

Accelerating Universe and Cosmological Equivalence

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Abstract The expansion of the universe is presently accelerating. Since the universe was earlier expanding gravitationally deceleratingly, and was therefore a dynamically viable universe, the question then arises as to why is the universe now expanding acceleratingly at all? The principle of equivalence recognizes the indistinguishability between gravitational freefall of a gravitational mass, and the impressional freefall of a stationary inertial mass with respect to an observer in an accelerating frame of reference. In other words, the gravitational effect of matter, in terms of freefall of gravitational mass, is equivalent to the inertial effect of motion, in terms of freefall of inertial mass. Thus, along with gravitational mass coexisting in magnitudinal equivalence with inertial mass, matter and its gravitational-freefall effect upon gravitational mass coexists in phenomenal equivalence with motion and its inertial-freefall effect upon inertial mass of objects. Gravitational mass of objects was in cosmological freefall when the universe was earlier expanding deceleratingly. The cosmological constant, which parametrizes the presently accelerating expansion of the universe, may be seen to be also acting upon inertial mass of objects in the universe, thereby rendering it in cosmological freefall. The accelerating expansion of the universe, therefore, manifests cosmological equivalence between the effect of motion, in terms of cosmological freefall of inertial mass, and the effect of matter, in terms of cosmological freefall of gravitational mass of objects.

Keywords Accelerating universe, Cosmological constant, Dark energy

1. Introduction

The presently observed accelerating expansion of the universe has come as a surprise, contrary to the expectation of a gravitationally decelerating expansion, which has been the dynamics of the universe since the big bang, before it started to expand acceleratingly $\sim 6 \times 10^9$ years ago [1].

It is possible to have a dynamically viable universe that was earlier expanding gravitationally deceleratingly – it is not necessary for the universe to have to expand acceleratingly for it to exist. The question therefore arises, as to why is the universe expanding acceleratingly now?

The accelerating expansion of the universe is parametrized by the cosmological constant. Attention has therefore been focused on the existence and origin of the cosmological constant. The cosmological constant was originally introduced by Einstein in the field equations of general relativity, to balance gravitation so as to account for what was then believed to be a static universe [2]. However, the instability of a static universe model due to matter density perturbations, and subsequent observational evidence in the 1920s of an expanding universe [3] rendered the cosmological constant redundant. Observations in the 1990s that the expansion of the universe is accelerating

[1,4], however, has required the resurrection of the cosmological constant.

The field equation containing the gravitational term and cosmological-constant term is

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (1)$$

The curvature of spacetime is represented by the metric tensor $g_{\mu\nu}$; the negative spacetime curvature gravitational

term is $-\frac{1}{2}Rg_{\mu\nu}$, the curvature of which is determined by

energy-momentum tensor $T_{\mu\nu}$ of matter and radiation on the right-hand side of equation (1); the positive cosmological-constant term is $\Lambda g_{\mu\nu}$ and the cosmological constant Λ determines the constancy of spacetime curvature $g_{\mu\nu}$.

It is possible to formulate the cosmological constant as an

energy density term $\frac{8\pi G\rho\Lambda}{c^4}$. By crossing over from

classical field theory of general relativity to quantum field theory of particle physics, the energy density of the cosmological constant may be sourced from the vacuum, representing zero-point energy, i.e., the lowest energy state of the particles in the universe. A naïve calculation, however, yields the infamous 120 orders of magnitude discrepancy between the predicted and observed value of the vacuum energy density [5].

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Be that as it may, aside from seeking the quantum vacuum origin of the cosmological constant to explain its existence, since the cosmological constant is a part of classical general relativity, we may also seek to understand its existence in terms of its role in the dynamics of the universe. However, since dynamically it is possible to have a viable universe that was earlier expanding gravitationally deceleratingly, therefore the presently accelerating expansion of the universe seems superfluous. Is there then a deeper physical significance to the accelerating expansion of the universe?

The principle of equivalence recognizes that gravitational freefall of gravitational mass of an object is indistinguishable from impressional freefall of inertial mass of a stationary object with respect to an observer in an accelerating frame of reference. To put it in another way, the gravitational effect of matter, in causing gravitational-mass freefall, is indistinguishable from the inertial effect of motion, in causing inertial-mass freefall.¹ Thus, along with gravitational mass coexisting magnitudinally equivalently with inertial mass in objects, matter and its gravitational-freefall effect upon gravitational mass coexists phenomenally equivalently with motion and its inertial-freefall effect upon inertial mass of objects.

While technically it is true that impressional freefall of inertial mass is equivalent to physical freefall of gravitational mass, from the physical point of view there should be scope for physical freefall of inertial mass as well. And indeed, the cosmological constant, which accelerates the expansion of the universe, can be seen to also play the role of causing physical cosmological freefall of inertial mass – and thereby manifesting cosmological equivalence with physical cosmological freefall of gravitational mass of objects.

2. Cosmological Equivalence

The description of earlier gravitationally decelerating expansion, and presently accelerating expansion of the universe due to the cosmological constant, is in the second Friedmann equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P/c^2) + \frac{\Lambda c^2}{3} \quad (2)$$

Here, the negative term with the gravitational constant G along with matter density ρ is responsible for the earlier gravitationally decelerating expansion, while the positive term with the cosmological constant Λ is responsible for the presently accelerating expansion of the universe.

Now, if we want to connect the dynamics of the universe

¹ The statement that 'gravitational effect of matter in causing gravitational-mass freefall is indistinguishable from the inertial effect of motion in causing inertial mass-freefall' reduces to the fact that both gravitational and inertial mass are in a locally inertial frame of reference, within the accelerating frame of reference of freefall, and therefore we may say that the effect of motion in the form of an accelerating frame of reference of freefall is (locally) inertial for inertial mass, just as the effect of matter in the form of a gravitational accelerating frame of reference of freefall is also (locally) inertial for gravitational mass.

with its discrete material constituents, we need to have a Newtonian formulation, which involves the description of gravitational deceleration acting upon gravitational mass of an object, whereby it would be in cosmological gravitational freefall in the earlier gravitationally decelerating expanding universe; we therefore re-cast equation (2) as an equation of motion of mass m on the surface of an expanding spherical distribution of matter M , with radius $R \equiv a$, and pressure $P = 0$; since $M = \frac{4\pi}{3}R^3\rho$, we have [6]:

$$\ddot{R} = -\frac{GM}{R^2} + \frac{\Lambda c^2}{3}R \quad (3)$$

When we insert mass m in equation (3), it is evident how the gravitational term $-\frac{GM}{R^2}$ and cosmological constant term $\frac{\Lambda c^2}{3}R$ act upon it:

$$\ddot{R}m = -\frac{GMm}{R^2} + \frac{\Lambda c^2 m}{3}R \quad (4)$$

Since $-\frac{GM}{R^2}$ is the gravitational term, we may infer that it is acting upon gravitational mass m_g of m ; and since gravitational mass is magnitudinally equivalent to inertial mass, we may then infer that the cosmological-constant term $\frac{\Lambda c^2}{3}R$ is acting upon inertial mass m_i of m :

$$\ddot{R}m = -\frac{GMm_g}{R^2} + \frac{\Lambda c^2 m_i}{3}R \quad (5)$$

We note that gravitational-mass deceleration, determined by the gravitational term $-\frac{GM}{R^2}$, is independent of the magnitude of m_g , and only depends upon distance $\frac{1}{R^2}$, whereby m_g is in freefall; similarly, inertial-mass acceleration, determined by the cosmological-constant term $\frac{\Lambda c^2}{3}R$, is independent of the magnitude of m_i , and only depends upon distance R , whereby m_i is in freefall.

Since both gravitational and inertial mass of objects are in freefall, there is therefore cosmological phenomenal equivalence between them; however, since gravitational-mass cosmological freefall decelerates as $\frac{1}{R^2}$, while inertial-mass cosmological freefall accelerates as R , there is therefore no magnitudinal equivalence, except momentarily, when in an expanding M , at a certain radial distance R_E , deceleration and acceleration are balanced.

With $\ddot{R} = 0$ in equation (3), we can then derive radius R_E :

$$R_E = \left(\frac{3GM}{\Lambda c^2} \right)^{1/3} \quad (6)$$

Thus, when, $R < R_E$, M is expanding deceleratingly, and when $R > R_E$, M is expanding acceleratingly. Since both gravitational term G and cosmological-constant term Λ are present as numerator and denominator in equation (6), however, neither is expansion of M purely decelerating when $R < R_E$ nor purely accelerating when $R > R_E$, but is a blend of both, with the former or the latter being the dominant dynamic, when $R < R_E$ or $R > R_E$. In terms of mass m , m_g of m would be the 'dominant participant' in decelerating freefall of m when $R < R_E$, while m_i of m is the 'dominant participant' in accelerating freefall of m when $R > R_E$.

3. Discussion

Our familiar notion of equivalence between gravitational-mass freefall and inertial-mass freefall is about how impressional inertial-mass freefall mimics physical gravitational-mass freefall. This, however, implies a bias in favour of gravitational-mass freefall as being the standard manifestation of freefall. Rather, if gravitational and inertial-mass freefall are to be truly equivalent, it should be mainly about how the fundamental phenomenon of freefall can be manifested by both gravitational and inertial mass – irrespective of the direction and magnitude of acceleration of inertial-mass freefall compared to gravitational-mass freefall.

Indeed, this is powerfully manifested by cosmological equivalence between gravitational and inertial-mass freefall, wherein the positive cosmological-constant acceleration which causes inertial-mass cosmological freefall is in the opposite direction to negative gravitational acceleration, which causes gravitational-mass cosmological freefall; and the acceleration/deceleration are also necessarily magnitudinally dissimilar, for the universe to be able to manifest the dual dynamics of earlier gravitationally decelerating and presently antigravitationally accelerating expansion; further, unlike local freefall of inertial mass, which is impressional, while freefall of gravitational mass is physical, cosmological freefall of inertial mass, due to it being accelerated by the cosmological constant, is physical, just like cosmological physical freefall of gravitational mass.

4. Conclusions

The presently accelerating expansion of the universe succeeded the earlier gravitationally decelerating expansion. Since it is possible to have a dynamically viable universe that was expanding gravitationally deceleratingly, the necessity of accelerating expansion seems unclear.

The principle of equivalence recognizes the indistinguishability between gravitational freefall of gravitational mass and impressional freefall of a stationary inertial mass, with respect to an observer in an accelerating frame of reference. Another way of stating it would be that the gravitational effect of matter, rendering freefall of gravitational mass, is equivalent to inertial effect of motion, rendering freefall of inertial mass of an object. Thus, along with gravitational mass coexisting in magnitudinal equivalence with inertial mass, matter and its gravitational-freefall effect upon gravitational mass coexists in phenomenal equivalence with motion and its inertial-freefall effect upon inertial mass of objects.

Previously, in the gravitationally decelerating expanding universe, gravitational mass of objects was in gravitational cosmological freefall. The presently accelerating expansion of the universe is parametrized by the cosmological constant. The cosmological constant may be seen to also act upon the inertial mass of objects in the universe, whereby it is in anti-gravitational cosmological freefall. The accelerating expansion of the universe is therefore manifesting cosmological equivalence between cosmological freefall of inertial mass and gravitational mass of objects.

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