American Journal of Mathematics and Statistics 2021, 11(4): 73-93
DOI: 10.5923/j.ajms.20211104.01

The Result of a Square Root Transformation on the Error Part of the Additive Time Series Model

C. A. Igbo[1,*], E. L. Otuonye[2]

[1]Department of Mathematics and Statistics, Federal Polytechnic Nekede, Owerri, Imo State
[2]Department of Statistics, Abia State University Uturu, Abia State

Abstract The square root transformation of the error component of the additive time series model was studied. The distribution of the transformed error component was established and shown that it is a proper probability density function. The properties of the distribution were investigated. It was shown that the mean of the transformed series $E(z) = \ell^{\frac{\sigma^2}{8}}$ is equal to one while the mean of the original series = 0 for $0 < \sigma \le 0.62$. Also, the variance of the transformed was established and given as $Var(z) = \ell^{\frac{\sigma^2}{2}} - \ell^{\frac{\sigma^2}{4}}$ and is four times that of the original series for $0 \le \sigma \le 0.18$ to one decimal place. The transformed series is normally distributed for $0 < \sigma \le 0.14$.

Keywords Additive time series model, Square root transformation, Error component, Means, Variance, Left truncation

1. Introduction

The normal distribution is a well known and most widely used in probability distribution. It is the most common distribution for independent randomly generated variables. The mathematical proof informed us that under certain condition, most distributions tend to the normal distribution. The normal distribution derives its usefulness from the famous Central Limit Theorem. (Hogg R.V and Graig, A.T, [1].

The limits of the normal random variable X is $-\infty < x < \infty$. However, there are situation where this random variable is constrained to be wholly positive or negative variables. This will now force us to study the behavior of the random variable under these restricted conditions. A typical illustration of this is the study of error part of the model. Its square root is constrained to positive values only. Iwueze [2] established the distribution of the truncated normal distribution and is given by;

$$f(x) = \ell^{-\frac{1}{2}\left(\frac{x-1}{\sigma^2}\right)^2} \quad , \quad x \ge 0, \quad \sigma^2 > 0$$

with mean E(x) and variance, Var(x) given by;

$$E(x) = 1 + \frac{\sigma\ \ell^{-\frac{1}{2}\sigma^2}}{\sigma\sqrt{2\pi}\left(1-\phi\left(-\frac{1}{\sigma}\right)\right)} \quad , \quad x \ge 0, \quad \sigma^2 > 0$$

$$Var(x) = \frac{\sigma^2}{2\left(1-\phi\left(-\frac{1}{\sigma}\right)\right)}\left[1 + \Pr\left(x_{(y)}^2 - \frac{1}{\sigma^2}\right)\right] - \left[\frac{\sigma\ell^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi\left(-\frac{1}{\sigma}\right)\right)}\right]^2$$

In the work, the implication of truncating the $N\left(1,\sigma^2\right)$ to the left was investigated. The distribution of the error part of the multiplicative time series model is given by $N\left(1,\sigma^2\right)$.

Okoroafor et al [3] studied the method of handling transformations of the error component when the time series model is additive that is $N\left(0,\sigma^2\right)$. In the work, he detailed the effect of truncating the normal distribution of the irregular component of the additive model and established the condition for a successful transformation. The mean and variance were derived and expressed as: $E(x) = \ell^{\frac{\sigma^2}{8}}$ and $Var(x) = \ell^{\sigma^2}\left(\ell^{\sigma^2} - 1\right)$.

Time series is a well arranged set of data that occur sequentially over time. However, the Trend (T_t), Seasonal

* Corresponding author:
celestine.igbo@yahoo.com (C. A. Igbo)
Received: Jun. 28, 2021; Accepted: Jul. 9, 2021; Published: Jul. 26, 2021
Published online at http://journal.sapub.org/ajms

(S_t), Cyclical (C_t) and error (e_t) are the components of time series. The trend and the cyclical components are usually combined to give the trend-cycle component especially in a short term series Chatfield [4].

For short time series especially, we have three mostly used models in the analysis of descriptive time series.

Multiplicative; $X_t = M_t.S_t.e_t$

Additive; $X_t = M_t + S_t + e_t$

Mixed; $X_t = M_t.S_t + e_t$

Where, M_t is a combination of Trend (T_t) and Cyclical (C_t). components.

Data Transformation: this is a mathematical tool used in converting data from one structure to another. In addition to stabilization of variance, it reduces the variance of the variable and ensures a normally distributed data and additivity of the seasonal effect. It can also be used to

(a) Ensue better organization and give easier and more appropriate utilization.
(b) It improves the quality of the data and in certain cases, reduces the error committed if not transformed.

There are several statistical literatures on the effects of transformations on the error component of the multiplicative time series model. Most of the work centered on establishing the condition under which the transformation would be successful and fulfill the purpose for such transformation. A transformation is successfully achieved when the conditions of the original series are not significantly altered.

Iwueze (2) studied the $N(1,\sigma^2)$ distribution when the values do not admit values less than zero. He showed that the truncated values are greater than zero if and only if $0 < \sigma \le 0.3$. He showed that the mean of the left truncated $N(1,\sigma^2)$ distribution is in terms of the cumulative distribution of the standard normal distribution while that of variance is in terms of the chi-square distribution with one degree of freedom.

Akpanta and Iwueze [5] investigated the effect of the logarithmic transformation on the seasonal and trend-cycle components of the multiplicative time series model. They obtained intervals within which the required seasonal indices of a purely multiplicative time series model would lie for a successful logarithmic transformation.

Iwu et al [6] studied the effect of the six transformations (logarithmic, square, square root, inverse, inverse square root and inverse square) on the trend-cycle (M_t) and seasonal (S_t) components of the multiplicative time series.

Bond and Fox [7] gave genuine reasons for adopting data transformation such as improvement of normality, stability of variance and conversion of scales to interval measurements.

Osborne [8] opined that data transformation are the application of a mathematical modification to the values of the variable and expressed that caution should be exercised in the choice of the type of data transformation to be adopted so that the fundamental structure of the series is not distorted and thereby rendering the interpretation difficult or intractable.

Iwueze and Nwogu [9] studied the relationship between the periodic mean and variance and showed a plot of the periodic mean and standard could determine the appropriate model to adopt in the analysis of time series model.

2. Methodology

In this work, we are interested in the properties of the square root transformation of the additive time model as against the multiplicative which has been extensively studied Hence, the following would be carried out;

(a) The probability density function of the square root transform of the error component would be derived and thereafter show that it is a proper probability density function.
(b) Establish the mean and variance of the distribution and see how it effectively compares with the mean and variance of the original series.
(c) The use of simulation would be used to see how the theoretical mean and variance compares with the simulated values.
(d) To find out if the square root transformation conforms to the normality assumption of the original series under specified condition.
(e) To investigate whether the square root transformation of the additive model compares favorably with the square root transformation of the multiplicative model.

3. Results

3.1. We Would First Find the Probability Density Function of the Square Root Transformation of the Error Component of the Additive Model

Let z be the transformed variable while y is the error variable. The probability density function of the truncated variable according to Okoroafor et al (3) is given by;

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{y} \ell^{-\frac{1}{2\sigma^2}(\ln(y))^2} \quad ; \quad 0 < y < \infty, \ \sigma^2 > 0$$

$$Let \ z = \sqrt{y} \ \Rightarrow z^2 = y; \ \frac{dy}{dz} = 2z$$

$$f(z) = f(y) \ in \ terms \ of$$

$$\left|\frac{dy}{dz}\right| = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{z^2} \ell^{-\frac{1}{2\sigma^2}\left(\ln\left(z^2\right)\right)^2} .2z$$

$$= \frac{2}{\sqrt{2\pi\sigma^2}} \frac{1}{z} \ell^{-\frac{1}{2\sigma^2}\left(\ln\left(z^2\right)\right)^2} \quad ; \quad 0 < z < \infty$$

Simplifying further, we have

$$f(z) = \frac{2}{\sqrt{2\pi\sigma^2}} \frac{1}{z} \ell^{-\frac{1}{2\sigma^2}(2\ln(z))^2}$$

3.2. We Now Show that It is a Proper PDF. For a Proper PDF, We have That

$$\int_0^\infty f(z)dz = 1 \ \ i.e \ \ \int_0^\infty \frac{2}{\sqrt{2\pi\sigma^2}} \frac{1}{z} \ell^{-\frac{1}{2\sigma^2}(2\ln(z))^2} dz = 1$$

$$Let \ \ p = 2\ln(z) \ \ i.e \ \ \ln(z) = \frac{p}{2} \ \ or \ \ z = \ell^{p/2} \ ,$$

$$\frac{dp}{dz} = \frac{2}{z} \Rightarrow \frac{zdp}{2} = dz$$

$$\int f(p)dp = \int_{-\infty}^{\infty} \frac{2}{\sqrt{2\pi\sigma^2}} \frac{1}{z} \ell^{-\frac{1}{2\sigma^2}p^2} z \frac{dp}{2}$$

Where the limit of p is $-\infty < p < \infty$

$$\int_{-\infty}^{\infty} f(p) \ dp \ = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}(p)^2} dp \ \ \text{Or}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}(p-0)^2} dp$$

This is the probability density of a normal distribution with mean 0 and the variance σ^2 whose integral within the limits = 1.

The graph forms of the square root transformed data for the specified values of σ are shown in appendix 1.

3.3. We then Find the $E(z)$

$$E(z) = \int_0^\infty z.\frac{2}{z} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}(2\ln(z))^2} dz$$

$$= \int_0^\infty \frac{2}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}(2\ln(z))^2} dz$$

$$Let \ \ p = 2\ln(z) \ ; \ \ \frac{p}{2} = \ln(z) \ \ \Rightarrow \ \ z = \ell^{p/2} \ ,$$

$$\frac{dp}{dz} = \frac{2}{z}; \ \ dz = \frac{zdp}{2}$$

the limits of p = $-\infty < p < \infty$

$$\int_0^\infty \frac{2}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}(2\ln(z))^2} dz$$

$$= \int \frac{2}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}p^2} . \frac{\ell^{\frac{p}{2}}}{2} dp$$

$$= \int \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left(p^2 - \sigma^2 p\right)} dp$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left[\left(p - \frac{\sigma^2}{2}\right) - \frac{\sigma^4}{4}\right]} dp$$

By completing the square we have

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left(p^2 - \frac{\sigma^2}{2}\right)} . \ell^{-\frac{1}{2\sigma^2}\left(-\frac{\sigma^4}{4}\right)} dp$$

$$= \ell^{\frac{\sigma^2}{8}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left(p - \frac{\sigma^2}{2}\right)^2} dp$$

But $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left(p - \frac{\sigma^2}{2}\right)^2} dp$ is the pdf of $N\left(\frac{\sigma^2}{2}, \sigma^2\right)$ whose integral within the limits = 1.

$$\therefore \ E(z) = \ell^{\frac{\sigma^2}{8}}$$

3.4. Variance of z: Var(z)

$$Let \ \ p = 2\ln(z) \ ; \ \ \frac{p}{2} = \ln(z) \ \ \Rightarrow \ \ z = \ell^{p/2} \ ,$$

$$\frac{dp}{dz} = \frac{2}{z}; \ \ dz = \frac{zdp}{2}$$

$$E\left(z^2\right) = \int \frac{2}{\sqrt{2\pi\sigma^2}} z \ell^{-\frac{1}{2\sigma^2}\left(p^2\right)} \ell^{\frac{p}{2}} \frac{dp}{2}$$

By substituting for z and simplifying, we have

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^p . \ell^{-\frac{1}{2\sigma^2}p^2} dp$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} . \ell^{-\frac{1}{2\sigma^2}p^2 + p} dp$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} . \ell^{-\frac{1}{2\sigma^2}\left(p^2 - 2\sigma^2 p\right)} dp$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} . \ell^{-\frac{1}{2\sigma^2}\left(p^2 - 2\sigma^2 p - \sigma^4 + \sigma^4\right)} dp$$

$$= \ell^{-\frac{1}{2\sigma^2}\left(-\sigma^4\right)} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left(p - \sigma^2\right)^2} dp$$

That is by completion of square

$$= \ell^{-\frac{\sigma^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left(p - \sigma^2\right)^2} dp$$

But $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left(p - \sigma^2\right)^2} dp$ is the pdf of $N\left(\sigma^2, \sigma^2\right)$

$$\therefore \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2\sigma^2}\left(p - \sigma^2\right)^2} dp = 1$$

$$\therefore E\left(z^2\right) = \ell^{-\frac{\sigma^2}{2}}$$

$$Var(z) = E\left(z^2\right) - \left(E(z)\right)^2 = \ell^{-\frac{\sigma^2}{2}} - \left(\ell^{\frac{\sigma^2}{8}}\right)^2$$

$$= \ell^{-\frac{\sigma^2}{2}} - \ell^{\frac{\sigma^2}{4}}$$

3.5. The Mean and Variance of the the Original Series and Square Root Transformed Series are Shown in Table 1 for $0 \leq \sigma \leq 0.5$

Table 1. Comparison of means and variances

S/no	σ	σ^2	E(X)	E(Y)	E(Z)	VAR(X)	VAR(Y)	VAR(Z)	VAR(X)/VAR(Z)
1	0.01	0.0001	0	1.00005	1.000013	0.0001	0.00010	0.000025	3.99985
2	0.02	0.0004	0	1.00020	1.000050	0.0004	0.00040	0.000100	3.99940
3	0.03	0.0009	0	1.00045	1.000113	0.0009	0.00090	0.000225	3.99865
4	0.04	0.0016	0	1.00080	1.000200	0.0016	0.00160	0.000400	3.99760
5	0.05	0.0025	0	1.00125	1.000313	0.0025	0.00251	0.000626	3.99625
6	0.06	0.0036	0	1.00180	1.000450	0.0036	0.00362	0.000901	3.99460
7	0.07	0.0049	0	1.00245	1.000613	0.0049	0.00494	0.001227	3.99266
8	0.08	0.0064	0	1.00321	1.000800	0.0064	0.00646	0.001604	3.99041
9	0.09	0.0081	0	1.00406	1.001013	0.0081	0.00820	0.002031	3.98787
10	0.10	0.0100	0	1.00501	1.001251	0.0100	0.01015	0.002509	3.98503
11	0.11	0.0121	0	1.00607	1.001514	0.0121	0.01232	0.003039	3.98189
12	0.12	0.0144	0	1.00723	1.001802	0.0144	0.01471	0.003619	3.97846
13	0.13	0.0169	0	1.00849	1.002115	0.0169	0.01733	0.004252	3.97473
14	0.14	0.0196	0	1.00985	1.002453	0.0196	0.02019	0.004936	3.97070
15	0.15	0.0225	0	1.01131	1.002816	0.0225	0.02327	0.005673	3.96639
16	0.16	0.0256	0	1.01288	1.003205	0.0256	0.02660	0.006462	3.96178
17	0.17	0.0289	0	1.01455	1.003619	0.0289	0.03018	0.007304	3.95688
18	0.18	0.0324	0	1.01633	1.004058	0.0324	0.03402	0.008199	3.95168
19	0.19	0.0361	0	1.01821	1.004523	0.0361	0.03811	0.009148	3.94620
20	0.20	0.0400	0	1.02020	1.005013	0.0400	0.04248	0.010151	3.94043
21	0.21	0.0441	0	1.02229	1.005528	0.0441	0.04712	0.011209	3.93437
22	0.22	0.0484	0	1.02450	1.006068	0.0484	0.05205	0.012322	3.92803
23	0.23	0.0529	0	1.02680	1.006634	0.0529	0.05728	0.013490	3.92140
24	0.24	0.0576	0	1.02922	1.007226	0.0576	0.06281	0.014715	3.91449
25	0.25	0.0625	0	1.03174	1.007843	0.0625	0.06865	0.015996	3.90730
26	0.26	0.0676	0	1.03438	1.008486	0.0676	0.07483	0.017334	3.89983
27	0.27	0.0729	0	1.03712	1.009154	0.0729	0.08134	0.018730	3.89208
28	0.28	0.0784	0	1.03998	1.009848	0.0784	0.08821	0.020185	3.88405
29	0.29	0.0841	0	1.04295	1.010568	0.0841	0.09544	0.021699	3.87575

S/no	σ	σ^2	E(X)	E(Y)	E(Z)	VAR(X)	VAR(Y)	VAR(Z)	VAR(X)/VAR(Z)
30	0.30	0.0900	0	1.04603	1.011314	0.0900	0.10304	0.023273	3.86717
31	0.31	0.0961	0	1.04922	1.012085	0.0961	0.11104	0.024907	3.85832
32	0.32	0.1024	0	1.05253	1.012882	0.1024	0.11945	0.026603	3.84921
33	0.33	0.1089	0	1.05596	1.013706	0.1089	0.12829	0.028361	3.83982
34	0.34	0.1156	0	1.05950	1.014555	0.1156	0.13756	0.030181	3.83017
35	0.35	0.1225	0	1.06316	1.015430	0.1225	0.14730	0.032066	3.82026
36	0.36	0.1296	0	1.06695	1.016332	0.1296	0.15752	0.034015	3.81008
37	0.37	0.1369	0	1.07085	1.017260	0.1369	0.16824	0.036030	3.79965
38	0.38	0.1444	0	1.07487	1.018214	0.1444	0.17948	0.038111	3.78895
39	0.39	0.1521	0	1.07902	1.019194	0.1521	0.19126	0.040259	3.77801
40	0.40	0.1600	0	1.08329	1.020201	0.1600	0.20362	0.042476	3.76681
41	0.41	0.1681	0	1.08768	1.021235	0.1681	0.21656	0.044763	3.75536
42	0.42	0.1764	0	1.09221	1.022295	0.1764	0.23013	0.047120	3.74366
43	0.43	0.1849	0	1.09686	1.023382	0.1849	0.24435	0.049548	3.73171
44	0.44	0.1936	0	1.10164	1.024495	0.1936	0.25924	0.052050	3.71953
45	0.45	0.2025	0	1.10655	1.025636	0.2025	0.27484	0.054625	3.70710
46	0.46	0.2116	0	1.11160	1.026803	0.2116	0.29119	0.057275	3.69444
47	0.47	0.2209	0	1.11678	1.027997	0.2209	0.30831	0.060002	3.68154
48	0.48	0.2304	0	1.12210	1.029219	0.2304	0.32624	0.062807	3.66840
49	0.49	0.2401	0	1.12755	1.030467	0.2401	0.34502	0.065690	3.65504
50	0.50	0.2500	0	1.13315	1.031743	0.2500	0.36470	0.068654	3.64145
51	0.51	0.2601	0	1.13889	1.033047	0.2601	0.38530	0.071700	3.62763
52	0.52	0.2704	0	1.14477	1.034378	0.2704	0.40689	0.074828	3.61360
53	0.53	0.2809	0	1.15079	1.035736	0.2809	0.42951	0.078042	3.59934
54	0.54	0.2916	0	1.15696	1.037122	0.2916	0.45320	0.081342	3.58487
55	0.55	0.3025	0	1.16329	1.038536	0.3025	0.47801	0.084729	3.57019
56	0.56	0.3136	0	1.16976	1.039978	0.3136	0.50402	0.088206	3.55530
57	0.57	0.3249	0	1.17639	1.041448	0.3249	0.53127	0.091775	3.54020
58	0.58	0.3364	0	1.18317	1.042947	0.3364	0.55982	0.095436	3.52489
59	0.59	0.3481	0	1.19012	1.044473	0.3481	0.58974	0.099191	3.50939
60	0.60	0.3600	0	1.19722	1.046028	0.3600	0.62110	0.103043	3.49368
61	0.61	0.3721	0	1.20448	1.047611	0.3721	0.65398	0.106993	3.47779
62	0.62	0.3844	0	1.21191	1.049223	0.3844	0.68844	0.111044	3.46170
63	0.63	0.3969	0	1.21951	1.050864	0.3969	0.72458	0.115196	3.44542
64	0.64	0.4096	0	1.22728	1.052533	0.4096	0.76247	0.119453	3.42896
65	0.65	0.4225	0	1.23522	1.054232	0.4225	0.80221	0.123816	3.41232
66	0.66	0.4356	0	1.24334	1.055960	0.4356	0.84389	0.128288	3.39550
67	0.67	0.4489	0	1.25163	1.057717	0.4489	0.88761	0.132870	3.37850
68	0.68	0.4624	0	1.26011	1.059503	0.4624	0.93348	0.137564	3.36133
69	0.69	0.4761	0	1.26877	1.061319	0.4761	0.98162	0.142375	3.34400
70	0.70	0.4900	0	1.27762	1.063165	0.4900	1.03214	0.147302	3.32649
71	0.71	0.5041	0	1.28666	1.065040	0.5041	1.08517	0.152350	3.30883
72	0.72	0.5184	0	1.29589	1.066946	0.5184	1.14084	0.157520	3.29101
73	0.73	0.5329	0	1.30532	1.068881	0.5329	1.19929	0.162815	3.27303
74	0.74	0.5476	0	1.31495	1.070847	0.5476	1.26068	0.168238	3.25491
75	0.75	0.5625	0	1.32478	1.072843	0.5625	1.32516	0.173792	3.23663
76	0.76	0.5776	0	1.33482	1.074870	0.5776	1.39290	0.179479	3.21821
77	0.77	0.5929	0	1.34508	1.076928	0.5929	1.46408	0.185301	3.19965
78	0.78	0.6084	0	1.35554	1.079017	0.6084	1.53888	0.191263	3.18095

S/no	σ	σ^2	E(X)	E(Y)	E(Z)	VAR(X)	VAR(Y)	VAR(Z)	VAR(X)/VAR(Z)
79	0.79	0.6241	0	1.36622	1.081136	0.6241	1.61750	0.197368	3.16212
80	0.80	0.6400	0	1.37713	1.083287	0.6400	1.70016	0.203617	3.14316
81	0.81	0.6561	0	1.38826	1.085469	0.6561	1.78707	0.210015	3.12407
82	0.82	0.6724	0	1.39962	1.087683	0.6724	1.87849	0.216564	3.10486
83	0.83	0.6889	0	1.41121	1.089929	0.6889	1.97464	0.223268	3.08552
84	0.84	0.7056	0	1.42305	1.092207	0.7056	2.07581	0.230131	3.06607
85	0.85	0.7225	0	1.43512	1.094516	0.7225	2.18228	0.237156	3.04651
86	0.86	0.7396	0	1.44745	1.096858	0.7396	2.29434	0.244347	3.02684
87	0.87	0.7569	0	1.46002	1.099233	0.7569	2.41231	0.251707	3.00707
88	0.88	0.7744	0	1.47285	1.101640	0.7744	2.53653	0.259240	2.98719
89	0.89	0.7921	0	1.48594	1.104080	0.7921	2.66736	0.266951	2.96721
90	0.90	0.8100	0	1.49930	1.106553	0.8100	2.80518	0.274842	2.94714
91	0.91	0.8281	0	1.51293	1.109060	0.8281	2.95040	0.282919	2.92698
92	0.92	0.8464	0	1.52684	1.111600	0.8464	3.10344	0.291186	2.90673
93	0.93	0.8649	0	1.54103	1.114173	0.8649	3.26476	0.299647	2.88640
94	0.94	0.8836	0	1.55550	1.116781	0.8836	3.43484	0.308306	2.86598
95	0.95	0.9025	0	1.57027	1.119422	0.9025	3.61421	0.317168	2.84549
96	0.96	0.9216	0	1.58534	1.122098	0.9216	3.80341	0.326238	2.82493
97	0.97	0.9409	0	1.60071	1.124808	0.9409	4.00303	0.335521	2.80430
98	0.98	0.9604	0	1.61640	1.127553	0.9604	4.21368	0.345021	2.78360
99	0.99	0.9801	0	1.63240	1.130333	0.9801	4.43602	0.354745	2.76283
100	1.00	1.0000	0	1.64872	1.133148	1.0000	4.67077	0.364696	2.74201

The comparison of the means and variances is shown in table 2.1.

Table 2.1. Comparison of the means and variances

S/no	Number of decimal places	E(Y) = E(Z) = 1	Var(Y) = 4VAR(Z)
1	4	$0 \le \sigma \le 0.01$	Not applicable
2	3	$0 \le \sigma \le 0.06$	$0 \le \sigma \le 0.01$
3	2	$0 \le \sigma \le 0.19$	$0 \le \sigma \le 0.05$
4	1	$0 \le \sigma \le 0.62$	$0 \le \sigma \le 0.18$

Depending on the level of accuracy demanded, we have the following conditions for the mean of the transformed to be equal to 1.0 and variance of the original series is equal to 4 times the variance of the square root transformed series as shown in table 2.1 above.

Table 2.2. Summary of the comparison of the means and variances of the additive and multiplicative

Number of decimal places	Equality of means of additive and multiplicative models	Equality of variances of additive and multiplicative model
1	$0 \le \sigma \le 0.29$	$0 \le \sigma \le 0.29$
2	$0 \le \sigma \le 0.14$	$0 \le \sigma \le 0.10$
3	$0 \le \sigma \le 0.04$	Not applicable
4	$0 \le \sigma \le 0.01$	$0 \le \sigma \le 0.04$

The abridged comparison of the means and variances of the multiplicative and additive models are shown in appendix 4. The condition under which the means and variances of the additive and multiplicative multiplicative models are equal is as given on table 2.2.

From **table 2.2**, it is seen that for $0 \le \sigma \le 0.29$ the means and variances of the two models are the same to one decimal place.

3.6. Simulated Examples

Simulations using Minitab were carried out for values of Z with mean 0 and various values of the standard deviation. For want of space, only simulations of some values of σ were shown. A summary of the analysis of the simulated values mentioned above is however presented on table 2. The corresponding values of the transformed series were obtained for σ = 0.01, 0.02, 0.03, 0.05, 0.1, 0.11, 0.12, 0.13, 0.14, 0.15, 0.16 and 0.17. A summary of the analysis of the simulated values mentioned above is however presented in appendix 2.

Also in this section, considering that the original distribution is $N\left(0,\sigma^2\right)$, we would also test for the normality of the square root distribution using Kolmogorov-Smirnov Goodness-of-Fit test (One sample case).

3.7. Test of Normality

A summary of the test of normality of the square root transformed series is shown on table 3 for specified values of σ.

Table 3. Summary of Kolmogorov-Smirnov Test of Normality for the transformed series (For the specified values of σ)

S/No	σ	Kolmogorov Smirnov value			Approximate p- value	Level of significance	Decision
		D^+	D^-	D			
1	0.01	0.038	0.023	0.038	> 0.15	0.05	Accept Normality
2	0.02	0.024	0.037	0.037	> 0.15	0.05	Accept Normality
3	0.05	0.022	0.033	0.033	> 0.15	0.05	Accept Normality
4	0.10	0.027	0.025	0.027	> 0.15	0.05	Accept Normality
5	0.11	0.030	0.027	0.030	> 0.15	0.05	Accept Normality
6	0.12	0.038	0.019	0.038	> 0.15	0.05	Accept Normality
7	0.13	0.042	0.020	0.042	0.085	0.05	Accept Normality
8	0.14	0.020	0.016	**0.020**	> 0.15	0.05	Accept Normality
9	0.15	0.062	0.031	0.062	< 0.01	0.05	Reject Normality
10	0.16	0.052	0.029	0.059	< 0.01	0.05	Reject Normality
11	0.17	0.069	0.029	0.069	< 0.01	0.05	Reject Normality

4. Summary and Conclusions

a) The probability density function of the square root transformed error component of the additive time series model was established and shown to be

$$f(z)=\begin{cases}\dfrac{2}{\sqrt{2\pi\sigma^2}}\dfrac{1}{z}\ell^{\frac{1}{2}(2\ln(z))^2}, & 0 \le \sigma \le \infty \\ \\ 0, \quad otherwise. \end{cases},$$

b) It was also proved that it is a proper probability density function as shown in **3.2.** The graph forms of the probability density function for the specified values of σ were shown in **appendix 1.** The comparative analysis of the mean and variances was carried out for σ to four decimal places and shown in **table 1**

c) The mean and variance of the distribution was shown on **3.3** and **3.4** to be

$$E(z)=\ell^{\frac{\sigma^2}{8}} \quad and \quad \text{var}\,iance,$$

$$Var(z)=\ell^{\frac{\sigma^2}{2}}-\ell^{\frac{\sigma^2}{4}} \quad respectively$$

d) The Kolmogorov Smirnov test of normality for the transformed series was carried out and summarized on …. It was shown that the series was normal for $0<\sigma\le 0.14$. Therefore, for the structure of normality of the original series not to be violated, all inferences on the transformed series should be done within this value of σ.

e) From **table 2.2**, it is seen that for $0 \le \sigma \le 0.29$ the means and variances of the two models are the same to one decimal place.

Appendix 1: Graph Forms of the Original, Truncated and Square Root Transformed Series for the Specified Values of σ

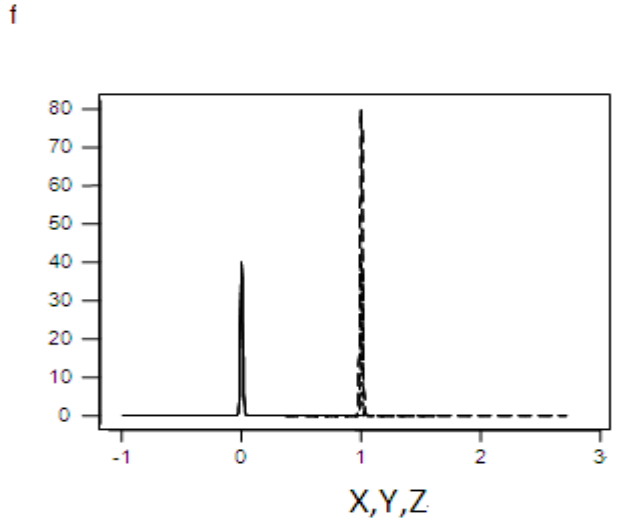


Figure 3.1. Graph of f(x), f(y), f(z) for α=0.01

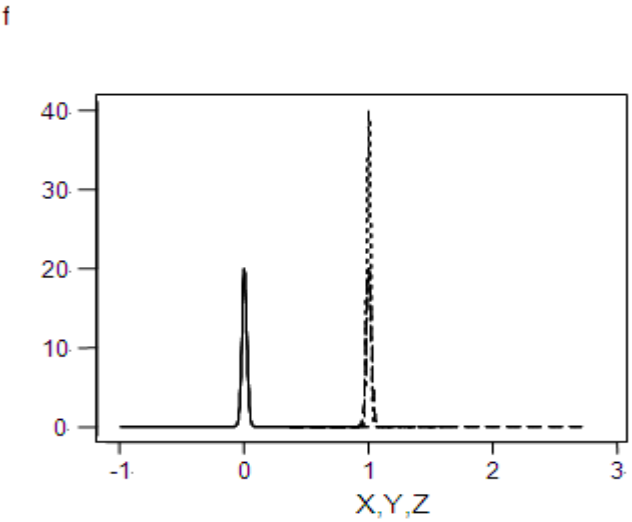


Figure 3.2. Graph of f(x), f(y),f(z) for α=0.02

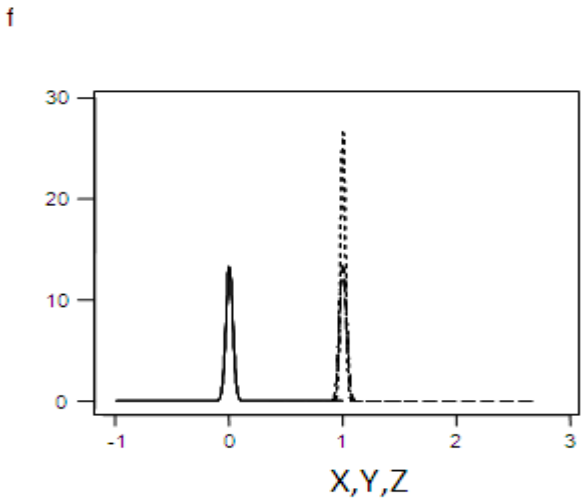


Figure 3.3. Graph of f(x), f(y),f(z) for α=0.03

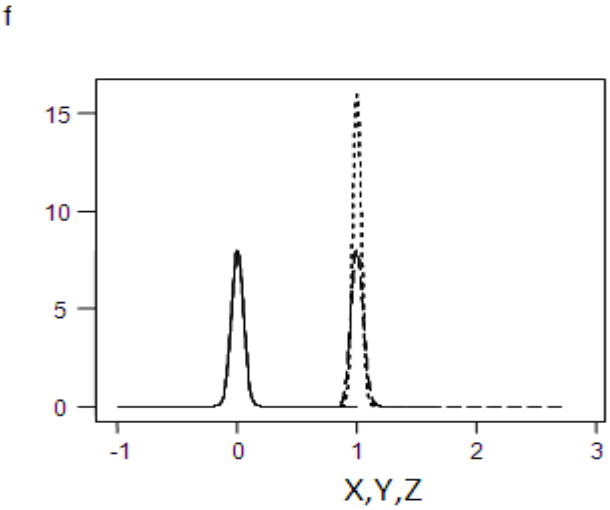


Figure 3.4. Graph of f(x), f(y),f(z) for α=0.05

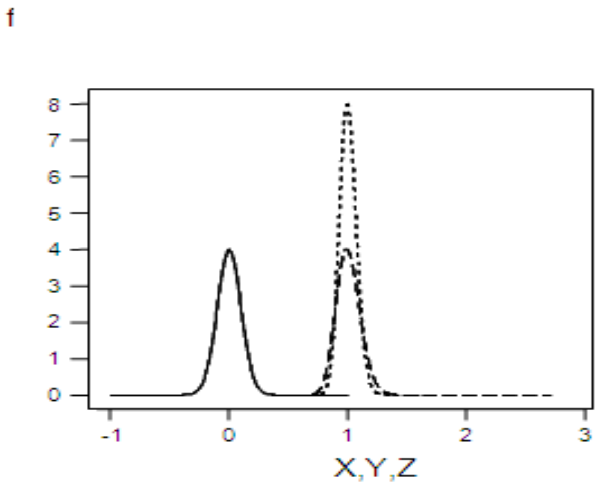


Figure 3.5. Graph of f(x), f(y),f(z) for α=0.1

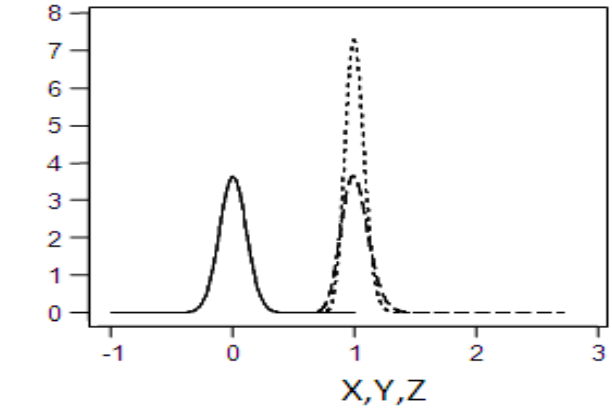


Figure 3.6. Graph of f(x), f(y),f(z) for α=0.11

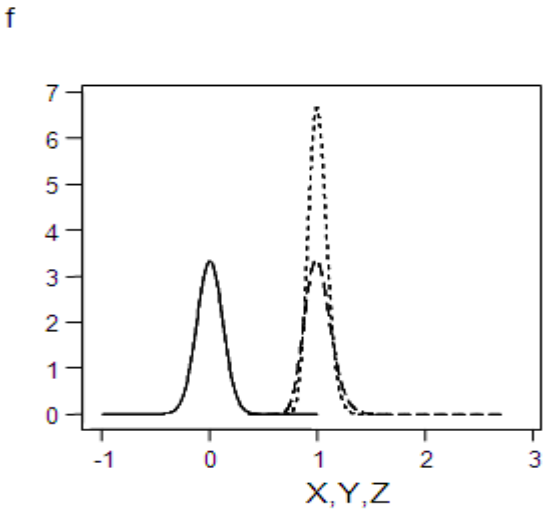


Figure 3.7. Graph of f(x), f(y),f(z) for α=0.12

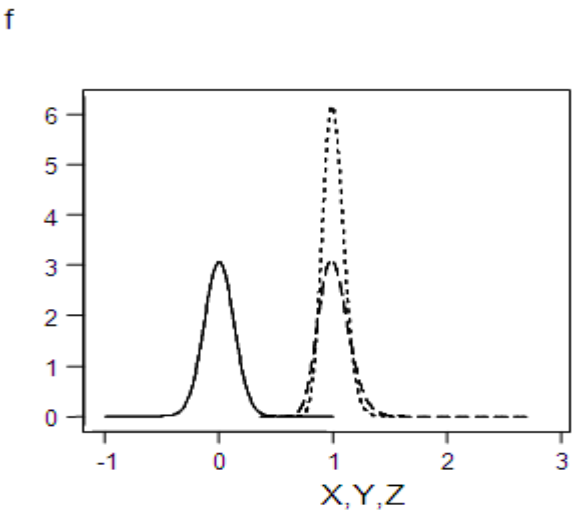


Figure 3.8. Graph of f(x), f(y),f(z) for α=0.13

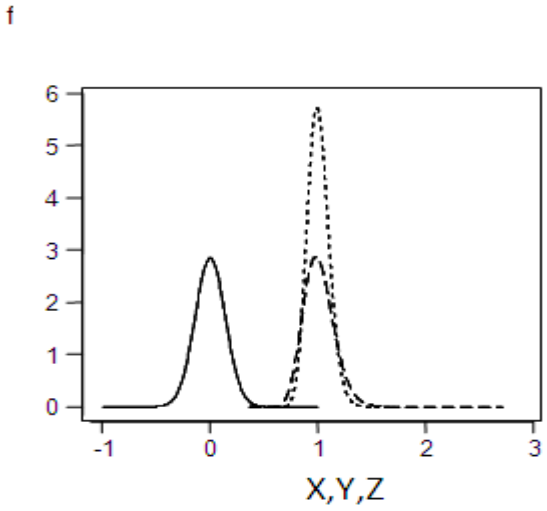


Figure 3.9. Graph of f(x), f(y),f(z) for α=0.14

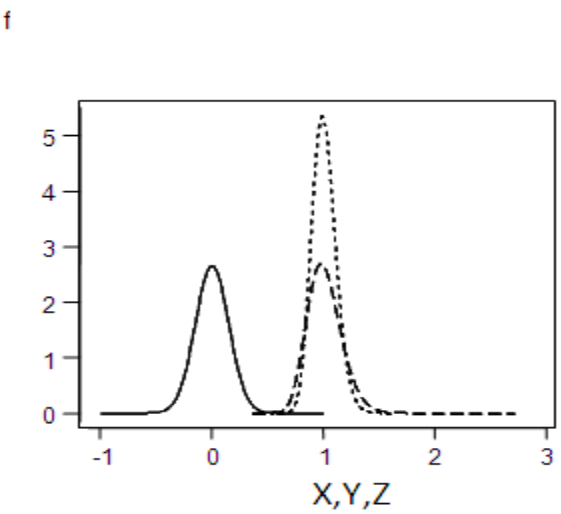


Figure 3.10. Graph of f(x), f(y),f(z) for α=0.15

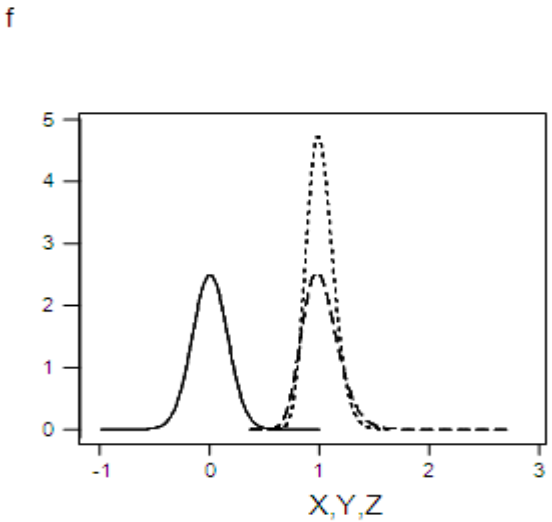


Figure 3.11. Graph of f(x), f(y),f(z) for α=0.16

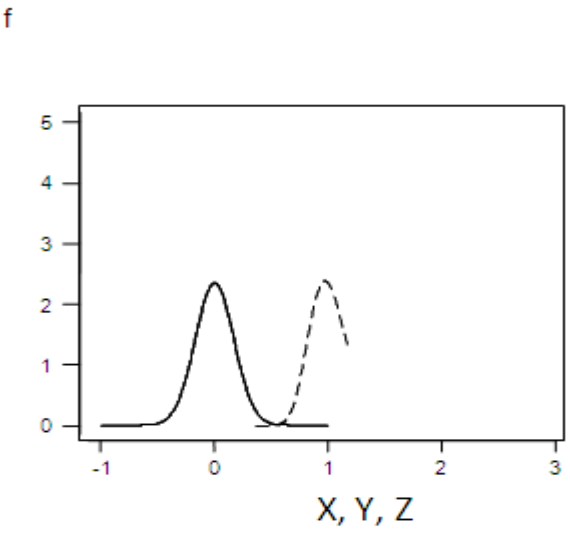
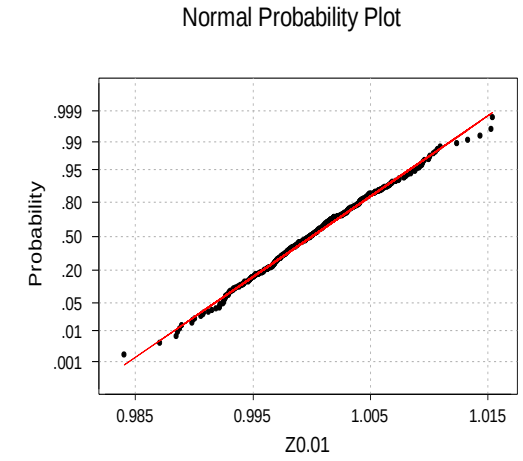


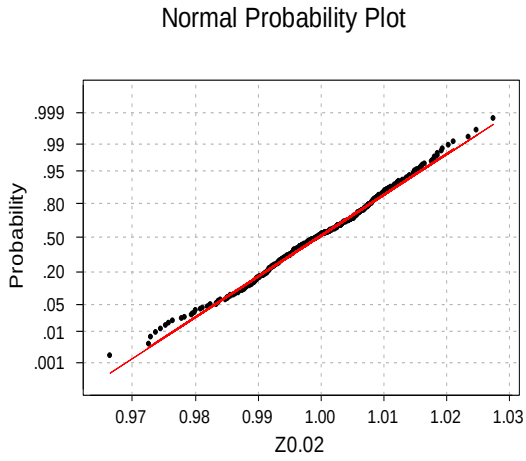
Figure 3.12. Graph of f(x), f(y),f(z) for α=0.17

Appendix 2: Kolmogorov-Smirnov Test of Normality for the Transformed Series (for Specified Values of σ)



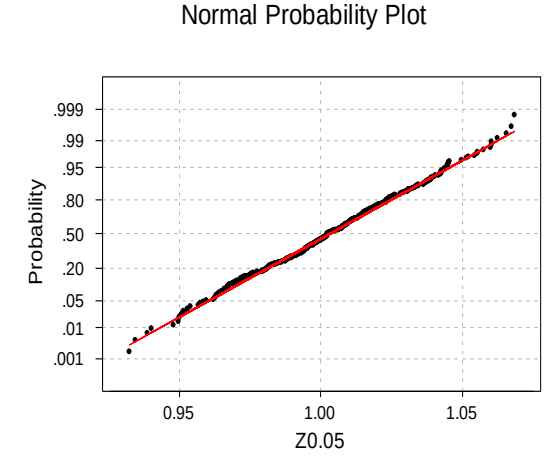
Average: 1.00006
StDev: 0.0050195
N: 400

Kolmogorov-Smirnov Normality Test
D+: 0.038 D-: 0.023 D : 0.038
Approximate P-Value > 0.15



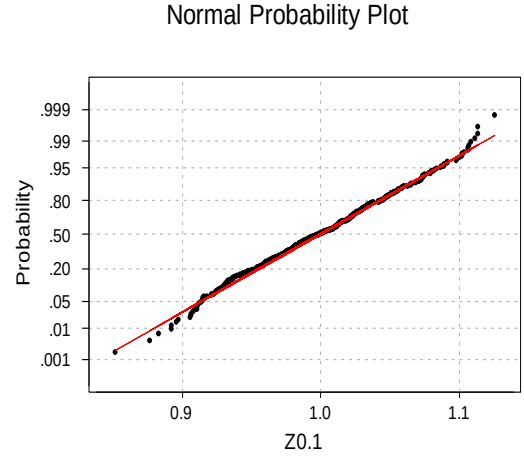
Average: 0.999515
StDev: 0.0098312
N: 400

Kolmogorov-Smirnov Normality Test
D+: 0.024 D-: 0.037 D : 0.037
Approximate P-Value > 0.15



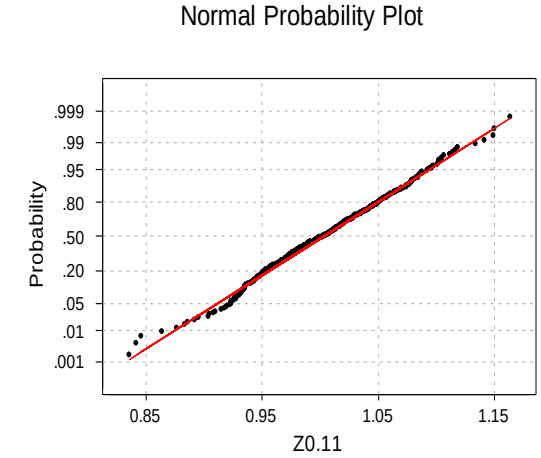
Average: 1.00309
StDev: 0.0255971
N: 400

Kolmogorov-Smirnov Normality Test
D+: 0.022 D-: 0.033 D : 0.033
Approximate P-Value > 0.15



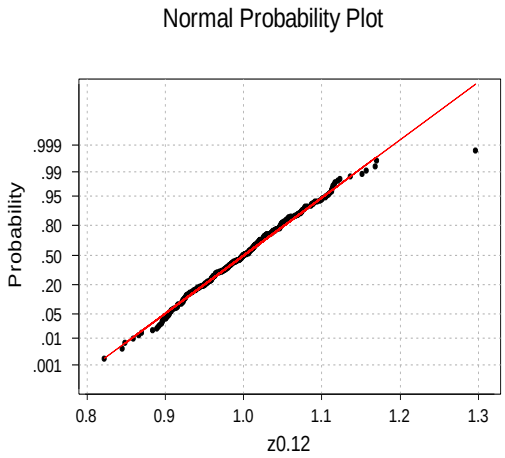
Average: 0.999186
StDev: 0.0512833
N: 400

Kolmogorov-Smirnov Normality Test
D+: 0.027 D-: 0.025 D : 0.027
Approximate P-Value > 0.15



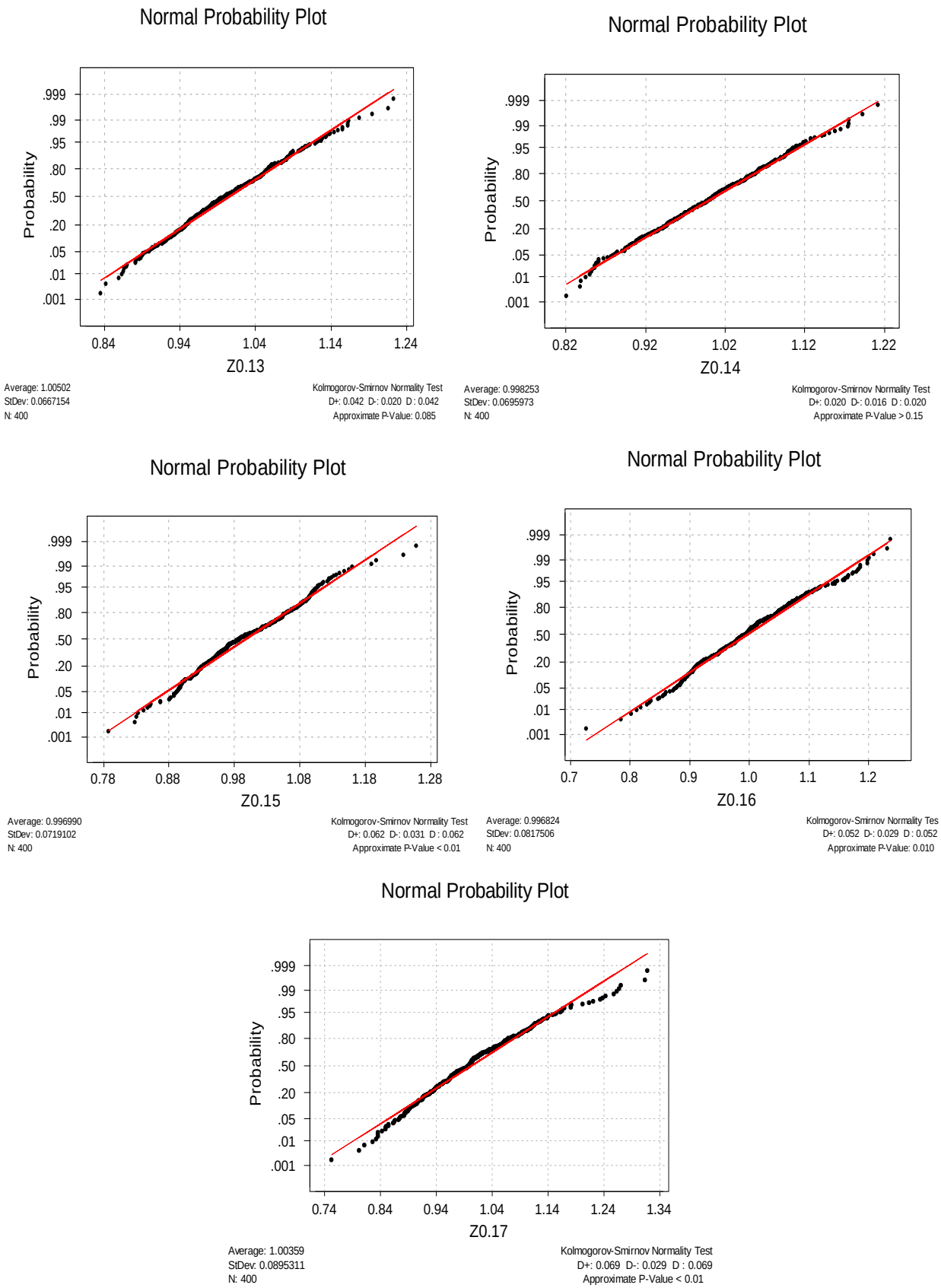
Average: 1.00296
StDev: 0.0547736
N: 400

Kolmogorov-Smirnov Normality Test
D+: 0.030 D-: 0.027 D : 0.030
Approximate P-Value > 0.15



Average: 1.00099
StDev: 0.0608148
N: 400

Kolmogorov-Smirnov Normality Test
D+: 0.038 D-: 0.019 D : 0.038
Approximate P-Value > 0.15


Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.84
0.94
1.04
1.14
1.24
Z0.13
Average: 1.00502
StDev: 0.0667154
N: 400
Kolmogorov-Smirnov Normality Test
D+: 0.042 D-: 0.020 D : 0.042
Approximate P-Value: 0.085
Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.82
0.92
1.02
1.12
1.22
Z0.14
Average: 0.998253
StDev: 0.0695973
N: 400
Kolmogorov-Smirnov Normality Test
D+: 0.020 D-: 0.016 D : 0.020
Approximate P-Value > 0.15
Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.78
0.88
0.98
1.08
1.18
1.28
Z0.15
Average: 0.996990
StDev: 0.0719102
N: 400
Kolmogorov-Smirnov Normality Test
D+: 0.062 D-: 0.031 D : 0.062
Approximate P-Value < 0.01
Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.7
0.8
0.9
1.0
1.1
1.2
Z0.16
Average: 0.996824
StDev: 0.0817506
N: 400
Kolmogorov-Smirnov Normality Tes
D+: 0.052 D-: 0.029 D : 0.052
Approximate P-Value: 0.010
Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.74
0.84
0.94
1.04
1.14
1.24
1.34
Z0.17
Average: 1.00359
StDev: 0.0895311
N: 400
Kolmogorov-Smirnov Normality Test
D+: 0.069 D-: 0.029 D : 0.069
Approximate P-Value < 0.01

Appendix 3: Simulated Values of Originl, Truncated and Square Root Series

S/no	X0.01	Y0.01	Z0.01	X0.02	Y0.02	Z0.02	X0.05	Y0.05	Z0.05
1	0.00110	1.00111	1.00055	-0.01392	0.98618	0.99307	-0.01899	0.98119	0.99055
2	-0.00007	0.99993	0.99997	0.03268	1.03322	1.01648	0.04624	1.04732	1.02339
3	0.00365	1.00365	1.00183	0.01611	1.01624	1.00809	0.04443	1.04543	1.02246
4	-0.01499	0.98512	0.99253	0.02918	1.02961	1.01470	-0.03849	0.96225	0.98094
5	-0.02537	0.97495	0.98740	0.00013	1.00013	1.00007	-0.02042	0.97979	0.98984
6	-0.01646	0.98367	0.99180	0.01622	1.01635	1.00814	0.06027	1.06212	1.03059
7	-0.00456	0.99545	0.99772	0.00135	1.00135	1.00068	-0.01090	0.98916	0.99456
8	-0.01582	0.98430	0.99212	-0.01617	0.98396	0.99195	-0.06944	0.93292	0.96588
9	-0.00634	0.99368	0.99684	-0.03574	0.96489	0.98229	0.05673	1.05837	1.02877
10	-0.00855	0.99149	0.99574	-0.03004	0.97040	0.98509	-0.00977	0.99028	0.99513
11	-0.00796	0.99207	0.99603	0.02287	1.02314	1.01150	-0.02510	0.97521	0.98753
12	0.00577	1.00578	1.00289	0.01135	1.01141	1.00569	0.00842	1.00846	1.00422
13	0.01914	1.01932	1.00961	-0.04915	0.95204	0.97572	0.03252	1.03306	1.01639
14	0.01172	1.01179	1.00588	-0.00063	0.99937	0.99969	-0.01384	0.98626	0.99311
15	-0.00185	0.99815	0.99908	0.01094	1.01100	1.00548	-0.01400	0.98609	0.99302
16	0.00956	1.00961	1.00479	-0.00115	0.99885	0.99942	0.01454	1.01465	1.00730
17	0.00621	1.00623	1.00311	0.01954	1.01973	1.00982	-0.05624	0.94531	0.97227
18	-0.02554	0.97478	0.98731	-0.01275	0.98733	0.99364	0.04077	1.04161	1.02059
19	-0.02165	0.97858	0.98923	-0.00341	0.99660	0.99830	0.04616	1.04725	1.02335
20	-0.01442	0.98568	0.99282	0.03123	1.03172	1.01574	-0.02254	0.97771	0.98879
21	0.00921	1.00925	1.00462	0.01047	1.01053	1.00525	0.08330	1.08687	1.04253
22	-0.00100	0.99900	0.99950	-0.01512	0.98500	0.99247	0.01734	1.01749	1.00871
23	0.01926	1.01944	1.00968	-0.01207	0.98800	0.99398	-0.08242	0.92088	0.95963
24	-0.00568	0.99433	0.99716	0.02693	1.02729	1.01356	0.04627	1.04735	1.02340
25	0.00611	1.00613	1.00306	-0.00078	0.99922	0.99961	0.04818	1.04936	1.02438
26	0.00241	1.00242	1.00121	-0.01957	0.98062	0.99026	-0.03335	0.96720	0.98346
27	0.00296	1.00297	1.00148	-0.04402	0.95693	0.97823	0.07682	1.07984	1.03916
28	0.01190	1.01197	1.00597	-0.01717	0.98298	0.99145	-0.03667	0.96399	0.98183
29	0.00392	1.00393	1.00196	-0.01659	0.98355	0.99174	0.03924	1.04002	1.01982
30	-0.00058	0.99942	0.99971	-0.03288	0.96765	0.98369	-0.07676	0.92611	0.96235
31	-0.00470	0.99531	0.99765	-0.02239	0.97786	0.98887	-0.08311	0.92025	0.95930
32	-0.01851	0.98166	0.99079	0.01639	1.01652	1.00823	0.02192	1.02216	1.01102
33	-0.00356	0.99644	0.99822	0.00331	1.00331	1.00166	0.07179	1.07442	1.03654
34	0.01110	1.01116	1.00557	0.01674	1.01688	1.00841	0.00133	1.00133	1.00066
35	0.00366	1.00366	1.00183	-0.00234	0.99766	0.99883	-0.01768	0.98247	0.99120
36	0.00561	1.00563	1.00281	0.03494	1.03555	1.01762	-0.00952	0.99053	0.99525
37	-0.00295	0.99706	0.99853	-0.00460	0.99541	0.99770	-0.06216	0.93973	0.96940
38	0.01621	1.01634	1.00814	-0.02381	0.97647	0.98816	0.03230	1.03283	1.01628
39	-0.00633	0.99369	0.99684	-0.00076	0.99924	0.99962	0.03323	1.03379	1.01676
40	0.00093	1.00093	1.00046	-0.00675	0.99327	0.99663	-0.00790	0.99213	0.99606
41	0.00131	1.00132	1.00066	0.01137	1.01144	1.00570	-0.00743	0.99260	0.99629
42	-0.02273	0.97753	0.98870	-0.01307	0.98702	0.99349	0.00714	1.00717	1.00358
43	-0.00731	0.99272	0.99635	0.01171	1.01178	1.00587	-0.01334	0.98674	0.99335
44	0.00571	1.00573	1.00286	0.01765	1.01781	1.00886	0.01455	1.01466	1.00730
45	-0.00324	0.99677	0.99838	0.00960	1.00964	1.00481	-0.03786	0.96285	0.98125
46	-0.00174	0.99826	0.99913	-0.01641	0.98372	0.99183	0.05579	1.05738	1.02829
47	-0.01969	0.98051	0.99021	-0.01754	0.98261	0.99127	-0.07409	0.92858	0.96363

S/no	X0.01	Y0.01	Z0.01	X0.02	Y0.02	Z0.02	X0.05	Y0.05	Z0.05
48	-0.00331	0.99669	0.99834	-0.02038	0.97983	0.98986	0.08586	1.08966	1.04387
49	0.00911	1.00915	1.00456	-0.00981	0.99024	0.99511	-0.00016	0.99984	0.99992
50	0.01470	1.01481	1.00738	0.04029	1.04111	1.02035	-0.03887	0.96188	0.98075
51	-0.00956	0.99048	0.99523	-0.05000	0.95123	0.97531	-0.06659	0.93558	0.96725
52	0.00781	1.00784	1.00391	0.00350	1.00351	1.00175	0.07471	1.07758	1.03806
53	-0.00019	0.99981	0.99991	-0.02414	0.97615	0.98800	0.05540	1.05697	1.02809
54	-0.00985	0.99020	0.99509	-0.00739	0.99264	0.99631	-0.04189	0.95897	0.97927
55	0.01082	1.01088	1.00543	-0.05180	0.94951	0.97443	0.00584	1.00585	1.00292
56	-0.01447	0.98563	0.99279	-0.01244	0.98764	0.99380	0.01291	1.01299	1.00647
57	-0.01651	0.98362	0.99178	-0.02133	0.97890	0.98939	-0.03612	0.96452	0.98210
58	0.00535	1.00537	1.00268	0.00094	1.00094	1.00047	0.08341	1.08698	1.04258
59	0.00892	1.00896	1.00447	-0.00906	0.99099	0.99548	0.06792	1.07028	1.03454
60	0.01118	1.01125	1.00561	0.01828	1.01845	1.00918	-0.06079	0.94102	0.97006
61	0.00045	1.00045	1.00023	0.01694	1.01708	1.00851	0.06101	1.06291	1.03097
62	0.00285	1.00285	1.00142	-0.03237	0.96815	0.98395	0.02313	1.02340	1.01163
63	-0.00783	0.99220	0.99609	0.01028	1.01033	1.00515	0.00811	1.00814	1.00406
64	-0.01926	0.98093	0.99042	-0.01509	0.98502	0.99248	0.00394	1.00394	1.00197
65	-0.02382	0.97646	0.98816	0.00663	1.00666	1.00332	-0.02847	0.97193	0.98586
66	-0.01021	0.98984	0.99491	0.00636	1.00638	1.00319	0.03573	1.03637	1.01802
67	-0.00609	0.99393	0.99696	0.02399	1.02429	1.01207	0.01322	1.01330	1.00663
68	0.00171	1.00172	1.00086	-0.02150	0.97873	0.98931	0.08161	1.08504	1.04165
69	0.00720	1.00723	1.00361	-0.00810	0.99193	0.99596	0.06676	1.06904	1.03394
70	0.00999	1.01004	1.00501	0.05402	1.05550	1.02738	0.02686	1.02722	1.01352
71	-0.00428	0.99573	0.99786	-0.01093	0.98913	0.99455	0.03578	1.03643	1.01805
72	0.01022	1.01027	1.00512	0.00670	1.00672	1.00335	-0.12640	0.88126	0.93875
73	0.00186	1.00187	1.00093	-0.00853	0.99150	0.99574	-0.06387	0.93813	0.96857
74	-0.00229	0.99771	0.99885	-0.01125	0.98881	0.99439	0.00036	1.00036	1.00018
75	-0.01112	0.98894	0.99446	-0.00898	0.99106	0.99552	-0.05372	0.94770	0.97350
76	-0.00563	0.99439	0.99719	-0.04477	0.95622	0.97786	-0.00344	0.99657	0.99828
77	0.00136	1.00136	1.00068	0.02815	1.02854	1.01417	-0.04588	0.95515	0.97732
78	0.00195	1.00195	1.00097	-0.01977	0.98042	0.99016	0.05775	1.05945	1.02930
79	-0.00442	0.99559	0.99779	0.01722	1.01737	1.00865	0.08752	1.09146	1.04473
80	-0.00303	0.99698	0.99849	-0.02822	0.97218	0.98599	0.04335	1.04430	1.02191
81	0.00450	1.00451	1.00225	0.00700	1.00702	1.00350	0.00328	1.00329	1.00164
82	0.00460	1.00462	1.00231	0.02318	1.02345	1.01166	0.00448	1.00449	1.00224
83	-0.00985	0.99020	0.99509	-0.02074	0.97947	0.98968	0.10622	1.11207	1.05455
84	0.00798	1.00801	1.00400	0.00453	1.00454	1.00227	-0.06617	0.93597	0.96746
85	0.00289	1.00290	1.00145	-0.02486	0.97545	0.98765	0.02186	1.02210	1.01099
86	0.00726	1.00729	1.00364	-0.01381	0.98628	0.99312	-0.06737	0.93485	0.96688
87	-0.00192	0.99809	0.99904	-0.01815	0.98202	0.99097	-0.02715	0.97322	0.98652
88	-0.00127	0.99873	0.99937	-0.04167	0.95919	0.97938	-0.01218	0.98790	0.99393
89	-0.00559	0.99442	0.99721	0.00381	1.00382	1.00191	-0.07003	0.93236	0.96559
90	0.01174	1.01181	1.00589	-0.00668	0.99334	0.99666	0.03288	1.03343	1.01658
91	0.00835	1.00839	1.00419	-0.00548	0.99454	0.99727	0.11652	1.12357	1.05999
92	0.02505	1.02537	1.01260	-0.00016	0.99984	0.99992	-0.03833	0.96239	0.98102
93	-0.00619	0.99383	0.99691	-0.02230	0.97795	0.98891	0.03502	1.03564	1.01766
94	-0.00416	0.99585	0.99792	0.01752	1.01767	1.00880	0.13246	1.14163	1.06847
95	-0.00589	0.99413	0.99706	0.00211	1.00211	1.00105	-0.05471	0.94676	0.97302
96	-0.01199	0.98808	0.99402	0.00746	1.00749	1.00374	0.04563	1.04669	1.02308

S/no	X0.01	Y0.01	Z0.01	X0.02	Y0.02	Z0.02	X0.05	Y0.05	Z0.05
97	-0.02231	0.97794	0.98891	-0.00888	0.99116	0.99557	0.03019	1.03065	1.01521
98	0.00874	1.00878	1.00438	0.01213	1.01220	1.00608	0.02962	1.03007	1.01492
99	-0.00423	0.99578	0.99789	0.02592	1.02626	1.01304	0.04817	1.04935	1.02438
100	-0.00128	0.99873	0.99936	0.00737	1.00739	1.00369	-0.03881	0.96193	0.98078

S/no	X0.10	Y0.10	Z0.10	X0.11	Y0.11	Z0.11	X0.12	Y0.12	Z0.12
1	-0.12153	0.88557	0.94104	-0.13143	0.87684	0.93640	0.17325	1.18916	1.09049
2	-0.03666	0.96400	0.98184	-0.10761	0.89797	0.94762	0.11321	1.11986	1.05824
3	0.00603	1.00605	1.00302	-0.14350	0.86632	0.93076	-0.18540	0.83077	0.91147
4	0.04440	1.04540	1.02245	0.13450	1.14396	1.06956	0.05275	1.05416	1.02673
5	0.00077	1.00077	1.00039	-0.05583	0.94570	0.97247	-0.08261	0.92071	0.95954
6	0.15849	1.17174	1.08247	0.02317	1.02344	1.01165	0.21713	1.24250	1.11468
7	-0.11452	0.89179	0.94435	0.16661	1.18129	1.08687	-0.04343	0.95750	0.97852
8	-0.09057	0.91341	0.95572	0.02082	1.02104	1.01046	0.20931	1.23283	1.11033
9	-0.08177	0.92148	0.95994	-0.12849	0.87942	0.93778	-0.16025	0.85193	0.92300
10	-0.01770	0.98246	0.99119	0.13432	1.14376	1.06947	-0.15002	0.86069	0.92773
11	-0.05666	0.94492	0.97207	0.22076	1.24702	1.11670	0.01544	1.01556	1.00775
12	-0.00216	0.99785	0.99892	-0.05835	0.94332	0.97125	0.07343	1.07619	1.03740
13	0.04176	1.04264	1.02110	0.04307	1.04401	1.02177	0.09066	1.09489	1.04637
14	-0.13210	0.87626	0.93608	0.02613	1.02647	1.01315	0.03351	1.03408	1.01690
15	0.20244	1.22439	1.10652	-0.16677	0.84640	0.92000	0.19093	1.21037	1.10017
16	0.09959	1.10472	1.05106	-0.13361	0.87493	0.93538	-0.17839	0.83661	0.91467
17	-0.22858	0.79566	0.89200	-0.09667	0.90786	0.95282	0.20623	1.22904	1.10862
18	0.01333	1.01342	1.00669	0.08344	1.08702	1.04260	0.01525	1.01536	1.00765
19	0.07067	1.07323	1.03597	0.07392	1.07672	1.03765	-0.03750	0.96319	0.98142
20	0.08509	1.08882	1.04346	-0.20001	0.81872	0.90483	0.00239	1.00239	1.00120
21	-0.00830	0.99174	0.99586	-0.15918	0.85284	0.92349	-0.00025	0.99975	0.99988
22	0.19741	1.21825	1.10374	-0.13446	0.87418	0.93498	0.09827	1.10326	1.05036
23	0.03875	1.03951	1.01956	-0.00243	0.99757	0.99879	-0.09336	0.91087	0.95439
24	0.14188	1.15244	1.07352	0.00185	1.00185	1.00092	-0.14963	0.86103	0.92792
25	-0.12860	0.87932	0.93772	-0.10244	0.90263	0.95007	0.14906	1.16074	1.07738
26	-0.07147	0.93102	0.96489	0.20229	1.22420	1.10644	0.15668	1.16962	1.08149
27	0.07060	1.07315	1.03593	-0.29246	0.74643	0.86396	-0.00037	0.99963	0.99981
28	-0.14445	0.86550	0.93032	0.00841	1.00844	1.00421	0.06989	1.07239	1.03556
29	-0.17924	0.83591	0.91428	0.03244	1.03297	1.01635	-0.07686	0.92602	0.96230
30	0.09016	1.09435	1.04611	0.27835	1.32095	1.14933	-0.00648	0.99354	0.99676
31	0.02020	1.02041	1.01015	-0.12449	0.88294	0.93965	-0.24683	0.78127	0.88390
32	0.09886	1.10392	1.05067	0.09706	1.10193	1.04973	0.02506	1.02538	1.01261
33	0.13674	1.14653	1.07076	-0.04882	0.95235	0.97589	0.03945	1.04023	1.01992
34	0.00407	1.00408	1.00204	-0.04167	0.95919	0.97938	0.01966	1.01986	1.00988
35	-0.02958	0.97086	0.98532	-0.13353	0.87500	0.93541	0.21426	1.23894	1.11308
36	0.04224	1.04314	1.02134	-0.14792	0.86250	0.92871	-0.13627	0.87260	0.93413
37	0.14029	1.15061	1.07267	0.04534	1.04638	1.02293	0.09527	1.09996	1.04879
38	0.09894	1.10400	1.05071	0.12426	1.13231	1.06410	-0.05497	0.94651	0.97289
39	0.05867	1.06043	1.02977	-0.09879	0.90593	0.95180	-0.02821	0.97218	0.98599
40	0.02054	1.02075	1.01032	-0.04061	0.96020	0.97990	-0.02798	0.97241	0.98611
41	0.08566	1.08943	1.04376	0.10206	1.10745	1.05236	0.04206	1.04296	1.02125
42	-0.18510	0.83102	0.91160	0.17562	1.19199	1.09178	0.08830	1.09232	1.04514
43	-0.12376	0.88359	0.93999	0.09005	1.09423	1.04605	-0.01907	0.98111	0.99051

S/no	X0.10	Y0.10	Z0.10	X0.11	Y0.11	Z0.11	X0.12	Y0.12	Z0.12
44	0.04301	1.04395	1.02174	0.03471	1.03532	1.01751	0.01412	1.01422	1.00708
45	-0.12963	0.87842	0.93724	0.01547	1.01559	1.00776	-0.08479	0.91871	0.95849
46	0.04641	1.04750	1.02348	0.15225	1.16445	1.07910	0.04927	1.05050	1.02494
47	-0.01664	0.98350	0.99172	-0.11831	0.88842	0.94256	-0.02382	0.97646	0.98816
48	0.13943	1.14962	1.07220	0.11279	1.11939	1.05801	0.22008	1.24617	1.11632
49	0.14130	1.15177	1.07320	0.18538	1.20368	1.09712	0.14878	1.16042	1.07723
50	0.09183	1.09618	1.04699	-0.07767	0.92527	0.96191	-0.16079	0.85147	0.92275
51	-0.19846	0.81999	0.90553	-0.04315	0.95777	0.97866	-0.22596	0.79775	0.89317
52	-0.04157	0.95928	0.97943	-0.10048	0.90440	0.95100	-0.17364	0.84060	0.91684
53	-0.01545	0.98467	0.99231	-0.02268	0.97757	0.98872	-0.00262	0.99738	0.99869
54	-0.00130	0.99870	0.99935	-0.00173	0.99827	0.99913	0.21606	1.24117	1.11408
55	-0.11285	0.89329	0.94514	-0.13943	0.86986	0.93266	-0.03954	0.96123	0.98042
56	-0.03372	0.96684	0.98328	-0.04057	0.96024	0.97992	-0.03551	0.96511	0.98240
57	-0.18443	0.83158	0.91191	0.14258	1.15325	1.07389	-0.00296	0.99704	0.99852
58	-0.07162	0.93088	0.96482	-0.13944	0.86984	0.93265	-0.04399	0.95697	0.97825
59	-0.02109	0.97913	0.98951	0.08410	1.08773	1.04294	-0.08337	0.92001	0.95917
60	-0.15339	0.85779	0.92617	0.05804	1.05975	1.02944	0.02885	1.02927	1.01453
61	-0.14177	0.86782	0.93157	-0.06123	0.94060	0.96985	0.10075	1.10600	1.05166
62	0.02360	1.02388	1.01187	-0.00635	0.99367	0.99683	-0.14676	0.86350	0.92925
63	0.10775	1.11377	1.05535	-0.10975	0.89606	0.94660	0.19870	1.21982	1.10445
64	0.08967	1.09381	1.04585	0.01477	1.01488	1.00741	0.29174	1.33875	1.15705
65	0.15631	1.16919	1.08129	-0.07274	0.92984	0.96428	0.09085	1.09511	1.04647
66	0.11399	1.12075	1.05865	0.08452	1.08820	1.04317	-0.07850	0.92450	0.96151
67	0.02502	1.02533	1.01259	-0.05109	0.95019	0.97478	0.07391	1.07671	1.03765
68	0.10370	1.10926	1.05322	-0.05129	0.95000	0.97468	0.25549	1.29109	1.13626
69	-0.03975	0.96103	0.98032	0.08223	1.08571	1.04197	-0.14967	0.86100	0.92790
70	-0.01706	0.98309	0.99151	-0.02196	0.97828	0.98908	0.02043	1.02064	1.01027
71	0.04762	1.04877	1.02409	-0.00813	0.99190	0.99594	-0.16427	0.84851	0.92115
72	-0.03854	0.96219	0.98091	0.09927	1.10437	1.05089	0.09294	1.09740	1.04757
73	-0.17537	0.83915	0.91605	-0.02060	0.97961	0.98975	0.05001	1.05128	1.02532
74	-0.09447	0.90986	0.95386	0.14342	1.15421	1.07434	0.03026	1.03072	1.01524
75	0.02997	1.03042	1.01510	0.02640	1.02675	1.01329	0.31475	1.36991	1.17043
76	-0.01035	0.98970	0.99484	0.08314	1.08669	1.04245	-0.13593	0.87291	0.93429
77	0.01325	1.01334	1.00665	0.04608	1.04716	1.02331	-0.19547	0.82245	0.90689
78	-0.08190	0.92136	0.95988	0.02562	1.02595	1.01289	0.03474	1.03535	1.01752
79	-0.18640	0.82994	0.91101	-0.05684	0.94475	0.97198	0.05443	1.05593	1.02759
80	0.03051	1.03098	1.01537	0.00748	1.00751	1.00375	0.05219	1.05357	1.02644
81	0.04930	1.05054	1.02496	0.07075	1.07332	1.03601	0.16583	1.18037	1.08645
82	-0.05146	0.94984	0.97460	0.16402	1.17824	1.08547	-0.28646	0.75092	0.86655
83	0.03966	1.04046	1.02003	0.14979	1.16159	1.07777	0.04227	1.04317	1.02136
84	-0.11096	0.89498	0.94603	0.02620	1.02655	1.01319	-0.11346	0.89274	0.94485
85	-0.16414	0.84862	0.92121	-0.05693	0.94466	0.97194	0.02100	1.02122	1.01055
86	0.08508	1.08880	1.04346	-0.02511	0.97520	0.98752	0.14568	1.15683	1.07556
87	0.04360	1.04457	1.02204	0.30218	1.35281	1.16310	0.05703	1.05869	1.02893
88	0.19413	1.21425	1.10193	0.05541	1.05697	1.02809	-0.16086	0.85141	0.92272
89	-0.14718	0.86313	0.92905	0.03612	1.03678	1.01822	0.21385	1.23844	1.11285
90	0.06627	1.06851	1.03369	-0.00564	0.99437	0.99718	0.10855	1.11466	1.05578
91	0.11557	1.12251	1.05948	0.12377	1.13176	1.06384	0.02508	1.02540	1.01262
92	0.10274	1.10821	1.05271	-0.11270	0.89341	0.94521	0.14228	1.15290	1.07373

S/no	X0.10	Y0.10	Z0.10	X0.11	Y0.11	Z0.11	X0.12	Y0.12	Z0.12
93	0.04042	1.04125	1.02041	-0.04780	0.95332	0.97638	0.04234	1.04325	1.02140
94	0.00342	1.00342	1.00171	0.14928	1.16099	1.07749	0.09652	1.10133	1.04944
95	-0.10748	0.89810	0.94768	-0.14591	0.86423	0.92964	0.11338	1.12006	1.05833
96	0.09166	1.09599	1.04690	0.07812	1.08126	1.03983	0.01434	1.01444	1.00719
97	-0.11932	0.88752	0.94208	-0.00812	0.99192	0.99595	-0.03860	0.96213	0.98088
98	0.09877	1.10381	1.05063	-0.03730	0.96339	0.98152	-0.10106	0.90388	0.95073
99	-0.05074	0.95052	0.97495	0.05992	1.06176	1.03042	-0.07280	0.92979	0.96426
100	0.14473	1.15573	1.07505	-0.09982	0.90500	0.95132	0.03703	1.03772	1.01869

S/no	X0.13	Y0.13	Z0.13	X0.14	Y0.14	Z0.14	X0.15	Y0.15	Z0.15
1	0.10696	1.11289	1.05494	-0.18299	0.83278	0.91257	-0.07592	0.92689	0.96275
2	0.02065	1.02087	1.01038	0.01432	1.01442	1.00719	0.16282	1.17682	1.08481
3	-0.01555	0.98457	0.99225	-0.03098	0.96950	0.98463	-0.06780	0.93445	0.96667
4	0.08545	1.08921	1.04365	-0.05297	0.94841	0.97386	-0.06808	0.93418	0.96653
5	0.05283	1.05425	1.02677	-0.07251	0.93006	0.96439	0.16461	1.17894	1.08579
6	0.07256	1.07526	1.03695	-0.10663	0.89886	0.94808	0.20585	1.22857	1.10841
7	-0.09942	0.90536	0.95150	-0.22456	0.79887	0.89379	-0.21702	0.80491	0.89717
8	0.06504	1.06720	1.03305	0.09460	1.09921	1.04843	-0.15806	0.85380	0.92401
9	-0.28976	0.74844	0.86512	-0.10595	0.89947	0.94841	-0.15392	0.85734	0.92593
10	0.01295	1.01304	1.00650	-0.09215	0.91197	0.95497	-0.24825	0.78016	0.88327
11	0.00740	1.00742	1.00370	0.04600	1.04707	1.02326	-0.02662	0.97373	0.98678
12	0.00643	1.00645	1.00322	0.10296	1.10845	1.05283	-0.01770	0.98245	0.99119
13	0.12881	1.13747	1.06652	-0.28390	0.75284	0.86767	-0.01222	0.98786	0.99391
14	0.02732	1.02769	1.01375	-0.07668	0.92618	0.96238	-0.23347	0.79178	0.88982
15	-0.13059	0.87758	0.93679	-0.05298	0.94840	0.97386	-0.21380	0.80751	0.89861
16	-0.17220	0.84181	0.91750	-0.02045	0.97976	0.98983	-0.15671	0.85495	0.92463
17	0.01880	1.01897	1.00944	0.08636	1.09020	1.04413	0.15329	1.16567	1.07966
18	0.11073	1.11710	1.05693	-0.00639	0.99363	0.99681	0.21707	1.24244	1.11465
19	-0.06969	0.93268	0.96576	-0.22210	0.80083	0.89489	-0.08258	0.92074	0.95955
20	0.11256	1.11914	1.05790	-0.20359	0.81580	0.90322	-0.21495	0.80658	0.89810
21	0.24029	1.27161	1.12766	0.11681	1.12391	1.06014	0.08971	1.09385	1.04587
22	-0.28075	0.75522	0.86903	0.16026	1.17381	1.08343	-0.09933	0.90544	0.95155
23	0.39057	1.47783	1.21566	-0.25346	0.77611	0.88097	-0.09566	0.90877	0.95329
24	-0.30549	0.73677	0.85835	0.04666	1.04776	1.02360	-0.17685	0.83791	0.91537
25	-0.00285	0.99715	0.99858	0.17194	1.18761	1.08978	-0.06296	0.93898	0.96901
26	0.22515	1.25252	1.11916	-0.08079	0.92239	0.96041	-0.07115	0.93132	0.96505
27	0.04113	1.04199	1.02078	0.07980	1.08307	1.04071	-0.13426	0.87436	0.93507
28	0.09868	1.10372	1.05058	-0.17521	0.83928	0.91612	-0.05739	0.94423	0.97171
29	0.03475	1.03536	1.01752	-0.11174	0.89427	0.94566	0.16449	1.17879	1.08572
30	-0.03511	0.96550	0.98260	0.00972	1.00977	1.00487	-0.06691	0.93528	0.96710
31	0.14649	1.15776	1.07599	0.02283	1.02309	1.01148	-0.02619	0.97415	0.98699
32	-0.02115	0.97908	0.98948	0.18060	1.19794	1.09450	0.23059	1.25934	1.12220
33	-0.06326	0.93870	0.96887	-0.20195	0.81713	0.90395	0.18166	1.19921	1.09508
34	0.11512	1.12201	1.05925	0.03944	1.04023	1.01991	-0.03385	0.96671	0.98322
35	-0.08514	0.91839	0.95833	-0.08035	0.92279	0.96062	-0.05103	0.95025	0.97481
36	-0.18860	0.82812	0.91001	0.08890	1.09297	1.04545	-0.05961	0.94213	0.97064
37	0.11247	1.11903	1.05784	-0.02892	0.97149	0.98564	0.12114	1.12878	1.06244
38	0.00352	1.00352	1.00176	-0.08237	0.92093	0.95965	-0.06408	0.93793	0.96847
39	-0.03880	0.96194	0.98079	0.04528	1.04632	1.02290	0.10441	1.11005	1.05359

S/no	X0.13	Y0.13	Z0.13	X0.14	Y0.14	Z0.14	X0.15	Y0.15	Z0.15
40	-0.07404	0.92863	0.96365	-0.04023	0.96057	0.98009	-0.20097	0.81793	0.90440
41	-0.11488	0.89147	0.94418	0.09292	1.09737	1.04755	-0.12545	0.88210	0.93920
42	-0.19439	0.82334	0.90738	0.03204	1.03256	1.01615	0.13729	1.14717	1.07106
43	0.02852	1.02893	1.01436	-0.16124	0.85108	0.92254	-0.15924	0.85279	0.92347
44	0.10927	1.11547	1.05616	0.06740	1.06972	1.03427	-0.06543	0.93666	0.96781
45	0.04932	1.05055	1.02497	0.18152	1.19904	1.09501	0.01587	1.01599	1.00796
46	0.15968	1.17314	1.08312	-0.05232	0.94902	0.97418	0.16731	1.18212	1.08725
47	0.09435	1.09894	1.04830	-0.32017	0.72602	0.85207	0.02190	1.02214	1.01101
48	-0.04312	0.95780	0.97867	-0.19732	0.82093	0.90605	-0.20374	0.81568	0.90315
49	-0.04776	0.95336	0.97640	0.30570	1.35757	1.16515	0.10138	1.10669	1.05199
50	0.08379	1.08740	1.04279	-0.04378	0.95716	0.97835	0.08793	1.09192	1.04495
51	-0.05682	0.94476	0.97199	0.17280	1.18863	1.09024	-0.02964	0.97080	0.98529
52	-0.04389	0.95706	0.97829	-0.08935	0.91453	0.95631	-0.34504	0.70819	0.84154
53	-0.01557	0.98455	0.99224	-0.31797	0.72762	0.85301	-0.13858	0.87059	0.93305
54	0.23142	1.26039	1.12267	-0.05039	0.95086	0.97512	-0.18249	0.83319	0.91279
55	-0.14667	0.86358	0.92929	0.02342	1.02370	1.01178	0.18714	1.20579	1.09809
56	0.07612	1.07909	1.03879	-0.21372	0.80758	0.89865	0.02297	1.02324	1.01155
57	0.16972	1.18497	1.08856	0.11552	1.12246	1.05946	0.13965	1.14987	1.07232
58	0.00127	1.00127	1.00063	0.07788	1.08099	1.03971	-0.07172	0.93079	0.96477
59	0.24117	1.27274	1.12816	0.04674	1.04785	1.02364	0.00708	1.00711	1.00355
60	-0.25326	0.77627	0.88106	-0.02691	0.97345	0.98663	-0.06493	0.93714	0.96806
61	0.02896	1.02938	1.01458	-0.18532	0.83084	0.91151	-0.12018	0.88676	0.94168
62	-0.06588	0.93625	0.96760	0.07853	1.08170	1.04005	-0.08479	0.91870	0.95849
63	-0.23041	0.79421	0.89119	0.05476	1.05628	1.02776	0.04713	1.04826	1.02385
64	0.16756	1.18241	1.08739	0.14338	1.15417	1.07432	-0.03400	0.96657	0.98314
65	-0.13395	0.87463	0.93522	-0.00343	0.99657	0.99829	0.06781	1.07016	1.03448
66	-0.01848	0.98169	0.99080	-0.07211	0.93043	0.96459	0.10651	1.11238	1.05470
67	-0.08351	0.91988	0.95911	-0.01709	0.98305	0.99149	0.05632	1.05794	1.02856
68	0.25085	1.28512	1.13363	-0.00646	0.99356	0.99677	0.07650	1.07950	1.03899
69	-0.23635	0.78950	0.88854	-0.10706	0.89847	0.94788	0.15829	1.17151	1.08236
70	-0.11645	0.89007	0.94344	0.18206	1.19969	1.09530	0.09130	1.09559	1.04671
71	-0.10204	0.90300	0.95026	0.26556	1.30416	1.14200	-0.21615	0.80561	0.89756
72	0.14336	1.15415	1.07431	-0.04099	0.95984	0.97971	-0.01631	0.98382	0.99188
73	0.03403	1.03462	1.01716	-0.05721	0.94440	0.97180	0.06581	1.06802	1.03345
74	-0.16796	0.84538	0.91945	0.03886	1.03962	1.01962	-0.01134	0.98872	0.99434
75	0.09318	1.09765	1.04769	-0.19315	0.82436	0.90794	-0.17806	0.83690	0.91482
76	0.12047	1.12803	1.06209	0.22927	1.25769	1.12147	-0.12823	0.87965	0.93790
77	0.00256	1.00257	1.00128	-0.24545	0.78235	0.88451	0.08696	1.09085	1.04444
78	-0.05129	0.95001	0.97468	0.11966	1.12712	1.06166	0.02692	1.02729	1.01355
79	-0.10692	0.89860	0.94794	-0.18222	0.83342	0.91292	0.02784	1.02823	1.01402
80	-0.14897	0.86159	0.92822	-0.17092	0.84289	0.91809	0.45836	1.58148	1.25757
81	0.07618	1.07916	1.03883	-0.09874	0.90598	0.95183	-0.00917	0.99087	0.99542
82	-0.01621	0.98393	0.99193	0.10173	1.10708	1.05218	0.23418	1.26387	1.12422
83	0.20048	1.22198	1.10543	0.03634	1.03701	1.01834	0.08320	1.08676	1.04248
84	0.00542	1.00543	1.00271	0.10658	1.11247	1.05474	-0.25216	0.77712	0.88154
85	0.00278	1.00278	1.00139	0.01268	1.01276	1.00636	-0.16540	0.84755	0.92063
86	-0.02436	0.97593	0.98789	0.02389	1.02418	1.01202	0.24092	1.27242	1.12802
87	-0.04041	0.96039	0.98000	0.24088	1.27237	1.12799	-0.15348	0.85771	0.92613
88	0.06000	1.06183	1.03045	-0.14871	0.86182	0.92834	0.12897	1.13765	1.06661

S/no	X0.13	Y0.13	Z0.13	X0.14	Y0.14	Z0.14	X0.15	Y0.15	Z0.15
89	-0.14311	0.86666	0.93095	0.09061	1.09484	1.04635	0.09307	1.09754	1.04763
90	-0.07732	0.92560	0.96208	0.12168	1.12940	1.06273	-0.12913	0.87886	0.93748
91	0.01575	1.01588	1.00791	0.15520	1.16790	1.08069	-0.28727	0.75031	0.86620
92	-0.03592	0.96472	0.98220	-0.15377	0.85747	0.92599	0.04044	1.04127	1.02043
93	-0.12357	0.88376	0.94008	0.02557	1.02590	1.01287	-0.07660	0.92626	0.96242
94	-0.00757	0.99246	0.99622	-0.27341	0.76078	0.87223	0.20391	1.22619	1.10733
95	0.03422	1.03481	1.01726	-0.05967	0.94207	0.97060	-0.03917	0.96158	0.98060
96	-0.12777	0.88006	0.93811	0.11219	1.11873	1.05770	0.03845	1.03920	1.01941
97	-0.10702	0.89851	0.94790	0.11465	1.12149	1.05900	-0.04700	0.95409	0.97678
98	-0.04151	0.95934	0.97946	0.01665	1.01679	1.00836	-0.06087	0.94094	0.97002
99	-0.11648	0.89005	0.94342	-0.00128	0.99873	0.99936	0.12500	1.13315	1.06450
100	0.15803	1.17120	1.08222	-0.09676	0.90778	0.95277	-0.17699	0.83779	0.91531

S/no	X0.16	Y0.16	Z0.16	X0.17	Y0.17	Z0.17
1	0.20481	1.22729	1.10783	0.11808	1.12534	1.06082
2	0.22678	1.25455	1.12007	-0.10687	0.89864	0.94797
3	-0.12906	0.87892	0.93751	0.18853	1.20748	1.09885
4	-0.03457	0.96602	0.98286	0.36891	1.44616	1.20256
5	-0.03086	0.96961	0.98469	-0.14548	0.86461	0.92984
6	-0.17789	0.83704	0.91490	0.03889	1.03966	1.01964
7	0.14994	1.16176	1.07785	-0.03332	0.96723	0.98348
8	-0.09890	0.90583	0.95175	-0.21322	0.80798	0.89888
9	-0.19755	0.82074	0.90595	0.03572	1.03637	1.01802
10	-0.13447	0.87418	0.93498	0.13000	1.13883	1.06716
11	-0.14881	0.86173	0.92830	0.11546	1.12239	1.05943
12	-0.04568	0.95534	0.97742	0.00851	1.00854	1.00426
13	-0.09254	0.91161	0.95478	-0.07573	0.92706	0.96284
14	-0.00605	0.99397	0.99698	-0.14321	0.86657	0.93090
15	-0.16120	0.85112	0.92256	0.18471	1.20286	1.09675
16	-0.10110	0.90384	0.95070	-0.10092	0.90400	0.95079
17	0.03303	1.03358	1.01665	-0.06654	0.93563	0.96728
18	0.08640	1.09024	1.04415	0.12154	1.12923	1.06265
19	-0.11187	0.89416	0.94560	0.00837	1.00840	1.00419
20	-0.13428	0.87435	0.93507	-0.01103	0.98904	0.99450
21	-0.04677	0.95431	0.97689	0.02907	1.02949	1.01464
22	-0.08744	0.91628	0.95722	0.20798	1.23119	1.10959
23	-0.20857	0.81174	0.90097	0.05417	1.05566	1.02745
24	-0.16735	0.84590	0.91973	0.18319	1.20104	1.09592
25	-0.18892	0.82785	0.90986	-0.16819	0.84519	0.91934
26	-0.19943	0.81919	0.90509	0.22012	1.24622	1.11634
27	0.10207	1.10746	1.05236	0.05192	1.05329	1.02630
28	-0.04040	0.96040	0.98000	-0.22996	0.79457	0.89139
29	-0.18864	0.82808	0.90999	0.00892	1.00896	1.00447
30	-0.27133	0.76236	0.87313	-0.19429	0.82342	0.90742
31	-0.06280	0.93913	0.96909	-0.01838	0.98179	0.99085
32	0.02445	1.02475	1.01230	-0.27285	0.76121	0.87247
33	0.01649	1.01663	1.00828	0.05601	1.05761	1.02840
34	0.10725	1.11321	1.05509	-0.17053	0.84322	0.91827
35	-0.07206	0.93047	0.96461	-0.13430	0.87433	0.93506

S/no	X0.16	Y0.16	Z0.16	X0.17	Y0.17	Z0.17
36	-0.00272	0.99728	0.99864	0.08178	1.08521	1.04174
37	0.11452	1.12133	1.05893	0.17508	1.19134	1.09148
38	-0.04234	0.95854	0.97905	-0.04937	0.95183	0.97562
39	0.20111	1.22276	1.10579	-0.09745	0.90715	0.95244
40	0.12838	1.13698	1.06629	-0.13239	0.87600	0.93595
41	-0.10800	0.89762	0.94743	0.06845	1.07085	1.03482
42	-0.00130	0.99870	0.99935	0.15890	1.17222	1.08269
43	-0.21939	0.80301	0.89611	-0.17374	0.84051	0.91679
44	0.08569	1.08947	1.04378	0.03901	1.03978	1.01970
45	0.03508	1.03570	1.01769	0.19835	1.21939	1.10426
46	-0.00452	0.99549	0.99774	-0.12532	0.88221	0.93926
47	-0.18267	0.83304	0.91271	-0.19619	0.82186	0.90656
48	0.09687	1.10172	1.04963	0.04480	1.04582	1.02265
49	-0.10306	0.90207	0.94977	0.13464	1.14412	1.06964
50	-0.26605	0.76640	0.87544	0.33230	1.39416	1.18075
51	0.00330	1.00330	1.00165	-0.12745	0.88034	0.93826
52	-0.18615	0.83015	0.91113	0.12706	1.13548	1.06559
53	0.41599	1.51587	1.23121	-0.24541	0.78238	0.88452
54	-0.03583	0.96481	0.98225	0.01311	1.01319	1.00658
55	-0.13236	0.87602	0.93596	-0.15437	0.85696	0.92572
56	-0.37725	0.68574	0.82810	-0.20396	0.81550	0.90305
57	-0.10233	0.90273	0.95012	-0.12283	0.88441	0.94043
58	-0.26200	0.76951	0.87722	-0.23148	0.79336	0.89071
59	0.15222	1.16442	1.07908	0.10160	1.10694	1.05211
60	-0.14813	0.86232	0.92861	-0.24270	0.78451	0.88573
61	0.17421	1.19030	1.09101	-0.07110	0.93137	0.96507
62	-0.02342	0.97685	0.98836	0.04253	1.04345	1.02149
63	-0.00677	0.99326	0.99662	0.21097	1.23488	1.11125
64	0.31898	1.37572	1.17291	0.21116	1.23511	1.11136
65	-0.13676	0.87218	0.93390	-0.05986	0.94190	0.97051
66	0.04734	1.04848	1.02395	0.19851	1.21959	1.10435
67	0.33802	1.40217	1.18413	-0.08984	0.91408	0.95607
68	-0.04862	0.95254	0.97598	-0.12760	0.88021	0.93819
69	0.00435	1.00436	1.00218	-0.23424	0.79117	0.88948
70	0.14098	1.15140	1.07303	0.03150	1.03201	1.01588
71	-0.09753	0.90707	0.95240	0.21414	1.23880	1.11301
72	0.13936	1.14953	1.07216	-0.14482	0.86518	0.93015
73	0.33579	1.39905	1.18281	-0.41842	0.65809	0.81122
74	-0.19800	0.82037	0.90574	0.08161	1.08503	1.04165
75	0.14805	1.15957	1.07683	-0.13014	0.87797	0.93700
76	-0.01811	0.98205	0.99098	0.26040	1.29744	1.13905
77	0.10326	1.10878	1.05299	0.10307	1.10857	1.05289
78	-0.24216	0.78493	0.88597	-0.01810	0.98207	0.99099
79	0.02353	1.02381	1.01183	0.10670	1.11260	1.05480
80	0.02623	1.02658	1.01320	0.25959	1.29640	1.13860
81	-0.03895	0.96180	0.98072	0.17150	1.18708	1.08953
82	0.16518	1.17960	1.08610	-0.22331	0.79987	0.89435
83	-0.26184	0.76964	0.87729	0.09059	1.09482	1.04634
84	-0.12069	0.88631	0.94144	-0.26834	0.76465	0.87444

S/no	X0.16	Y0.16	Z0.16	X0.17	Y0.17	Z0.17
85	0.03062	1.03109	1.01543	-0.18060	0.83477	0.91365
86	0.42496	1.52953	1.23674	-0.08157	0.92167	0.96004
87	0.09682	1.10166	1.04960	0.13107	1.14005	1.06773
88	-0.48354	0.61659	0.78523	0.17833	1.19522	1.09326
89	0.27675	1.31883	1.14841	0.03399	1.03458	1.01714
90	-0.04103	0.95980	0.97970	0.21368	1.23822	1.11276
91	-0.02752	0.97286	0.98633	0.08525	1.08898	1.04354
92	0.15533	1.16804	1.08076	-0.07742	0.92550	0.96203
93	0.01949	1.01968	1.00979	0.00036	1.00036	1.00018
94	0.34285	1.40896	1.18700	-0.05236	0.94899	0.97416
95	0.00840	1.00844	1.00421	0.02444	1.02474	1.01229
96	0.33153	1.39310	1.18030	0.01348	1.01357	1.00676
97	-0.03152	0.96897	0.98436	0.30629	1.35838	1.16550
98	0.09862	1.10365	1.05055	-0.27052	0.76298	0.87349

Appendix 4: Comparative Analysis of the Means and Variances of the Square Root Transformations of the Additive and Multiplicative Models

σ	Additive E(Z)	Multiplicative E(Y)	Multiplicative VAR(Y)	Additive VAR(Z)	Mean Ratio E(Z)/E(Y)	Variance ratio VAR(Z)/VAR(Y)
0.01	1.00001	0.99999	0.000025	0.000025	1.00002	1.00000
0.02	1.00005	0.99995	0.000100	0.000100	1.00010	1.00000
0.03	1.00011	0.99989	0.000225	0.000225	1.00022	1.00000
0.04	1.00020	0.99980	0.000400	0.000400	1.00040	1.00000
0.05	1.00031	0.99969	0.000625	0.000626	1.00062	1.00160
0.06	1.00045	0.99955	0.000900	0.000901	1.00090	1.00111
0.07	1.00061	0.99939	0.001225	0.001227	1.00122	1.00163
0.08	1.00080	0.99920	0.001599	0.001604	1.00160	1.00313
0.09	1.00101	0.99899	0.002024	0.002031	1.00203	1.00346
0.10	1.00125	0.99875	0.002498	0.002509	1.00250	1.00440
0.11	1.00151	0.99849	0.003023	0.003039	1.00303	1.00529
0.12	1.00180	0.99820	0.003597	0.003619	1.00361	1.00612
0.13	1.00211	0.99789	0.004221	0.004252	1.00423	1.00734
0.14	1.00245	0.99755	0.004894	0.004936	1.00492	1.00858
0.15	1.00282	0.99719	0.005617	0.005673	1.00564	1.00997
0.16	1.00321	0.99680	0.006390	0.006462	1.00643	1.01127
0.17	1.00362	0.99639	0.007212	0.007304	1.00726	1.01276
0.18	1.00406	0.99595	0.008084	0.008199	1.00814	1.01423
0.19	1.00452	0.99549	0.009005	0.009148	1.00907	1.01588
0.20	1.00501	0.99500	0.009975	0.010151	1.01006	1.01764
0.21	1.00553	0.99449	0.010994	0.011209	1.01110	1.01956
0.22	1.00607	0.99395	0.012063	0.012322	1.01219	1.02147
0.23	1.00663	0.99339	0.013180	0.013490	1.01333	1.02352
0.24	1.00723	0.99281	0.014344	0.014715	1.01452	1.02586
0.25	1.00784	0.99221	0.015556	0.015996	1.01576	1.02828
0.26	1.00849	0.99159	0.016813	0.017334	1.01704	1.03099
0.27	1.00915	0.99096	0.018115	0.018730	1.01836	1.03395
0.28	1.00985	0.99032	0.019459	0.020185	1.01972	1.03731
0.29	1.01057	0.98968	0.020843	0.021699	1.02111	1.04107

0.30	1.01131	0.98904	0.022264	0.062470	1.02252	2.80587
0.31	1.01208	0.98841	0.023721	0.067540	1.02395	2.84727
0.32	1.01288	0.98780	0.025209	0.072790	1.02539	2.88746
0.33	1.01371	0.98722	0.026728	0.078210	1.02683	2.92614
0.34	1.01455	0.98667	0.028273	0.083800	1.02826	2.96396
0.35	1.01543	0.98616	0.029843	0.089540	1.02968	3.00037
0.36	1.01633	0.98570	0.031435	0.095420	1.03108	3.03547
0.37	1.01726	0.98529	0.033048	0.101430	1.03245	3.06917
0.38	1.01821	0.98493	0.034680	0.107560	1.03379	3.10150
0.39	1.01919	0.98464	0.036329	0.113800	1.03509	3.13248
0.40	1.02020	0.98441	0.037993	0.120130	1.03636	3.16190
0.41	1.02123	0.98425	0.039673	0.126550	1.03758	3.18983
0.42	1.02229	0.98416	0.041367	0.133040	1.03875	3.21609
0.43	1.02338	0.98414	0.043074	0.139600	1.03987	3.24093
0.44	1.02450	0.98419	0.044795	0.146220	1.04095	3.26420
0.45	1.02564	0.98431	0.046529	0.152900	1.04198	3.28612
0.46	1.02680	0.98451	0.048276	0.159610	1.04296	3.30620
0.47	1.02800	0.98478	0.050036	0.166370	1.04389	3.32501
0.48	1.02922	0.98512	0.051810	0.173170	1.04476	3.34240
0.49	1.03047	0.98553	0.053597	0.180000	1.04560	3.35840
0.50	1.03174	0.98601	0.055398	0.186860	1.04638	3.37305

REFERENCES

[1] Hogg. R. V. and Craig, A. T. (1978). Introduction to Mathematical Statistics. 4th ed., Macmillan Publishing Company, New York.

[2] Iwueze, I. S. (2007), Some Implications of Truncating the $N(1,\sigma^2)$ Distribution to the Left at Zero. *Journal of Applied Sciences* (7)2; 189 – 195.

[3] I B Okoroafor, E.L Otuonye, D.C Chikezie, and A.C Akpanta (2021), Method of Handling Transformation when the Time Series Model is Additive, *Advanced Journal of Statistics and Probability* (9)1 pp17-31.

[4] Chatfield, C. (1980). The Analysis of Time Series: Theory and Practice, Chapman and Hall, London.

[5] Iwueze, I. S. and Akpanta A. C. (2007). *Effect of the logarithmic transformation on the trend-cycle component of the multiplicative Time series Model*. Journal of Applied Sciences. (7)17, 2414-2422.

[6] Iwu H.C, Iwueze I.S, and Nwogu E.C (2009), Trend Analysis of Transformations of the Multiplicative Time Series Model, Journal of the Nigerian Statistical Association, Volume (21)5, pp 40-54.

[7] Box, G. E. P. and Cox, D. R. (1964), Analysis of Transformations. J. Roy Statistics Society B – 26, 211 – 243, discussion 244 – 252.

[8] Osborne, J. (2002), Notes on the Use of Data Transformations, J. Practice Practical Assessment, Research and Evaluation, 8 (6).

[9] Iwueze I S and E C Nwogu (2004), Buys-Ballot Estimates for Time Series Decomposition, Global Journal of Mathematical Sciences, Vol.(3)2, pp83-98.

[10] Otuonye, E. L., Iwueze, I. S. and Ohakwe, J. (2011), *The Effect of Square Root Transformation on the Error Component of the Multiplicative Time Series Mode*l,. International Journal of Statistics and Systems. (6)4; 461 – 476.