_o = 0, \Theta_o = 1, \Phi_o = 1 \text{ at } y = 0 \tag{23}$$

$$u_o = 0, \Theta_o = 0, \Phi_o = 0 \text{ at } y \rightarrow \infty \quad (24)$$

for the first order

$$\frac{\partial u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - M_1^2 u_1 = -A \frac{\partial u_o}{\partial y} = -Gr \Theta_1 - Gc \Phi_1 \quad (25)$$

$$\frac{1}{Pr} \frac{\partial^2 \Theta_1}{\partial y^2} + \frac{\partial \Theta_1}{\partial y} + \lambda^2 \Theta_1 = -A \frac{\partial \Theta_o}{\partial y} \quad (26)$$

$$\frac{1}{Sc} \frac{\partial^2 \Phi_1}{\partial y^2} + \frac{\partial \Phi_1}{\partial y} - \delta^2 \Phi_1 = -A \frac{\partial \Phi_o}{\partial y} \quad (27)$$

with the boundary conditions.

$$u_1 = 0, \Theta_1 = 1, \Phi_1 = 1 \text{ at } y = 0 \quad (28)$$

$$u_1 = 0, \Theta_1 = 0, \Phi_1 = 0 \text{ at } y \rightarrow \infty \quad (29)$$

Additionally, we express the heat transfer rate (Nu), mass transfer rate (Sh) and skin friction as

$$Nu = k \frac{2q_w \sqrt{vt}}{(T_w - T_\infty)} = -\Theta'|_{y=0}, \quad (30)$$

$$Sh = D \frac{2s_w \sqrt{vt}}{(C_w - C_\infty)} = -\Phi'|_{y=0}, \quad (31)$$

$$C_f = \mu u'|_{y=0} \quad (32)$$

where

$$q_w = -k \frac{\partial T}{\partial y} \Big|_{y=0}, \quad s_w = -D \frac{\partial C}{\partial y} \Big|_{y=0}$$

We investigate the unsteady MHD natural convective flow over a vertical porous plate under the influence of Hartmann, Grashof, Prandtl and Schmidt numbers, heat source parameter, thermal radiation and chemical reaction rate, and the results are shown graphically in Figure 1 – Figure 20. The results are obtained using Mathematica 11.0 computational software. For constant value of $\varepsilon = 0.1$, $A = 0.1$; $\chi^2 = 0.1$ and varied values of $M^2 = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2$; $Gr = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0$; $Pr = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2$; $Sc = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2$; $Ra^2 = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2$; $N^2 = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2$; $\delta^2 = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2$, we obtained the

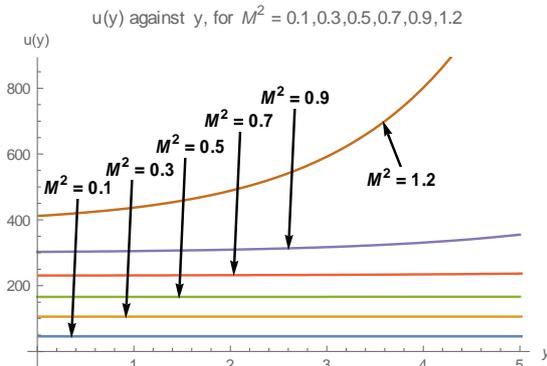


Figure 2. Velocity ($u(y)$)-Hartmann Number (M^2) Profiles

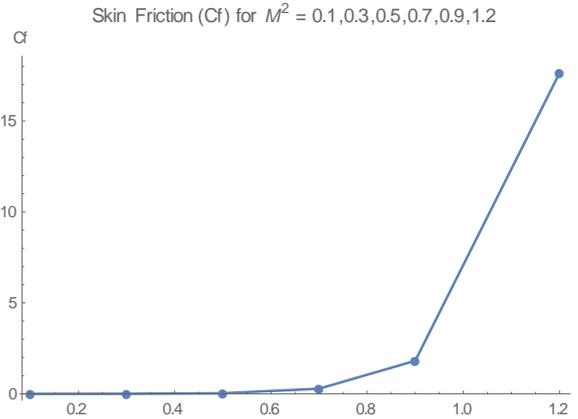


Figure 3. Skin friction (C_f)-Hartmann Number (M^2) Profile

The influences of Hartmann number on the flow are given in Figure 2 and Figure 3. They show that increase in Hartmann number increases both velocity and skin friction. Figure 2 shows that Hartmann number increases the velocity. The fluid is assumed electrolytic; therefore, its particles exist as ions. The motion of the particles in the electric field produces electric currents, which in turn, is affected by the magnetic field to give a mechanical force (the Lorentz force) that modifies the flow. This accounts for what is seen in Figure 2. This result agrees with [23-25]. Furthermore, Figure 3 shows that increase in Hartmann number increases the skin friction. The stress on the wall depends on the strength of the velocity. Thus, the increase in velocity enhances the skin friction.

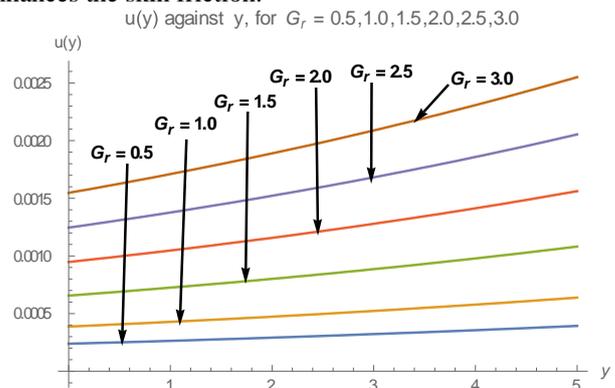


Figure 4. Velocity ($u(y)$)-Grashof Number (Gr) Profile

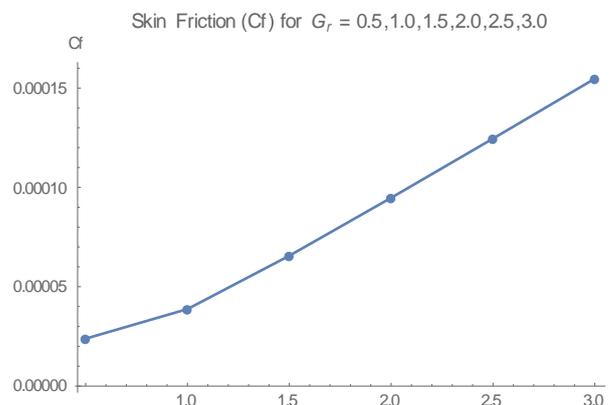


Figure 5. Skin Friction (C_f)-Grashof Number (Gr) Profile

The effects of convective currents (the Grashof numbers) are shown in Figure 4 and Figure 5. They depict that increase in Grashof number increases the velocity and skin friction. Grashof number arises from the gradient between the environmental temperature or concentration and that of the fluid at equilibrium. The gradient increases when the environmental temperature or concentration increases. Importantly, these differential increases energize the fluid particles. The energy gained tends to loose them from the grip of viscosity to become buoyant. Buoyancy enhances velocity. Similarly, increase in Grashof number increases the skin friction, as seen in Figure 5. This may arise from the increase in the velocity. These results agree with [23- 25].

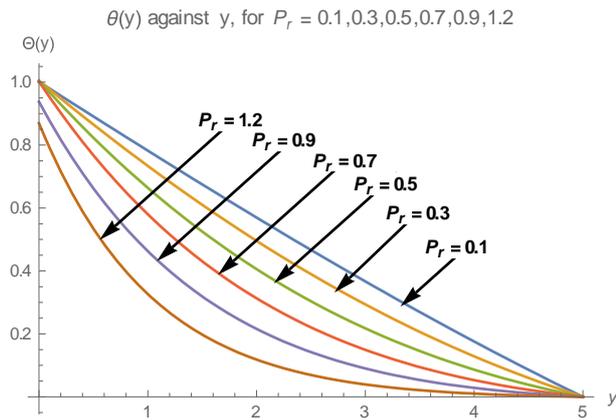


Figure 6. Temperature ($\theta(y)$)-Prandtl Number (Pr) Profiles

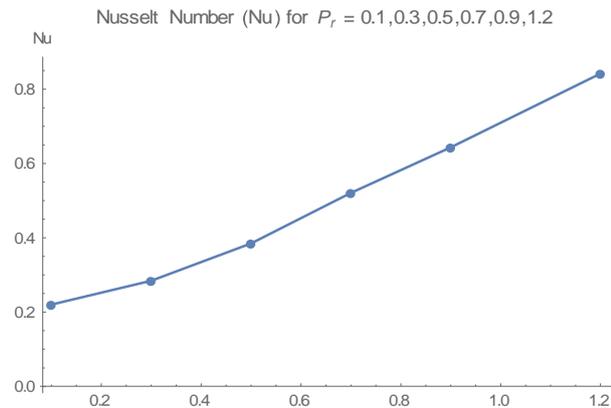


Figure 7. Nusselt Number (Nu)-Prandtl Number (Pr) Profile

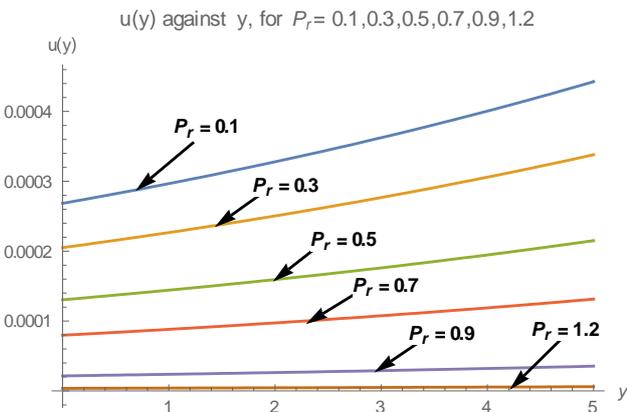


Figure 8. Velocity ($u(y)$)-Prandtl Number (Pr) Profiles

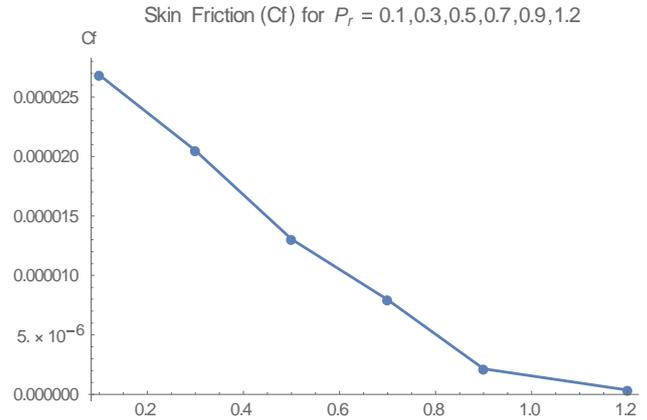


Figure 9. Skin Friction (Cf)-Prandtl Number Profile

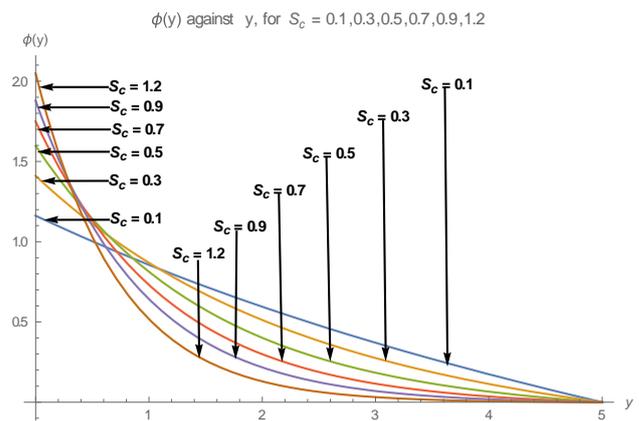


Figure 10. Concentration ($\phi(y)$)-Schmidt Number (Sc) Profiles

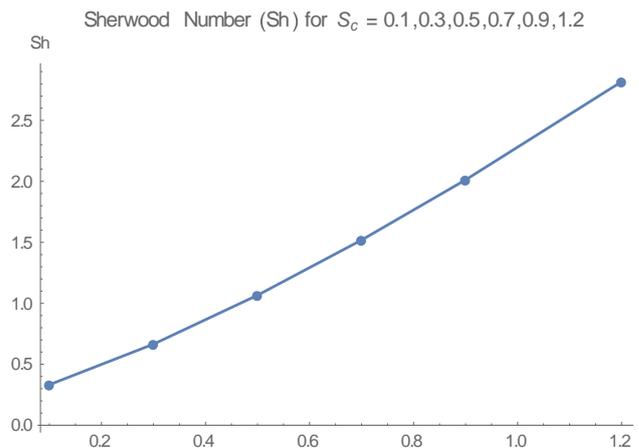


Figure 11. Sherwood Number (Sh)-Schmidt Number (Sc) Profile

The roles of Prandtl number on the flow variables are shown in Figure 6 - Figure 9. The results show that the increase in Prandtl number decreases the temperature, velocity and skin friction but increases the Nusselt number. Prandtl number portrays the interacting relationship between the kinetic viscosity/momentum diffusion and the thermal diffusion coefficient. Prandtl number is small ($Pr \ll 1$) when the thermal diffusivity coefficient is much higher than the momentum diffusivity; it is large ($Pr \gg 1$) when the momentum diffusivity is much higher than the thermal diffusivity coefficient. Heat diffuses faster when the Prandtl

number is small ($Pr \ll 1$). Figure 6 shows that the temperature decreases as Prandtl number increases. More so, Figure 7 shows that increase in Prandtl number increases the rate at which heat is transferred to the fluid. Furthermore, Figure 8 depicts that the velocity decreases with the increase in Prandtl number. These results align with [23,24]. Similarly, Figure 9 depicts that the skin friction decreases as the Prandtl number increases. This is possible, as the decrease in velocity affects the force on the wall. These results are in consonance with [24].

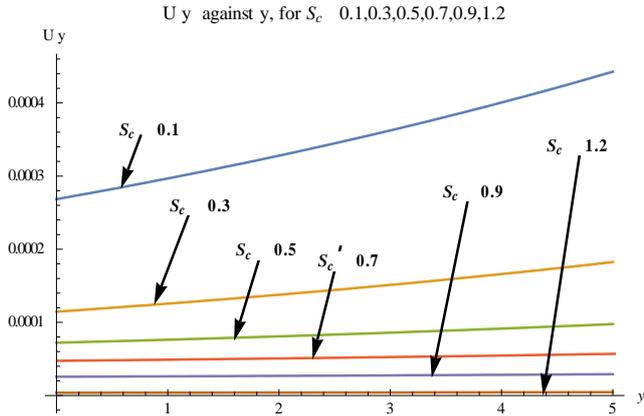


Figure 12. Velocity ($u(y)$)-Schmidt Number (Sc) Profiles

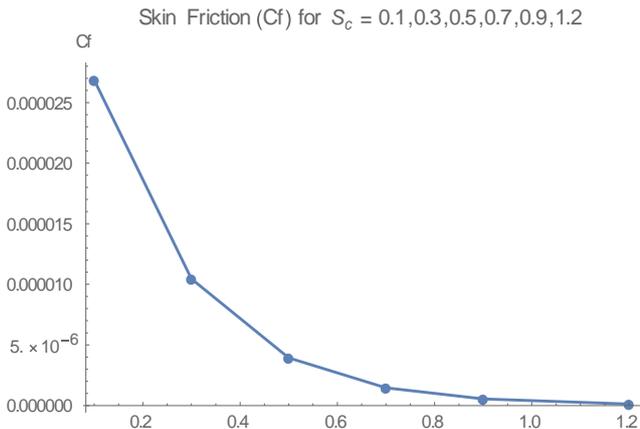


Figure 13. Skin Friction (C_f)-Schmidt Number (Sc) Profile

The significance of Schmidt number on the flow is seen in Figure 10 – Figure 13. The results show that increase in Schmidt number decreases the concentration in part, velocity, and skin friction and concentration (in part), but increases the Sherwood number. Schmidt number arises from the interacting relationship between the momentum diffusivity and chemical diffusivity coefficient of the fluid. Its value is small when the chemical diffusivity coefficient is higher than the momentum diffusivity. The reverse occurs when the momentum diffusivity is higher than the chemical diffusivity coefficient. Figure 10 shows that the concentration increases in the region $y \leq 0.5$ but decreases in the region $y \geq 0.5$ as the Schmidt number increases. The concentration profiles are twisted at $y=0.5$. Additionally, Figure 11 shows that the Sherwood number increases as the Schmidt number increases. Similarly Figure 12 shows that the velocity

decreases as the Schmidt number increases. This may partly be due to the decrease in the concentration, velocity being a function of concentration. This result agrees with [24]. More so, Figure 13 depicts that the skin friction decreases as the Schmidt number increases. This may be due to the decrease in the velocity. This result agrees with [24].

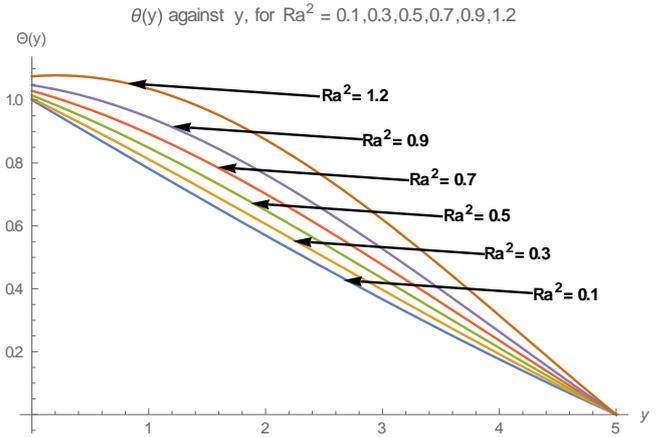


Figure 14. Temperature ($\theta(y)$)-Raleigh Number (Ra^2) Profiles

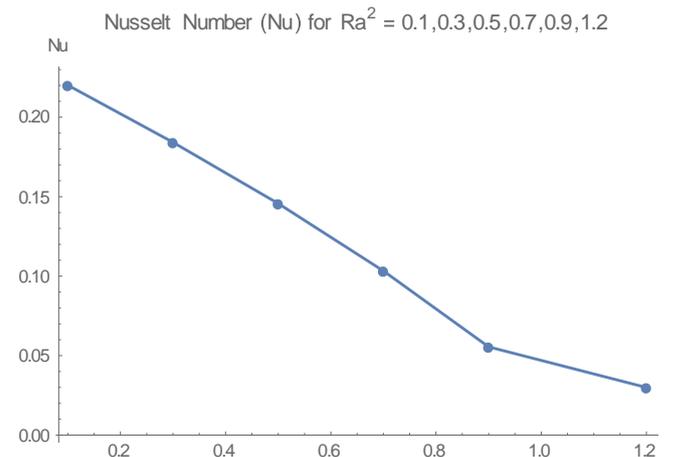


Figure 15. Nusselt Number (Nu)-Raleigh Number (Ra^2) Profile

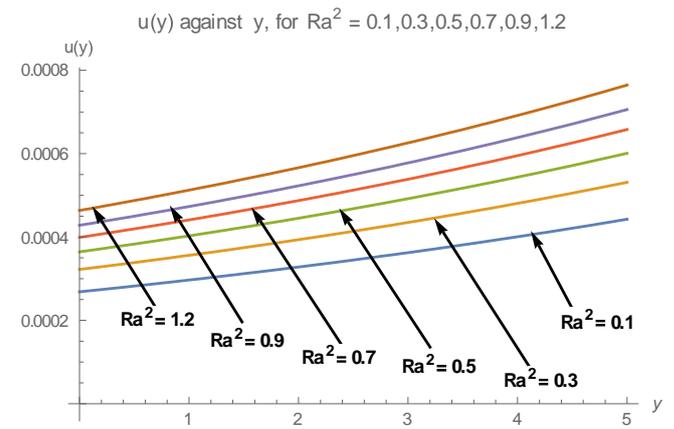


Figure 16. Velocity ($u(y)$)-Raleigh Number (Ra^2) Profile

The effects of Raleigh number (Ra^2) on the flow are shown in Figure 14 - Figure 17. The results show that increase in Raleigh number increases the fluid temperature,

velocity and skin friction but decreases the heat transfer rate (Nusselt number). Raleigh number is the product of thermal convection (Grashof number) and Prandtl number. It is associated with buoyancy-driven flow. Figure 14 shows that increase in the Raleigh number increases the temperature of the fluid. Furthermore, Figure 15 shows that increase in Raleigh number decreases the rate at which heat is transferred to the fluid. Additionally, Figure 16 depicts that increase in Raleigh number increases the flow velocity. This may arise from the increase in the temperature, which is a function of velocity. More so, Figure 17 depicts that increase in Raleigh number increases the skin friction. This may be due to the increase in the velocity, which is a function of force.

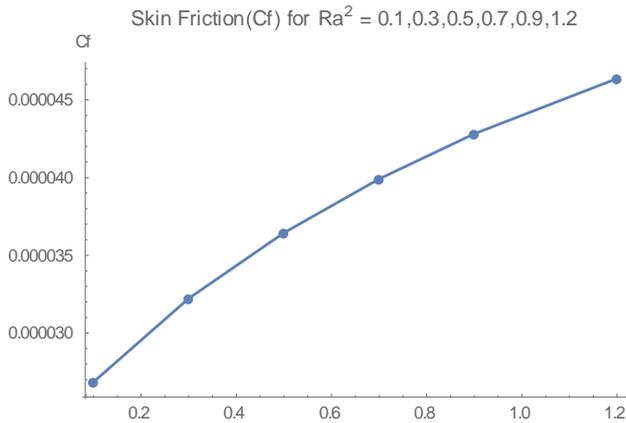


Figure 17. Skin Friction (Cf)-Raleigh Number (Ra^2) Profile

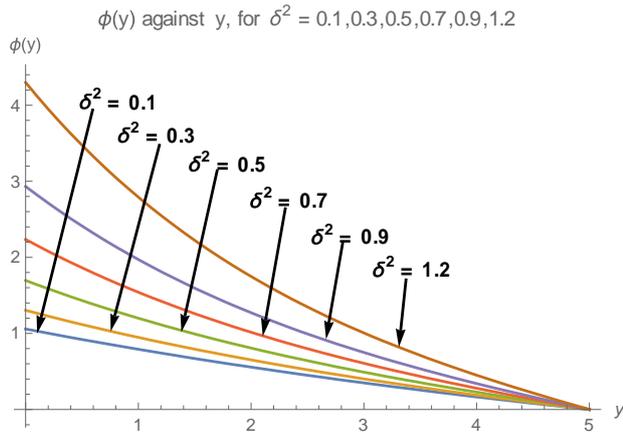


Figure 18. Concentration ($\phi(y)$)-Chemical Reaction rate (δ^2) Profile

In a similar development, the increase in the Heat source parameter (N^2) increases the fluid temperature, velocity and force on the wall but decreases the rate at which heat is transferred to the fluid. Some of these results agree with [25].

The importance of chemical reaction rate on the flow is seen in Figure 18 – Figure 21. The results show that increase in the rate of chemical reaction increases the concentration, Sherwood number, velocity and skin friction. Chemical reaction rate depends on the nature and order of the reactants, concentration, temperature, electromagnetic radiation and the likes. Higher rate of chemical reaction enhances the concentration, Sherwood number, velocity and skin friction,

as seen in Figure 18 - Figure 21. Figure 18 depicts that increase in chemical reaction rate increases the fluid concentration while Figure 19 shows that the Sherwood number increases as the rate of chemical reaction increases. Additionally, Figure 20 shows that increase in the rate of chemical reaction increases the flow velocity. The increase in the velocity may stem from the increase in the concentration, which is a function of velocity. More so, Figure 21 shows that increase in the rate of chemical reaction increases the skin friction. This may be due to the increase in the velocity. These results are in agreement with [24].

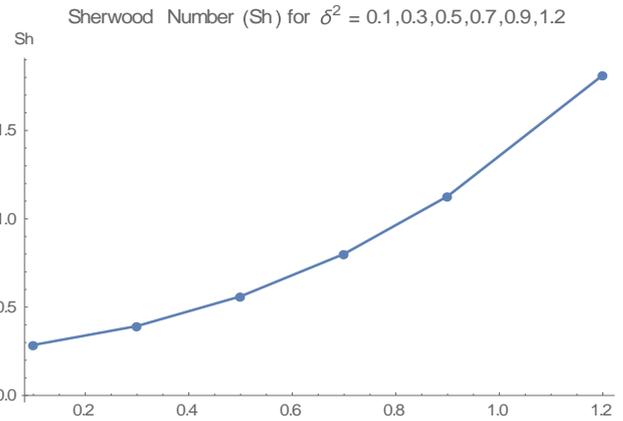


Figure 19. Sherwood Number-Chemical Reaction (δ^2) Profile

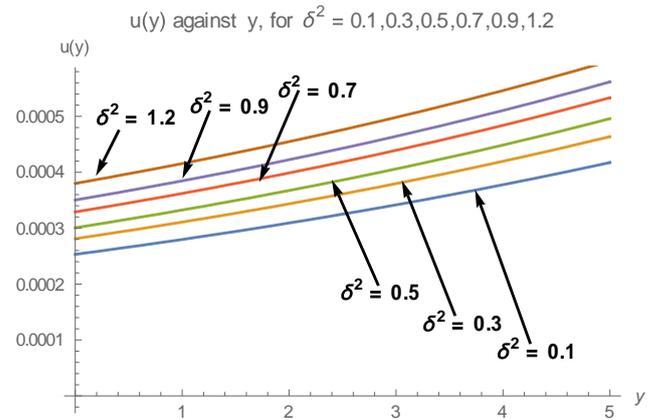


Figure 20. Velocity ($u(y)$)-Chemical Reaction rate (δ^2) Profiles

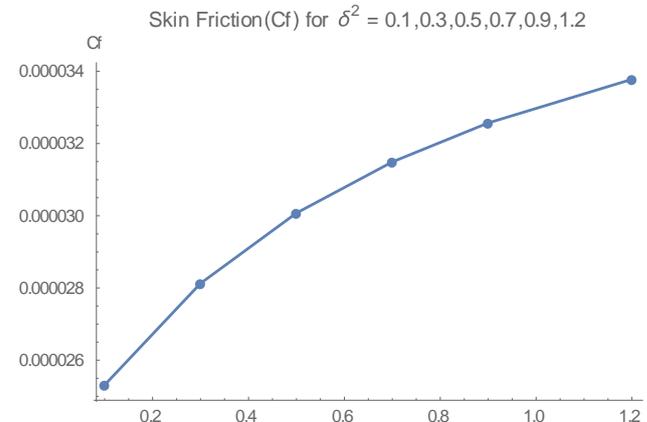


Figure 21. Skin Friction (Cf)-Chemical Reaction (δ^2) Profile

3. Conclusions

The roles of magnetic field, convection, Prandtl number, Schmidt number, Raleigh number, heat source and chemical reaction rate parameters in an unsteady viscous, incompressible, Newtonian flow past a vertically accelerating porous plate are considered. The equations governing the problem are simplified using the oscillating perturbation series expansion solutions. The evolving equations are solved and computed using the Mathematica 11.0 computational software. The results show that increase in:

- Hartmann number increases the velocity and skin friction;
- Grashof number increases the velocity and skin friction;
- Prandtl number decreases the temperature, velocity and skin friction but increases the Nusselt number,
- Schmidt number decreases the velocity and skin friction but increases the rate of mass transfer;
- Raleigh number increases the temperature, velocity and skin friction but decreases the rate of heat transfer;
- the heat source parameter increases the temperature, velocity and skin friction but decreases the rate of heat transfer.
- the chemical reaction rate increases the concentration, Sherwood number, velocity and skin friction..

These results are benchmarked with some existing reports in literature, and are in good agreement.

REFERENCES

- [1] Alam, M.S., Ferdows, M., Ota, M., (2006), Dufour and Soret effects on an unsteady free convective and mass transfer past a semi-infinite vertical porous plate in a porous medium, *Int. J. Appl. Mech. and Eng.* 11(3), 535-545.
- [2] Awad, F.G., Sibanda, P., Narayana, P.M., Motsa, S.S., (2011), Convection from a semi-finite plate in a fluid saturated porous medium with cross-diffusion and radiative heat transfer. *Int. J. Phys. Sc.* 6 (12), 4910-4923. DOI: 10.5897/IJPS11.295.
- [3] Srinivasacharya, D., Mallikarjuna, B., Bhuvanavijaya, R., (2015), Soret and Dufour effects on mixed convection along a vertical wavy surface in a porous medium with variable properties. *Ain Shams Eng. J.* 6(2), 553-564. <http://dx.doi.org/10.1016/j.asej.2014.11.007>.
- [4] Sattar Abdus, M.D., (1994), Free convection and mass transfer flow through a porous medium past an infinite vertical plate with time-dependent temperature and concentration. *Indian J. Pure and Appl. Maths.* 759-766.
- [5] Singh, K.D., Verma, G.N., Kumar, S. (1995), Effects of mass transfer on a 3-D unsteady forced and free convective flow past an infinite vertical plate with periodic suction, *Proceedings Nat. Acad. Sc. , India.* 65(A), 293-308.
- [6] Bakier, A.Y., (2001), Thermal radiation effects on a mixed convection from a vertical surface in a saturated porous medium, *Int. Comm. Heat and Mass Transf.* 28(1), 119-126.
- [7] Halem Attia, (2011), On the effectiveness of porosity on the unsteady mixed convective flow along an infinite vertical porous plate with heat and mass transfer, periodic suction and oscillatory free stream velocity, *Tankang J. Sc. and Eng.* 14(4), 285-291.
- [8] Gnaneshwar, M., Reddy, B.N., Bhaskar Reddy, (2010), Soret and Dufour effects on steady MHD free convection flow past a semi infinite moving vertical plate in a porous medium with viscous dissipation. *Int.J. Appl. Maths and Mech.* 6(1), 1-20.
- [9] Shateyi, S., Motsa, S., Sibanda, P., (2010), The effects of thermal radiation, Hall currents, Soret, and Dufour on MHD flow by mixed convection over a vertical surface in a porous medium. *Mathemat. Problems in Eng.* 1-20. Doi:10.1155/2010/627475.
- [10] Pattnaik, J.R., Dash, G.C., Singh, S., (2017), Diffusion-thermal effects with hall currents on unsteady hydrodynamic flow past an infinite vertical plate. *Alexandria Eng. J.* 56, 13-25.
- [11] Okuyade, W.I.A., Abbey, T.M., Gimlaabel, A.T., (2018), Unsteady MHD free convective chemically reacting flow over a vertical plate with thermal radiation, cross-diffusion and constant suction effects. *Alexander Eng. J.* 3863-3871. <https://doi.org/10.1016/j.aej.2018.02.006>.
- [12] Gersten, K., and Gross, J.F., (1974), Flow and heat transfer along a vertical wall with periodic suction. *ZAMP* 25, 399-408.
- [13] Bansal, J.L., Dave, A., Jat, R.N., (1990), Heat and mass transfer in unsteady hydrodynamic free convective flow past an accelerated vertical plate, *Proceeding Nat. Acad. Sc., India.* 60(A). 211-226.
- [14] Alagoa, K.D., Tay, G., Abbey, T.M., (1998), Radiation and free convective effects on an MHD flow through a porous medium between parallel plates with time-dependent suction. *Astrophys. and Space Sc.* 260(4), 455-468.
- [15] Abdel-Naby, M.A., (2003), Finite difference solution of radiation effects on MHD unsteady free convective flow over a vertical plate with variable surface temperature. *J. Appl. Maths.* 2, 65-86.
- [16] Sarangi, K.C., and Jose, C.B., (2004), Unsteady MHD free convective flow and mass transfer through a porous medium with variable suction and constant heat flux. *J. Indian Acad. Maths.* 26, 115-126.
- [17] Sudheer Babu, M., Narayana, P.V. Satya, (2009), Effects of chemical reaction and thermal radiation on heat and mass transfer through a porous medium with variable suction in the presence of uniform magnetic field, *Journal of Heat and Mass Transf.* V, 219-234.
- [18] Pal, D., and Talukdar, B., (2010), Pertubation analysis of unsteady MHD convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. *Comm. Non-linear and Num. Simulat.* 15(7), 1813-1830.
- [19] Kesavaiah, D. Ch., Narayana, .P.V. Satya, Venkataramana, S., (2011), Effects of chemical reaction and radiation absorption on an unsteady MHD free convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. *J. Appl. Maths and Mech.* 7, 652-699.

- [20] Anand, Rao J., and Shivaiah, S., (2011), Chemical reaction effects on an unsteady MHD flow past a vertical porous plate with constant suction. *Indust. Chem. Eng.* 17, 249-257.
- [21] Narayana, P.V. Satya, Kesavaiah, D. Ch., Venkataramana, S., (2011), Viscous dissipation and thermal radiation effects on an unsteady MHD convection flow past a semi-infinite vertical permeable moving porous plate. *Int. J. Math Archives*, 2(4), 476-487.
- [22] Anand, Rao J., and Shivaiah, S., (2012), Chemical reaction effects on an unsteady MHD flow past a vertical porous plate with viscous dissipation, *J. Appl. Maths and Mech. (Engineering Edition)*. 32(8), 1065-1078.
- [23] Gundagani, M., Sivaiah Sheri, Ajit Paoul, Reddy, M.C.K., (2013), Unsteady MHD free convective flow past a vertical porous plate. *Int. J. Appl. Sc. and Eng*, 11(3), 267-275.
- [24] Sarada, K., and Shanker, B., (2013), The effect of chemical reaction on an unsteady MHD free convective flow past an infinite vertical porous plate with a variable suction. *Int. J. Modern Eng. Res.* 3(2), 725-735.
- [25] Devi, R.L.V.R., Neeraja, A., Reddy, N. Bhasakar, (2016), Effects of radiation on unsteady MHD mixed convection flow past an accelerating vertical porous plate with suction and chemical reaction. *Int. J. Tech. Res. and Applic.* 4(2), 1-8.
- [26] Cogley, A.C.L., Vincenti, W.G., Gilles, E.S., (1968), Differential approximation for radiation heat transfer in a non-linear equation gray gas near equilibrium, *Am. Aeronautic J.* 6,551-553.
- [27] Israel-Cookey, C., Ogulu, A., Omubo-Pepple, V.B., (2003), Influence of viscous dissipation and radiation on an unsteady MHD free convection flow past an infinite vertical plate in a porous medium with time-dependent suction. *Int. J. Heat Mass Transf.* 48, 2305-2311.