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Using Transportation Model for Aggregate Planning: A Case Study in Soft Drinks Industry

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Abstract In this paper, aggregate planning strategies are discussed and a special structure of transportation models are investigated for the aggregate planning purpose of Baghdad Company for soft drinks. For the purpose of achieving the aims of this study, we will develop an optimal total production plan by determining the quantities of production necessary to meet the variable demand for a period of time in the medium term and at the lowest cost using the transportation model and then comparing the company's plan with the proposed plan by adopting specific criteria for determining the best. The study reached a set of conclusions, the most important of which is the development of an optimal production plan for the family of the product under study for the year 2018, and recommended the need to work according to the optimal plan proposed, which is better than the company's plan, since the total cost of production of the company and the unsold production cost of the company, Is greater than the corresponding costs reached by the optimal solution of the transport model.

Keywords Aggregate planning, Transportation model, Seasonal ARIMA, the bottom-up approach

1. Introduction

Aggregate production planning is concerned with determining the quantity and timing of production in the intermediate future to meet forecasted demand, it usually covers a time period ranging from 3 to 18 months. The main objective of aggregate planning is to minimize the total cost over the planning horizon. The plan must take into account the various ways a firm can cope with demand fluctuations as well as the cost associated with them. Typically a firm can cope with demand fluctuations by, changing the size of the work force by hiring and firing (thus allowing changes in the production rate), varying the production rate by introducing overtime and (or) outside subcontracting, accumulating seasonal inventories and planning backorders.

These ways of absorbing demand fluctuations can be combined to create a large number of alternative production planning options. Costs relevant to aggregate production planning are basic production costs (material costs, direct labor costs, and overhead costs), costs associated with changes in the production rate and Inventory related costs. There are two extreme aggregate production plans, the just-in-time production plan and the production-smoothing plan.

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Sultana etal. in 2014 discussed aggregate planning strategies and then investigated a special structure of transportation model for the aggregate planning purpose of “Bangladesh Cable Shilpa Ltd, Khulna”. For this transportation problem, all the unit costs, supplies, demands & other values are taken from a case study. They determined forecasting demand values by using Single Exponential Smoothing Forecasting technique. TORA software is used to find the optimum cost by using transportation method. Chada in 2015 designed and developed a computer program to facilitate exploration of aggregate planning methods to obtain desirable cost attributes. The first phase is to design and develop a spread sheet with different cases such that each case will use a constant strategy throughout the period for that particular case and also sensitivity analysis is performed on the input variables with the help of macros in the spreadsheet. The second phase is developed in such a way that there is flexibility to select different strategies for each period. Rosemary etal. in 2016 examined transportation model of quantitative techniques for management. she argued that, managers must be aware of the quantity of available supplies, the quantities demanded and location to find the cost of transporting one unit of commodity from place to another. Conclusions were drawn and recommendation made that the optimal solution to a transportation problems must consist of integer values for the decision variables as long as all supply and demand values are integers. Jamalnia in 2017 proposed a novel decision model to aggregate production planning decision making problem based on mixed chase and level strategy under uncertainty where the market demand acts as the main source of uncertainty. Jamalnia appraised also the performance of different aggregate production planning strategies in presence of uncertainty. Damghani etal. in 2017 proposed a multi-period multi-product multi-objective aggregate production planning (APP) model for an uncertain multi-echelon supply chain considering financial risk, customer satisfaction, and human resource training. They considered three conflictive objective functions and several sets of real constraints. Some parameters of the proposed model are assumed to be uncertain and handled through a two-stage stochastic programming (TSSP) approach. The proposed TSSP is solved using the goal attainment technique, the modified ε-constraint method, and STEM method. They applied the whole procedure in an automotive resin and oil supply chain as a real case study wherein the efficacy and applicability of the proposed approaches are illustrated in comparison with existing experimental production planning method.

2. Statistical Tools

2.1. Seasonal ARIMA Model

SARIMA models are the most general class of models for forecasting a time series, since there too many phenomena that are follow to.. Seasonal ARIMA models are expressed in factored form by the notation ARIMA(p,d,q)(P,D,Q)$_s$, where p is the number of autoregressive terms, d is the number of nonseasonal differences needed for stationarity, q is the number of lagged forecast errors in the prediction equation, P is the order of the seasonal autoregressive part, D is the order of the seasonal differencing, Q is the order of the seasonal moving-average process and s is the length of the seasonal cycle.

Given a dependent time series $\{y_t : 1 \le t \le n\}$, mathematically the ARIMA seasonal model is written as,

$$C + \left(1 - \sum_{i=1}^{p} \varphi_i B^i\right)\left(1 - \sum_{i=1}^{P} \varphi_i^* B^{Si}\right)(1-B)^d \left(1-B^S\right)^D y_t = \left(1 - \sum_{i=1}^{q} \theta_i B^i\right)\left(1 - \sum_{i=1}^{Q} \theta_i^* B^{Si}\right) e_t \quad (1)$$

where, C is a constant, θ's and Φ's are weighting parameters for the different nonseasonal lagged terms, θ^*'s and Φ^*'s are weighting parameters for the different seasonal lagged terms, B is the back shift operator ($B^j y_t = y_{t-j}$) and e represents a random error term.

2.2. Transportation Model

This model can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control and many others.

The transportation problem can be put in general form as shown in table (1)

Table 1. Matrix of Transportation

Sources (Supply from)	Destinations (Demand for)				Total supply (Capacity available)
	B_1	B_2	…	B_n	
A_1	x_{11} c_{11}	x_{12} c_{12}		x_{1n} c_{1n}	s_1
A_2	x_{21} c_{21}	x_{22} c_{22}		x_{2n} c_{2n}	s_2
⋮			⋮		⋮
A_m	x_{m1} c_{m1}	x_{m2} c_{m2}		x_{mn} c_{mn}	s_m
Total demand	d_1	d_2	…	d_n	

Where,

A_i – periods of production, i = 1,2, …,m (each period of production may contains production in regular time, production in overtime and production under subcontract), (the beginning inventory may be taken as first period of production)

B_j – periods of demand, j = 1,2, …,n

m – total number of containers produced

n – total number of containers demanded

c_{ij} –unit costs for the transport between i-th period of production and the j-th period of demand,

i = 1,2, …,m, j = 1,2, …,n

x_{ij} – number of containers supplied from i-th period of production to j-th period of demand,

i = 1,2, …,m, j = 1,2, …,n

s_i – available number of produced containers at i-th period of production, i = 1,2, …,m

d_j – demand for containers at j-th period of demand, j = 1,2, …,n.

It is worth mentioning that the costs concluded the regular production cost per unit, overtime cost per unit, subcontracting cost per unit, holding cost per unit period, Backorder cost per unit per period and inventory carrying cost. c_{ij} contains some of costs which are compatible with situation of x_{ij} . The mathematical model for the solution of the above transportation problem can be summarized as follows:

Criteria function, Min $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

with constraints,

$$\sum_{j=1}^{n} x_{ij} = s_i \text{ , } i = 1,2, \dots,m \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = d_j \text{ , } j = 1,2, \dots,n \quad (3)$$

$x_{ij} \ge 0$; i = 1,2, …,m, j = 1,2, …,n.

To be conveniently correlated the equations (2) and (3) should fulfill the following condition:

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \; .$$

There are some assumptions in the transportation model, that are, total quantity of the item available at different sources is equal to the total requirement at different destinations, item can be transported conveniently from all sources to destinations, the unit transportation cost of the item of the item from all sources to destinations is certainly and precisely known, the transportation cost on a given route is directly proportional to the number of units shipped on that route and the objective is to minimize the total transportation cost.

The first step to solve transportation problem is to find out the initial feasible solution. The Vogel approximation method (VAM) is an iterative procedure for computing that basic feasible solution. This method is preferred over the other methods, because the initial basic feasible solution obtained by this method is either optimal or very close to the optimal solution. The steps in VAM are as follows,

1. Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (penalty) along the side of the table against the corresponding row.
2. Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (penalty) against the corresponding column
3. Identify the maximum penalty. If it is along the side of the table, make maximum allotment to the box having minimum cost of transportation in that row. If it is below the table, make maximum allotment to the box having minimum cost of transportation in that column.
4. If the penalties corresponding to two or more rows or columns are equal, you are at liberty to break the tie arbitrarily.
5. Repeat the above steps until all restrictions are satisfied.

After computing an initial basic feasible solution, one can used the modified distribution method (MODI) is for finding the optimal solution of a transportation problem. It is provides a minimum cost solution. The steps in MODI are as follows,

1. Determine the values of dual variables, u_i and v_j, using $u_i + v_j = c_{ij}$
2. Compute the opportunity cost using $c_{ij} – (u_i + v_j)$.
3. Check the sign of each opportunity cost. If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimal solution. On the other hand, if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimal solution and further savings in transportation cost are possible.
4. Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.
6. Draw a closed path or loop for the unoccupied cell selected in the previous step. Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.
7. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.
8. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs, and subtract it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.

3. Case Study

To determine the optimal aggregate production plan, this will be according to the following subsections,

3.1. Prepare the Necessary Data

The data needed to achieve the aims of the study will be prepared as follows:

1. For the purpose of conducting quantitative analysis appropriate to the objectives of the study, the monthly data for the demand and production of 250 ml (Pepsi, Miranda, Seven up, Green apple, peace, Shani, Lemon) for the period from the beginning of 2012 until the end of 2017 was obtained from Baghdad Company of soft drinks. This data is shown in tables (1-2) in annex respectively.
2. Through the personal interview with the director of production operations and the administrator of the first line of production in Dijla plant, we learned that there are two of works shift, the first one starts from seven in the morning and ends at three in the afternoon (eight hours) and then extend for another two hours as an additional time until 5 pm. The second shift of the work begins from 7 pm to 3 am, followed by two additional working hours until 5 am. It is also worth mentioning that:
   a. The work shifts in Both of Fridays and Saturdays treated as additional shifts.
   b. Work is continuing for all days of the week except public holidays approved by the government and those granted to certain necessities. Table (3) In annex shows these holidays over the months from the beginning of 2012 until the end of 2017.
3. Based on Table (3) In annex, the actual and additional working hours are in Table (4) In annex, were calculated for January 2012, for example, as follows:
   a. The number of regular working days (including additional working hours) is ((number of days of the month (31 days minus (8 days of Fridays and

Saturdays plus an official holidays) 1 day of january (1 January) Remaining 22 days, 31 - 8 - 1 = 22.

b. The additional working days (overtime) are (the number of Fridays and Saturdays, which is 8 days, minus a public holiday on Friday, (6th of January), which will be 8 - 1 = 7.

c. The regular working hours are equal to the number of regular working days of 22 days multiplied by 16 hours (8 working hours for each shift), (22 x 16 = 352).

d. The number of working hours for overtime is equal to the number of regular working days of 22 days multiplied by 4 hours (2 extra hours for each shift) plus 7 overtime days multiplied by 20 hours (10 overtime hours in each shift) (22 x 4) + (7 x 20) = 228. And so on for the rest of the months in the period under study from the beginning of 2012 until the end of 2017.

4. According to table (4) In annex, we calculated the quantities of production within the regular and overtime hours of each of the products under study. These quantities are shown in tables from (5-10) In annex. for example, for December 2012, the calculations are as follows:

   a. The ratio of the number of regular work hours to the total number of working hours is, 352 / (580) = 0.606897.

   b. We multiply the above ratio by the number of containers produced of Pepsi in December 2012, the production quantity in regular time is, 0.606897 × 2199.327 = 1334.764.

   c. The ratio of the number of overtime hours to the total number of working hours is

   228 / (580) = 0.393103.

   d. We multiply the above ratio by the number of containers produced of Pepsi in December 2012, the production quantity in overtime is, 0.393103 × 2199.327 = 864.5631. And so on for the rest of the months and products for the period from 2012 to 2017.

It is worth mentioning that the single container contains 110 boxes, each box contains 30 metal cans of 250 ml.

3.2. Demand and Production Forecasts

For the purpose of planning the aggregate production of the products of Baghdad Company for soft drinks and specifically for 250 ml metal cans for the period from March to December of 2018, firstly we will forecast the demand for those products as well as the quantities of production at the regular and overtimes in those months according to the following steps:

1. Based on the demand data in Table (1) In annex and the quantities of production within the regular and overtimes in Tables (5-10) In annex, the Akaike criterion was used to determine the degree of appropriate model of data among several estimated models. In addition, the root of mean square error $\sqrt{\mathbf{MSE}}$ is used to determine the advantage of a model among several models were estimated.

   a. The models mentioned in Table (11) In annex, are appropriate to represent the demand on containers for each product, and then the total demand for the sum of all products.

   b. The models mentioned in Table (12) In annex, are appropriate to represent quantities of production on containers for each product within the regular time, and then the total quantities of production for the sum of all products.

   c. The models mentioned in Table (13) In annex, are appropriate to represent quantities of production on containers for each product within the overtime, and then the total quantities of production for the sum of all products.

It should be noted that the criterion for selecting the appropriate model is the least value of √MSE and the lowest value of the AIC criterion, for example SARIMA (1.0,0) x (2,0,0) 2, is a model which was chosen to represent demand. It was used to forecast future values for Pepsi, with value of √MSE equal to 2901.56 and value of Akiaki criterion equal to 1156.113, which are the lowest among all corresponding values of the models selected to represent the demand for Pepsi product.

SARIMA (1,0,0) x (2.0.0) 2 model, which is represented the demand on the Pepsi, can be written according to the equation (1) as follows,

$$\mathrm{c} + (1-\emptyset \mathrm{B})(1-\emptyset^{*}\,\mathrm{B}-\emptyset_2^{*}\,\mathrm{B}^2)\mathrm{y_t} = \mathrm{e_t} \qquad (4)$$

The parameters values listed in Table (6) are substituted in the above equation to be,

$$3762.39 + (1-0.124865\,B)$$
$$(1-0.10239\,\mathrm{B} - 0.120968\,\mathrm{B}^2)y_t = e_t \qquad (5)$$

by writing,

$$(1-\emptyset_1\mathrm{B})(1-\emptyset_1^{*}\mathrm{B}-\emptyset_2^{*}\,\mathrm{B}^2)$$
$$= 1-\emptyset_1^{*}\,B-\emptyset_2^{*}\,B^2-\emptyset_1\,B+\emptyset_1\emptyset_1^{*}\,B^2+\emptyset_1\emptyset_2^{*}B^3$$
$$= 1-(\emptyset_1^{*}+\emptyset_1)B-(\emptyset_2^{*}-\emptyset_1\emptyset_1^{*})B^2+\emptyset_1\emptyset_2^{*}B^3$$

Equation (2) will be:

$$c+(1-(\emptyset_1^{*}+\emptyset_1)B-(\emptyset_2^{*}-\emptyset_1\emptyset_1^{*})B^2+\emptyset_1\emptyset_2^{*}B^3)y_t = e_t$$
$$c+y_t-(\emptyset_1^{*}+\emptyset_1)y_{t-1}-(\emptyset_2^{*}-\emptyset_1\emptyset_1^{*})y_{t-2}+\emptyset_1\emptyset_2^{*}y_{t-3} = e_t$$
$$3762.39+y_t-(0.10239+0.124865)y_{t-1} - \big(0.120968 - (0.10239)(0.124865)\big)y_{t-2} + (0.124865)(0.120968)y_{t-3} = e_t$$
$$y_t-0.227y_{t-1}-0.108y_{t-2}+0.015y_{t-3} = e_t \qquad (6)$$

Equation (6) above represents another form of the model in equation (4), and so it is for other models, whether for demand or production.

2. Before using the models mentioned in Tables (11-13) In annex, for the purposes of forecasting, Box-Piers test were used to confirm the ability of chosen models to forecast. The Box-Piers test results that are shown in Tables (14-16) In annex, indicate that the ability of chosen models to forecast, where P value is the critical point between accept and reject the hypothesis that the model is able to forecast (there is no pattern for the residuals). The rule is accept the hypothesis where the chosen significant level is less than p. For example, p value for demand model on Pepsi product is 0.765096, Since this value is greater than all the common significant levels 0.01, 0.05 and 0.10, we accept the forecasts resulting from this model, and that the residuals are random.
3. Using the models mentioned in Tables (11-13), the forecasts for the demand for the products under consideration, as well as the quantities of production at the regular and overtime for the months from March to December 2018, were obtained. Tables (17-19) In annex, shows the values of these forecasts.

3.3. Using the Transportation Model

After obtaining the previous data and information related with products under consideration beside the following information,

1. Beginning inventory obtained from the company's stores for each product are in table (2),
2. The production cost of single container in regular and overtime are 660,000 Iraqi dinars and 673000 respectively (from the final accounts of the company).
3. The cost of storage of one container is 3000 Iraqi dinars per month (from the company's accounts).

Table (2). Beginning inventory (containers) of the products under study on 1/3/2018

Lemon	Shani	Green apple	Seven up	Mirnda	Pepsi	product
303	465.33	559.04	1598.53	279.2	2727.7	Beginning inventory

POM software was used to obtain the optimal value for the costs resulting from satisfying the demand of the products under consideration through the production of the company. The Vogel method was used to get initial basic feasible solution. The results are as follows,

1. product (Pepsi): Table (20) In annex, represents the optimal solution for the transportation model. It is also includes the data and information that represent the analysis requirements to use the mentioned model. It was found that the lowest optimal cost value is 34975750000 Iraqi dinar.
2. Product (Miranda): Table (21) In Annex, represents the optimal solution for the transportation model. It also includes the data and information that represent the analysis requirements to use the mentioned model. It was found that the lowest optimal cost value is 4850998000 Iraqi dinar.
3. Product (Seven up): Table (22) In Annex, represents the optimal solution for the transportation model. It also includes the data and information that represent the analysis requirements to use the mentioned model. It was found that the lowest optimal cost value is 15484670000 Iraqi dinar.
4. Product (green apple): Table (23) In Annex, represents the optimal solution for the transportation model. It also includes the data and information that represent the analysis requirements to use the mentioned model. It was found that the lowest optimal cost value is 2437385000 Iraqi dinar.
5. Product (Shani): Table (24) In Annex, represents the optimal solution for the transportation model. It also includes the data and information that represent the analysis requirements to use the mentioned model. It was found that the lowest optimal cost value is 3582915000 Iraqi dinar.
6. Lemon product: Table (25) In Annex, represents the optimal solution for the transportation model. It also includes the data and information that represent the analysis requirements to use the mentioned model. It was found that the lowest optimal cost value is 742554400 Iraqi dinar.
7. The sum of products: Table (26) In Annex, represents the optimal solution for the transportation model. It also includes the data and information that represent the analysis requirements to use the mentioned model. It was found that the lowest optimal cost value is 62074272400 Iraqi dinar.

3.4. Determination of Optimum Aggregate Production Plan

In this section, we will discuss the results and determine the aggregate production plan Proposed for each product and for all products at once. For the purpose of arranging and clarifying the results analysis, we will presenting it in accordance with the following paragraphs,

1. Any company that wishes to sell all its production in order to maximize its profits (equivalently, reduce its losses due to the possibility of not selling part of its production). Therefore, the Baghdad Company for soft drinks, when produced at regular and overtime, assumes that all the production will sold according to planned time. For this purpose, we multiplied the production cost of the container at the regular time which is 660000 iraqi dinar, by the quantity of production at the regular time to obtain the production cost at the regular time (row 1 in Table 3). We combined the result with the cost of production in overtime (row 2 In Table 28) which is calculated by multiplying the production cost of the container which is 673000 iraqi dinar, by the quantity of production at overtime (row 3 in Table 3) (where the company assumes that all of its production will be sold).

For example, the forecasted production of containers of Pepsi in the regular time of the last ten months of 2018 is 31610.94 containers, which if it is multiplied by the cost of production of the single container at the regular time, which is 660,000 Iraqi dinars, then the output will be about 20863220400 Iraqi dinars. This result represent the cost of Pepsi product production at the regular time.

In addition, the forecasted production of containers of Pepsi in overtime of the last ten months of 2018 is 23464.61 containers, which if it is multiplied by the cost of production of the single container at overtime, which is 673,000 Iraqi dinars, then the output will be about 15791682530 Iraqi dinars. This result represent the cost of Pepsi product production at overtime. By combining the above two results, we get the total cost of Pepsi product production in the company. Similarly, calculations are done for the other company's products.

The POM software of Heizer's book was then used to obtain the optimal solution (lowest possible cost) for each of products under consideration and then for sum of all of them. Row 4 in Table (3) includes this. The difference between the cost of producing the product (for each product and the total) and the cost calculated using the transport model is mentioned in row (5). It is clear that the use of the transport model to determine the optimal production cost is better than the mechanism calculated by the company currently due to the difference between the costs the products have total aggregate.

For example, the difference between the cost calculated according to the transportation model and the cost calculated according to the company's mechanism of the Pepsi product is 1679152930 Iraqi dinar. This amount represents a waste of the company's money for the year under planning.

It should also be noted that the total cost of production according to the company's mechanism is greater than the cost calculated according to the transportation model, although the total cost of production according to the company's mechanism is multiplied by the cost of production only (because the company usually assumes that all its production will be sold without storage to reach the ideal situation), while that the total cost calculated according to transportation model multiplied by production cost plus storage. This is due to the fact that the production of the company is actually greater than the demand, which was shown through the transportation model itself, as there was a dummy column for the purpose of balancing production and demand quantities, which contains unsold quantities of production, which represent the difference between production and demand.

Tables (20-26) In annex, show that there is a quantity of production going to dummy demand column, which means that the production is greater than the demand, which is means also that the company bears the additional costs resulting from the production of these quantities without real demand. The explanation is the same for other products.

2. In the sixth row of Table (3), we find the cost of non-sold production according to the company's mechanism, which is calculated as follows for each product, by adding the quantity of containers produced in the regular time to the quantity of containers produced in overtime and quantity of beginning inventory of production and then subtract the demand from the result. Then the result multiplied by the cost of production and storage, which is 700 thousand Iraqi dinars. For example, for seven up product, the result is, (10543.63 + 14228.95 + 1598.53 - 24879.34) × 700000 = 1044239000 Iraqi dinars.

The seventh row of table (3) includes the cost of unsold production according to the optimal solution of the transportation model. It is calculated by multiplying the cost of production and storage by corresponding quantities of unsold products in the dummy balance column. For example, from the transportation table of the seven up product, the quantities of unsold output are 1053.64, 431.26, 3.44 and 3.43 respectively of production in the overtime of the first, second, third and fourth production periods. By multiplying these quantities by corresponding production and storage costs, we will obtain the cost of the unsold production of the seven up product according to the optimal solution of the transportation model as follows, 1053.64 (700000) +431.26 (697000) +3.44 (694000) +3.43 (691000) = 1042893710.

We note that the cost of unsold production of seven up product according to the optimal solution of the transportation model is less than the cost of production that is unsold according to company's mechanism. The difference is, 1044239000 - 1042893710 = 1345290 Iraqi dinars. similarly, calculations are done for the rest of the products and sum of those products.

It is noted that the cost of production, that is unsold, according to the optimal solution of the transportation model is less than the corresponding cost according to company's mechanism for each of products as well as the sum of those products. row 8 of Table (3) includes the differences between the costs.

3. In the ninth row of table (3), we find the losses of unsold production according to company's mechanism, which is calculated for each product as follows; the quantity of containers produced in regular time plus the quantity of containers produced in overtime plus quantity of containers of beginning inventory minus the demand for that product, then the result is multiplied by the cost of storage only, which is (27000 dinars), for example, the losses for seven up product will be, (10543.63 + 14228.95 + 1598.53 - 24879.34) × 27000 = 40277790 Iraqi dinars.

It is worth mentioning that to calculate the loss, we multiplied by the cost of storage only, on the basis that the unsold production will be sold in the next year, 2019 as beginning inventory.

The contents of the tenth row of table (3) are the cost of unsold production according to the optimal solution of the transportation model and is calculated by multiplying the cost of storage by the corresponding quantity of unsold containers in dummy column.

Table (3). Production costs and losses according to company's mechanism and to the transportation model (in Iraqi dinar)

Sum	Product						
	Lemon	Shani	Green apple	Seven up	Mirnda	pepsi	
37428434406	563032206	2163800760	1587927000	9391107000	2859347040	20863220400	**production Cost at regular time (1)**
28340889085	434200420	1640041776	1191036366	7095862990	2188065003	15791682530	**production Cost at overtime (2)**
65769323491	997232626	3803842536	2778963366	16486969990	5047412043	36654902930	**Total production cost according to company's mechanism (2)+(1)=(3)**
62074272400	742554400	3582915000	2437385000	15484670000	4850998000	34975750000	**optimal cost according to the transportation model (4)**
3695051091	254678226	220927536	341578366	1002299990	196414043	1679152930	**(4)-(3)=(5)**
3853963820	267115520	230776000	357179900	1044239000	204352400	1750301000	**Cost of unsold production according to company's mechanism (6)**
3846698830	264226640	230439020	355535230	1042893710	204058660	1749545570	**Cost of unsold production according to according to transportation model (7)**
7264990	2888880	336980	1644670	1345290	293740	755430	**(6)-(7)=(8)**
148652890	10303027	8901360	13776939	40277790	7882164	67511610	**Losses of unsold production (inventory) according to company's mechanism (9)**
141838464	7860000	8569764	12136980	38932500	7583040	66756180	**Losses of unsold production according to optimal solution of the transportation model (inventory) (10)**
6814426	2443027	331596	1639959	1345290	299124	755430	**(9)-(10)=(11)**

Table (4). The optimal aggregate production plan for the months from March to December 2018

Sum	Product						Production time	Production periods
	Limon	Shani	Green apple	Seven up	Mirnda	Pepsi		
5690.9	51.32	335.77	236.3	1489.45	431.17	3146.89	**regular time**	**1**
42.65	–	–	–	–	42.65	–	**Overtime**	
5659.292	85.087	265.06	234.435	1479.75	434.15	3160.81	**regular time**	**2**
3224.95	–	113.38	–	622.58	332.84	2156.15	**Overtime**	
5647.27	85.33	323.36	242.66	1401.5	433.28	3161.14	**regular time**	**3**
3932.63	–	239.32	–	1051.11	315.93	2326.27	**Overtime**	
5699	85.31	366.31	242.97	1408.04	433.43	3162.94	**regular time**	**4**
4231.75	–	278.17	186.59	1051.11	331.8	2384.08	**Overtime**	
5658.26	85.32	333.66	241.48	1401.44	433.38	3162.98	**regular time**	**5**
4128.285	–	248.825	177.28	1054.51	316.24	2331.43	**Overtime**	
5644.91	85.32	319.32	241.42	1402.25	433.39	3163.21	**regular time**	**6**
4208.08	57.76	233.17	172.3	1054.51	325.98	2364.36	**Overtime**	
5666.96	85.32	331.18	241.69	1412.16	433.39	3163.22	**regular time**	**7**
4208.4	59.9	246.46	181.71	1054.51	321.96	2343.86	**Overtime**	
5675.04	85.32	340.07	241.7	1411.31	433.39	3163.25	**regular time**	**8**
4234.893	59.5	253.91	183.72	1054.51	325.503	2357.75	**Overtime**	
5668.49	85.32	333.33	241.65	1411.55	433.39	3163.25	**regular time**	**9**
4212.26	60.36	248.94	179.46	1054.51	322.4	2346.59	**Overtime**	
5665.54	85.32	330.43	241.65	1411.5	433.39	3163.25	**regular time**	**10**
4215.803	60.17	245.053	178.42	1054.51	323.96	2353.69	**Overtime**	
93315.36	**1116.657**	**5385.718**	**3665.435**	**23280.81**	**7291.623**	**52575.12**	**optimal quantity of production that meet demand**	
5932.8	**303**	**465.33**	**559.04**	**1598.53**	**279.2**	**2727.7**	**beginning inventory**	
99248.16	**1419.657**	**5851.048**	**4224.475**	**24879.34**	**7570.823**	**55302.82**	**Total demand**	

For example, from the seven up transportation table, the quantities of unsold containers of production are 1053.64, 431.26, 3.44 and 3.43 respectively of production in the overtime of the first, second, third and fourth production periods, these quantities are multiplied by the corresponding storage cost, we will get the unsold production losses for seven up product according to the optimal solution of the transportation model,

1053.64 (27000) +431.26 (24000) +3.44 (21000) +3.43 (18000) = 38932500 Iraqi dinars.

We note that the unsold production losses of the seven up product according to the optimal solution of the transportation model are less than the unsold production losses according to company's mechanism. The difference is, 40277790-38932500 = 1345290 Iraqi dinars. These differences are positive for all products and then for their sum. Row 11 in table (3) contains these differences.

4. Our main goal has been achieved, based on all of the above, Baghdad Soft Drinks Company can adopt the aggregate plan of production in Table (4) for all products under consideration (Pepsi, Miranda, Seven up, Green Apple, Shani, Lemon) and then their sum.

4. Conclusions, Recommendations and Future Studies

Before stating the conclusions, it should be stated that,

1. The company is not designed an aggregate production plan but it is put an estimated annual plan in the light of achieved actual sales by using the method of the seasonal exponential smoothing, which is one of the simple forecasting methods. This plan is adopted for production purposes.
2. The company did not specify the quantities of production in the regular time and the overtime required to meet the demand but the company adopted the method of production and storage directly, this leads to increase costs because of most of inventory production will be from production in overtime, without taking into account that need to meet the most demand from production in the regular time, while the remaining demand will be met from the production in overtime. This mean that the inventory at the end of the plan period must be zero or at least close to zero in the optimal aggregate production planning.
3. The company did not specify from which Production batches (periods) should meet the sequential monthly demand so that costs are as low as possible.
4. The company bears a cost of storage higher than the cost of the best plan (which is supposed to bear), since the unsold production was used as a beginning inventory in the beginning of the year.

Then, an aggregate total production plan for all the studied products (Pepsi Cola, Miranda, Green apple, Seven up, Shani and Limon) has been putted, and thus for their total sum of 250ml metal cans category, in both regular time and overtime. production in 2018 under this plan fully covers the forecasted demand, taking into consideration the beginning inventory, since, it is well known that production is as much as demand in an optimal aggregate production plan. Some of the results of this optimal plan are as follows,

1. The total cost of production under the company's mechanism for each product and then for the total sum is greater than the cost reached according to the optimal solution of the transportation model.
2. The cost of unsold production under the company's mechanism for each product and then for the total sum is greater than the unsold production cost reached according to the optimal solution of the transportation model.
3. Losses of unsold production (inventory) according to the company's mechanism for each product and then for the total sum is greater than the losses of unsold production according to the optimal solution of transportation model.

Based on the above, the bottom-up approach was used to assemble the plans of the six products under consideration to put the aggregate production plan. So, we are highly recommended to work according to the proposed optimal aggregate production plan.

As future studies, we suggest using the optimization models along with an appropriate forecasting models to put an aggregate production plan for other families of products in Baghdad Soft Drinks Company and for other commodity products of other companies, also for services sector such as the health sector or Municipal sector. Also, we suggest studying the relationship between the use of transportation model to plan an aggregate production and achieve competitive dimensions.

Appendix

Table (1). Actual demand on produced containers for products under study

Lemon	Shani	Green apple	Seven up	Mirnda	Pepsi	Product / Month	Year
376.5273	0	625.1	2415.736	334.5273	3188.791	**Jan.**	**2012**
216.6091	0	400.3818	2361.2	961.4727	4119.927	**Feb.**	
201.5636	519.9273	689.6727	2738.591	272.3909	2233.736	**Mar.**	

347.8818	0.109091	1023.245	3008.191	1232.245	3561.8	**Apr.**	
203.5545	439.6364	0.836364	1913.736	684.8	4660.664	**May**	
43.8	538.8091	0	3151.773	832.2091	3821.182	**Jun.**	
0	227.9727	0	4880.227	793.3727	6201.118	**Jul.**	
0	426.0727	1374.464	2975.582	729.1182	4325.591	**Aug.**	
293.2727	406.7273	277.5455	3139.7	184.7545	4517.409	**Sep.**	
0.472727	456.2182	311.7	2934.836	1086.255	5487.418	**Oct.**	
0	327	619.2909	1654.209	750.9818	2499.909	**Nov.**	
0	237.1818	220.7091	850.5182	96.36364	4189.173	**Dec.**	
0	269	205.5727	2817.682	508.0091	4828.409	**Jan.**	**2013**
0	378.0182	394.6091	2712.918	595.4545	4188.936	**Feb.**	
0	416.4636	444.2909	2604.8	565.8909	5219.409	**Mar.**	
217.4545	534.0455	518.2	2061.536	718.3636	3307.564	**Apr.**	
0.863636	184.3273	384.3364	840.3182	600.9727	3027.755	**May**	
0	0	2.381818	583.6545	616.6909	1773.845	**Jun.**	
348	641.2364	0.109091	3930.664	1095.027	4504.264	**Jul.**	
132.1636	466.2909	217	4593.2	994.6273	6027.891	**Aug.**	
0	1117.055	365.8909	3724.555	610.8727	6750.9	**Sep.**	
0	269.3091	0.390909	2021.673	731.6091	5497.773	**Oct.**	
0	588.4909	296	3568.391	538.3727	5339.209	**Nov.**	
0	246	301	452.2091	213.4364	1186.773	**Dec.**	
0	309.4091	32.6	948.9636	337.1273	5506.745	**Jan.**	**2014**
0	579.6455	636.0273	1529.255	404.0727	4316.264	**Feb.**	
0	222.0273	302.6	1574.609	546.5091	2866.391	**Mar.**	
0	465.6636	238.8636	1570.309	700.2818	2656.264	**Apr.**	
0	736.9545	661.0091	4316.355	710.1818	6370.409	**May**	
0	643.8636	292.0364	1803.927	538.8273	3569.755	**Jun.**	
0	477.8545	7.672727	2242.864	611.0818	2886.718	**Jul.**	
0	889.1818	708.8273	2744.282	756.6091	4744.627	**Aug.**	
0	907.4909	582.9091	2916.655	1027.536	5837.836	**Sep.**	
0	762.7818	599.0364	3138.155	875.1091	6024.427	**Oct.**	
0	382.0182	336.7636	1770.345	447.1909	4600.118	**Nov.**	
0	230.1545	25.18182	895.6	273.1818	1599.218	**Dec.**	

Lemon	**Shani**	**Green apple**	**Seven up**	**Mirnda**	**Pepsi**	**Product / Month**	**Year**
0	419.2364	503.9455	1451.227	595.0545	4172.036	**Jan.**	**2015**
248	541.3909	85.88182	2420.855	461.5091	4843.491	**Feb.**	
207.2455	491.6545	660.7636	2114.227	779.9818	5079.564	**Mar.**	
270.9182	518.1909	418.9273	2154.227	730.9	4427.327	**Apr.**	
227.0545	631.7	452.0636	2449.564	725.2636	5549.145	**May**	
373.1545	530.6364	447.5545	2437.291	683.4091	5161.527	**Jun.**	
210.0364	594.5636	457.0818	2373.618	811.6455	5333.673	**Jul.**	
369.4545	896.1364	611.3273	2542.418	1003.318	6641.418	**Aug.**	
308.2636	734.2182	499.3636	2857.155	888.8091	6935.145	**Sep.**	
274.0818	684.2	458.7273	2311.209	930.7455	5601.245	**Oct.**	
90	329.6818	273	1605	464.9091	3481.7	**Nov.**	
999.5727	2536.209	1873.6	9942.518	3104.745	23330.21	**Dec.**	
118.0091	589.3545	200.4364	1824.155	654.4273	5182.427	**Jan.**	**2016**
178.0182	427.5	497.1273	1903.173	565.6	4876.145	**Feb.**	
206.1364	472.4545	343.0909	1878.4	731.2455	5625.264	**Mar.**	

150.1182	706.4182	376.1727	2107.991	584.2364	5966.182	**Apr.**	
293.0091	630.2364	439.0091	2852.682	1186.164	7373.791	**May**	
378.3636	894.3909	630.4	2420.318	836.5909	6150.127	**Jun.**	
349.4	1311.9	395.3545	3068.864	1340.155	8873.027	**Jul.**	
278.9182	809.9727	552.0182	3173.309	1200.945	9114.691	**Aug.**	
220.8	583.6091	594.5455	3240.864	619.8273	8758.8	**Sep.**	
0	1078.718	401.0364	2586.373	892.2727	8705.891	**Oct.**	
0	779.6364	0.009091	1101.973	982.5364	4934.173	**Nov.**	
0	384.9364	460.8818	1225.818	372.9273	2729.255	**Dec.**	
108	467.4091	572.7727	1727.209	465.5091	6078.382	**Jan.**	
72.03636	800.4636	202.0818	2416.436	1050.936	9121.1	**Feb.**	
0.418182	1033.118	2.381818	3171.818	873.0727	9157.655	**Mar.**	
333	675	955.1091	1770.618	806.7818	5145.164	**Apr.**	
309.0091	799.5545	557.0091	2441.3	1024.836	9105.082	**May**	
191	845.0455	292.0455	2098.209	607.1	7503.2	**Jun.**	
218.2455	860.1273	682	3392.164	1240.045	9749.055	**Jul.**	**2017**
321	1227.009	533.0091	2897.173	1180.091	10344.14	**Aug.**	
280.5909	462.7091	619.5	2414.082	917.2818	5923.055	**Sep.**	
146.7455	1285.036	722.0273	2768.691	1010.109	7581.618	**Oct.**	
92.01818	591	387	1504.164	280.8273	4879.9	**Nov.**	
86.26364	558	184	1403.955	982.5091	5611.564	**Dec.**	

Source: Sales Department

Table (2). Actual production for produced containers

Lemon	**Shani**	**Green apple**	**Seven up**	**Mirnda**	**Pepsi**	**Product / Month**	**Year**
733.0182	0	652.6636	1752.545	0	2199.327	**Jan.**	
0	0	0	2457.8	1233.864	4619.164	**Feb.**	
0	520.0364	700.6727	2640.436	0	1726.736	**Mar.**	
0	0	1013.082	3008.173	1298.355	3570.236	**Apr.**	
0	439.6364	0	1913.691	675.7818	4658.491	**May**	
0	538.8091	0	3178.309	1445.336	3820.136	**Jun.**	
0	227.9727	0	4866.145	220.0455	6212.182	**Jul.**	**2012**
0	426.0727	1374.464	2962.773	632.2273	4309.091	**Aug.**	
293.7455	406.7273	294.3545	3184.382	184.7545	4518.209	**Sep.**	
0	456.2182	294.8909	3200.009	1169.945	5502.5	**Oct.**	
0	350.2909	726.1818	2070.273	667.2909	2492.6	**Nov.**	
0	558.7455	773.6636	292.7455	487.4091	4859.745	**Dec.**	
0	0	0	2650.227	429.9636	4421.955	**Jan.**	
0	402.1818	446.8727	2721.655	308.5182	3933.718	**Feb.**	
0	459.2727	0	2597.273	1034.127	5521.418	**Mar.**	
218.3182	575.5455	660.5455	2059.791	278.0364	3150.2	**Apr.**	
0	0	182.2364	843.5455	547.5545	2914.227	**May**	
0	0	0	658.2545	706.7091	1790.482	**Jun.**	
480.1636	641.2364	0	3853.636	1098.955	4446.673	**Jul.**	**2013**
0	466.3273	217.8727	4615.409	901.4182	6037.691	**Aug.**	
0	1179.955	365.3909	4727.036	609.7545	6757.527	**Sep.**	
0	207.1455	0	1156.964	731.4364	5481.655	**Oct.**	
0	587.8545	296.0909	3650.745	538.6455	5332.918	**Nov.**	
0	289.2727	333.5091	667.4091	304.5364	2586.936	**Dec.**	
0	347.1909	0	1050.818	630.8545	5132.182	**Jan.**	**2014**

0	691.1091	764.1455	1105.327	307.0727	3304.164	**Feb.**	
0	284.9091	294.4091	1884.273	331.6182	3347.582	**Mar.**	
0	290.7	38.36364	1125.918	647.4273	2222.155	**Apr.**	
0	928.3182	661.1273	4424.755	861.0091	6641.236	**May**	
0	640.3091	299.6727	1862.509	367.6455	3456.682	**Jun.**	
0	290.3273	0	2090.664	611.7091	3107.936	**Jul.**	
0	994.3273	708.8364	2727.809	755.9818	4731.555	**Aug.**	
0	810.8273	808.1818	2916.745	1076.6	5879.645	**Sep.**	
19.03636	908.2636	512.7545	3301.027	982.2364	6174.9	**Oct.**	
0	294.9091	222.8636	1865.673	311.7182	4004.073	**Nov.**	
0	293.8818	0	805.3182	246.4636	2023.464	**Dec.**	

Lemon	**Shani**	**Green apple**	**Seven up**	**Mirnda**	**Pepsi**	**Product / Month**	**Year**
0	288.6273	589.8273	1482.455	693.5091	3737.582	**Jan.**	**2015**
0	549.7636	0	2334.773	533.0545	5055.482	**Feb.**	
0	569.9636	849.5545	2426.055	747.4091	5602.7	**Mar.**	
0	569.9182	393.1	2073.745	609.6	5027.545	**Apr.**	
0	682.2182	402.9091	2664.118	840.3545	4950.745	**May**	
0	363.8091	471.4091	2252.618	849	5401.927	**Jun.**	
0	746.0727	518.2545	2403.691	581.1364	4734.573	**Jul.**	
0	994.2273	584.6636	2716.973	1228.191	7568.664	**Aug.**	
0	685.9636	513.9182	2698.9	874.0091	6178.136	**Sep.**	
0	461.4636	293.6091	2067.7	678.9909	5521.591	**Oct.**	
0	343.7727	439.4818	1428.564	648.5364	3014.045	**Nov.**	
0	2508.855	1856.582	10060.11	2994.027	24477.23	**Dec.**	
0	680.1455	59.2	1899.473	810.9909	4450.964	**Jan.**	**2016**
140.6273	400.8364	574.0364	2112.4	359.1727	5087.909	**Feb.**	
292.6091	502.8727	424.0727	1655.745	913.5727	6224.882	**Mar.**	
0	576.7091	221.1818	1862.527	359.6091	4756.827	**Apr.**	
296.7273	749.1364	436.1091	3112.809	1344.209	8251.636	**May**	
441.1091	992.1182	735.2545	2159.345	736.7364	6843.8	**Jun.**	
5042.636	1156.6	290.4	3068.827	1282.018	7305.436	**Jul.**	
0	654.3091	585.5636	3627.191	1233.764	9134.618	**Aug.**	
220.7	537.5545	589	3263.7	611.0455	8124.736	**Sep.**	
0	1077.345	401.3818	2549.155	888.4273	8738.473	**Oct.**	
0	781.5	0	1269.482	1212.155	4909.982	**Nov.**	
0	561.4909	577.0545	1098.791	239.4182	3458.782	**Dec.**	
108.5455	288.5727	456.7273	1816.645	361.2636	5472.536	**Jan.**	**2017**
71.90909	801.2364	204	2292.664	1214.618	9178.245	**Feb.**	
0	1028.218	0	3394.618	725.2636	9069.491	**Mar.**	
630.9545	1194.536	1309.082	2105.509	1268.518	6785.591	**Apr.**	
357.2727	383.2727	553.3636	1879.445	750.2273	8110.491	**May**	
0	803.4273	63.95455	2200.164	603.9727	6763.218	**Jun.**	
291.0545	796.4091	729.2636	3285.264	1130.145	9733.445	**Jul.**	
728.3	1292.9	364.4182	2896.782	1139.373	10338.15	**Aug.**	
0	396.8182	618.5909	2413.873	946.3455	6313.045	**Sep.**	
0	1889.073	1095.745	2986.882	934.7636	7957.8	**Oct.**	
362.5545	602.0091	367.3636	1285.973	277.8	4108.518	**Nov.**	
0	0	0	1564.291	1270.091	5593.427	**Dec.**	

Source: production operations Department

Table (3). Public holidays over the period studied 2012-2017

No. holidays in which there is no work	No. Fridays and Saturdays	No. days	Year	No. holidays in which there is no work	No. Fridays and Saturdays	No. days	month		Year
3	8	31	2013	2	8	31	Jan.	1	2012
-	8	28		-	8	29	Feb.	2	
1	10	31		1	10	31	Mar.	3	
-	8	30		-	8	30	Apr.	4	
1	9	31		1	8	31	May	5	
-	9	30		-	10	30	Jun.	6	
1	8	31		1	8	31	Jul.	7	
3	10	31		7	9	31	Aug.	8	
-	8	30		-	9	30	Sep.	9	
5	8	31		6	8	31	Oct.	10	
2	10	30		3	9	30	Nov.	11	
-	8	31		-	9	31	Dec.	12	
16	104	365		21	104	366	sum		
3	10	31	2015	3	9	31	Jan.	1	2014
-	8	28		-	8	28	Feb.	2	
1	8	31		1	9	31	Mar.	3	
-	8	30		-	8	30	Apr.	4	
1	10	31		1	10	31	May	5	
-	8	30		-	8	30	Jun.	6	
5	9	31		6	8	31	Jul.	7	
-	9	31		-	10	31	Aug.	8	
4	8	30		-	8	30	Sep.	9	
2	10	31		7	9	31	Oct.	10	
-	8	30		1	9	30	Nov.	11	
-	8	31		-	8	31	Dec.	12	
16	104	365		19	104	365	sum		
2	8	31	2017	2	10	31	Jan.	1	2016
-	8	28		-	8	29	Feb.	2	
1	9	31		1	8	31	Mar.	3	
-	9	30		-	10	30	Apr.	4	
1	8	31		1	8	31	May	5	
5	9	30		-	8	30	Jun.	6	
1	9	31		5	10	31	Jul.	7	
-	8	31		-	8	31	Aug.	8	
6	10	30		5	9	30	Sep.	9	
-	8	31		2	9	31	Oct.	10	
1	8	30		-	8	30	Nov.	11	
-	10	31		1	10	31	Dec.	12	
17	104	365		17	106	366	sum		

Table (4). The number of hours worked at regular time and overtime

Total hours worked	no. working hours in overtime	no. working hours in regular time	no. extra working days	no. regular working days	year	Total hours worked	no. working hours in overtime	no. working hours in regular time	no. extra working days	no. regular working days (Including extra hours)	month	year
560	240	320	8	20	**2013**	580	228	352	7	22	**Jan.**	**2012**
560	240	320	8	20		580	244	336	8	21	**Feb.**	
600	280	320	10	20		600	280	320	10	20	**Mar.**	
600	248	352	8	22		600	248	352	8	22	**Apr.**	
600	264	336	9	21		600	232	368	7	23	**May**	
600	264	336	9	21		600	280	320	10	20	**Jun.**	
600	248	352	8	22		600	232	368	7	23	**Jul.**	
560	240	320	8	20		480	208	272	7	17	**Aug.**	
600	248	352	8	22		600	264	336	9	21	**Sep.**	
520	216	304	7	19		500	196	304	6	19	**Oct.**	
560	272	288	10	18		540	236	304	8	19	**Nov.**	
620	252	368	8	23		620	268	352	9	22	**Dec.**	
560	256	304	9	19	**2015**	560	256	304	9	19	**Jan.**	**2014**
560	240	320	8	20		560	240	320	8	20	**Feb.**	
600	232	368	7	23		600	248	352	8	22	**Mar.**	
600	248	352	8	22		600	248	352	8	22	**Apr.**	
600	264	336	9	21		600	280	320	10	20	**May**	
600	248	352	8	22		600	248	352	8	22	**Jun.**	
520	216	304	7	19		500	228	272	8	17	**Jul.**	
620	268	352	9	22		620	284	336	10	21	**Aug.**	
520	200	320	6	20		600	248	352	8	22	**Sep.**	
580	260	320	9	20		480	208	272	7	17	**Oct.**	
600	248	352	8	22		580	260	320	9	20	**Nov.**	
620	252	368	8	23		620	252	368	8	23	**Dec.**	
580	228	352	7	22	**2017**	580	260	320	9	20	**Jan.**	**2016**
560	240	320	8	20		580	244	336	8	21	**Feb.**	
600	264	336	9	21		600	248	352	8	22	**Mar.**	
600	264	336	9	21		600	280	320	10	20	**Apr.**	
600	248	352	8	22		600	248	352	8	22	**May**	
500	244	256	9	16		600	248	352	8	22	**Jun.**	
600	248	352	8	22		520	232	288	8	18	**Jul.**	
620	252	368	8	23		620	252	368	8	23	**Aug.**	
480	208	272	7	17		500	244	256	9	16	**Sep.**	
620	252	368	8	23		580	260	320	9	20	**Oct.**	
580	244	336	8	21		600	248	352	8	22	**Nov.**	
620	284	336	10	21		600	280	320	10	20	**Dec.**	

Table (5). The quantity of production (s) at regular time N and overtime O for studied products and their total by the months for 2012

Dec.	Nov.	Oct.	Sep.	Aug.	Jul.	Jun.	May	Apr.	Mar.	Feb.	Jan.	month		
0.567742	0.562963	0.608	0.56	0.566667	0.613333	0.533333	0.613333	0.586667	0.533333	0.57931	0.606897	**Percentage of regular working hours**		
0.432258	0.437037	0.392	0.44	0.433333	0.386667	0.466667	0.386667	0.413333	0.466667	0.42069	0.393103	**Percentage of overtime hours**		
2759.081	1403.241	3345.52	2530.197	2441.818	3810.138	2037.406	2857.208	2094.539	920.9261	2675.929	1334.764	N	**pepsi**	product
2100.664	1089.359	2156.98	1988.012	1867.273	2402.044	1782.73	1801.283	1475.698	805.8103	1943.234	864.5631	O		
276.7226	375.6601	711.3268	103.4625	358.2621	134.9612	770.8461	414.4795	761.7013	0	714.79	0	N	**Mirnda**	
210.6865	291.6308	458.6186	81.292	273.9652	85.08424	674.4903	261.3023	536.6532	0	519.0737	0	O		
166.2039	1165.487	1945.606	1783.254	1678.905	2984.569	1695.098	1173.73	1764.795	1408.233	1423.829	1063.614	N	**Seven up**	
126.5416	904.7859	1254.404	1401.128	1283.868	1881.576	1483.211	739.9605	1243.378	1232.204	1033.971	688.9317	O		
439.2413	408.8135	179.2937	164.8385	778.8627	0	0	0	594.3413	373.6921	0	396.0993	N	**Green apple**	
334.4223	317.3684	115.5972	129.516	595.6009	0	0	0	418.7405	326.9806	0	256.5643	O		
317.2232	197.2008	277.3807	227.7673	241.4412	139.8233	287.3648	269.6436	0	277.3527	0	0	N	**Shani**	
241.5222	153.0901	178.8375	178.96	184.6315	88.14945	251.4442	169.9927	0	242.6836	0	0	O		
0	0	0	164.4975	0	0	0	0	0	0	0	444.8662	N	**Lemon**	
0	0	0	129.248	0	0	0	0	0	0	0	288.152	O		
3958.472	3550.403	6459.127	4974.017	5499.289	7069.492	4790.715	4715.061	5215.376	2980.204	4814.548	3239.343	N	**Total**	
3013.837	2756.234	4164.437	3908.156	4205.338	4456.854	4191.876	2972.539	3674.469	2607.678	3496.279	2098.211	O		

Table (6). The quantity of production (s) at regular time N and overtime O for studied products and their total by the months for 2013

Dec.	Nov.	Oct.	Sep.	Aug.	Jul.	Jun.	May	Apr.	Mar.	Feb.	Jan.	month		
0.593548	0.514286	0.584615	0.586667	0.571429	0.586667	0.56	0.56	0.586667	0.533333	0.571429	0.571429	**Percentage of regular working hours**		
0.406452	0.485714	0.415385	0.413333	0.428571	0.413333	0.44	0.44	0.413333	0.466667	0.428571	0.428571	**Percentage of overtime hours**		
1535.472	2742.644	3204.66	3964.416	3450.109	2608.715	1002.67	1631.967	1848.117	2944.756	2247.839	2526.831	N	**pepsi**	product
1051.464	2590.275	2276.995	2793.111	2587.582	1837.958	787.812	1282.26	1302.083	2576.662	1685.879	1895.123	O		
180.7571	277.0177	427.609	357.7227	515.0961	644.72	395.7571	306.6305	163.1147	551.5345	176.2961	245.6935	N	**Mirnda**	
123.7793	261.6278	303.8274	252.0319	386.3221	454.2345	310.952	240.924	114.9217	482.5927	132.2221	184.2701	O		
396.1396	1877.526	676.3787	2773.195	2637.377	2260.8	368.6225	472.3855	1208.411	1385.212	1555.231	1514.416	N	**Seven up**	

271.2695	1773.219	480.5849	1953.842	1978.032	1592.836	289.632	371.16	851.3802	1212.061	1166.423	1135.812	O		
197.9538	152.2753	0	214.3627	124.4987	0	0	102.0524	387.52	0	255.3558	0	N	**Green apple**	
135.5553	143.8156	0	151.0282	93.37403	0	0	80.184	273.0255	0	191.5169	0	O		
171.6974	302.3252	121.1004	692.24	266.4727	376.192	0	0	337.6533	244.9455	229.8182	0	N	**Shani**	
117.5754	285.5294	86.04503	487.7145	199.8545	265.0444	0	0	237.8921	214.3273	172.3636	0	O		
0	0	0	0	0	281.696	0	0	128.08	0	0	0	N	**Lemon**	
0	0	0	0	0	198.4676	0	0	90.23818	0	0	0	O		
2482.02	5351.788	4429.748	8001.936	6993.553	6172.123	1767.049	2513.036	4072.896	5126.448	4464.54	4286.94	N	**Total**	
1699.644	5054.466	3147.452	5637.728	5245.165	4348.541	1388.396	1974.528	2869.54	4485.642	3348.405	3215.205	O		

Table (7). The quantity of production (s) at regular time N and overtime O for studied products and their total by the months for 2014

Dec.	**Nov.**	**Oct.**	**Sep.**	**Aug.**	**Jul.**	**Jun.**	**May**	**Apr.**	**Mar.**	**Feb.**	**Jan.**	month		
0.593548	0.551724	0.566667	0.586667	0.541935	0.544	0.586667	0.533333	0.586667	0.586667	0.571429	0.542857	**Percentage of regular working hours**		
0.406452	0.448276	0.433333	0.413333	0.458065	0.456	0.413333	0.466667	0.413333	0.413333	0.428571	0.457143	**Percentage of overtime hours**		
1201.024	2209.144	3499.11	3449.392	2564.197	1690.717	2027.92	3541.993	1303.664	1963.915	1888.094	2786.042	N	**pepsi**	product
822.4401	1794.929	2675.79	2430.253	2167.357	1417.219	1428.762	3099.244	918.4905	1383.667	1416.07	2346.14	O		
146.2881	171.9824	556.6006	631.6053	409.6934	332.7697	215.6853	459.2048	379.824	194.5493	175.4701	342.4639	N	**Mirnda**	
100.1755	139.7357	425.6358	444.9947	346.2884	278.9393	151.9601	401.8042	267.6033	137.0688	131.6026	288.3906	O		
477.9953	1029.337	1870.582	1711.157	1478.297	1137.321	1092.672	2359.869	660.5387	1105.44	631.6156	570.4442	N	**Seven up**	
327.3229	836.3361	1430.445	1205.588	1249.513	953.3426	769.8371	2064.885	465.3795	778.8327	473.7117	480.374	O		
0	122.9592	290.5609	474.1333	384.1436	0	175.808	352.6012	22.50667	172.72	436.6545	0	N	**Green apple**	
0	99.90439	222.1936	334.0485	324.6928	0	123.8647	308.5261	15.85697	121.6891	327.4909	0	O		
174.4331	162.7085	514.6827	475.6853	538.8612	157.938	375.648	495.103	170.544	167.1467	394.9195	188.4751	N	**Shani**	
119.4487	132.2006	393.5809	335.1419	455.466	132.3892	264.6611	433.2152	120.156	117.7624	296.1896	158.7158	O		
0	0	10.78727	0	0	0	0	0	0	0	0	0	N	**Lemon**	
0	0	8.249091	0	0	0	0	0	0	0	0	0	O		
1999.74	3696.13	6742.324	6741.973	5375.192	3318.746	3887.733	7208.771	2537.077	3603.771	3526.753	3887.425	N	**Total**	
1369.387	3003.106	5155.895	4750.027	4543.317	2781.89	2739.085	6307.675	1787.486	2539.02	2645.065	3273.621	O		

Table (8). The quantity of production (s) at regular time N and overtime O for studied products and their total by the months for 2015

Dec.	Nov.	Oct.	Sep.	Aug.	Jul.	Jun.	May	Apr.	Mar.	Feb.	Jan.	month		
0.593548	0.586667	0.551724	0.615385	0.567742	0.584615	0.586667	0.56	0.586667	0.613333	0.571429	0.542857	**Percentage of regular working hours**		
0.406452	0.413333	0.448276	0.384615	0.432258	0.415385	0.413333	0.44	0.413333	0.386667	0.428571	0.457143	**Percentage of overtime hours**		
14528.42	1768.24	3046.395	3801.93	4297.048	2767.904	3169.131	2772.417	2949.493	3436.323	2888.847	2028.973	N	**pepsi**	product
9948.809	1245.805	2475.196	2376.206	3271.616	1966.669	2232.797	2178.328	2078.052	2166.377	2166.635	1708.609	O		
1777.1	380.4747	374.6157	537.8517	697.2955	339.7413	498.08	470.5985	357.632	458.4109	304.6026	376.4764	N	**Mirnda**	
1216.927	268.0617	304.3752	336.1573	530.8954	241.3951	350.92	369.756	251.968	288.9982	228.4519	317.0327	O		
5971.162	838.0907	1140.8	1660.862	1542.539	1405.235	1321.536	1491.906	1216.597	1487.98	1334.156	804.761	N	**Seven up**	
4088.948	590.473	926.9	1038.038	1174.433	998.4562	931.0822	1172.212	857.1481	938.0744	1000.617	677.6935	O		
1101.971	257.8293	161.9912	316.2573	331.9381	302.9796	276.56	225.6291	230.6187	521.0601	0	320.1919	N	**Green apple**	
754.6107	181.6525	131.6179	197.6608	252.7256	215.275	194.8491	177.28	162.4813	328.4944	0	269.6353	O		
1489.127	201.68	254.6006	422.1315	564.4645	436.1656	213.4347	382.0422	334.352	349.5777	314.1506	156.6834	N	**Shani**	
1019.728	142.0927	206.863	263.8322	429.7628	309.9071	150.3744	300.176	235.5662	220.3859	235.613	131.9439	O		
0	0	0	0	0	0	0	0	0	0	0	0	N	**Lemon**	
0	0	0	0	0	0	0	0	0	0	0	0	O		
24867.78	3446.315	4978.403	6739.032	7433.285	5252.025	5478.741	5342.593	5088.693	6253.352	4841.756	3687.086	N	**Total**	
17029.02	2428.085	4044.952	4211.895	5659.433	3731.702	3860.022	4197.752	3585.216	3942.33	3631.317	3104.914	O		

Table (9). The quantity of production (s) at regular time N and overtime O for studied products and their total by the months for 2016

Dec.	Nov.	Oct.	Sep.	Aug.	Jul.	Jun.	May	Apr.	Mar.	Feb.	Jan.	month		
0.533333	0.586667	0.551724	0.512	0.593548	0.553846	0.586667	0.586667	0.533333	0.586667	0.57931	0.551724	**Percentage of regular working hours**		
0.466667	0.413333	0.448276	0.488	0.406452	0.446154	0.413333	0.413333	0.466667	0.413333	0.42069	0.448276	**Percentage of overtime hours**		
1844.684	2880.523	4821.226	4159.865	5421.838	4046.088	4015.029	4840.96	2536.975	3651.931	2947.478	2455.704	N	**pepsi**	product
1614.098	2029.459	3917.246	3964.871	3712.78	3259.349	2828.771	3410.676	2219.853	2572.951	2140.431	1995.26	O		
127.6897	711.1307	490.1668	312.8553	732.2984	710.0408	432.2187	788.6027	191.7915	535.9627	208.0725	447.4433	N	**Mirnda**	
111.7285	501.0239	398.2605	298.1902	501.4652	571.9773	304.5177	555.6064	167.8176	377.6101	151.1003	363.5476	O		
586.0218	744.7627	1406.43	1671.014	2152.913	1699.658	1266.816	1826.181	993.3479	971.3707	1223.735	1047.985	N	**Seven up**	
512.7691	524.7192	1142.724	1592.686	1474.278	1369.169	892.5295	1286.628	869.1794	684.3748	888.6648	851.4878	O		

307.7624	0	221.452	301.568	347.5604	160.8369	431.3493	255.8507	117.9636	248.7893	332.5452	32.66207	N	**Green apple**	
269.2921	0	179.9298	287.432	238.0033	129.5631	303.9052	180.2584	103.2182	175.2834	241.4912	26.53793	O		
299.4618	458.48	594.3975	275.2279	388.3641	640.5785	582.0427	439.4933	307.5782	295.0187	232.2087	375.2527	N	**Shani**	
262.0291	323.02	482.948	262.3266	265.945	516.0215	410.0755	309.643	269.1309	207.8541	168.6277	304.8928	O		
0	0	0	112.9984	0	2792.845	258.784	174.08	0	171.664	81.46683	0	N	**Lemon**	
0	0	0	107.7016	0	2249.792	182.3251	122.6473	0	120.9451	59.16044	0	O		
3165.619	4794.896	7533.673	6833.529	9042.974	10050.05	6986.24	8325.168	4147.656	5874.736	5025.507	4359.047	N	**Total**	
2769.917	3378.222	6121.109	6513.207	6192.471	8095.871	4922.124	5865.459	3629.199	4139.019	3649.475	3541.726	O		

Table (10). The quantity of production (s) at regular time N and overtime O for studied products and their total by the months 2017

Dec.	**Nov.**	**Oct.**	**Sep.**	**Aug.**	**Jul.**	**Jun.**	**May**	**Apr.**	**Mar.**	**Feb.**	**Jan.**	month		
0.541935	0.57931	0.593548	0.566667	0.593548	0.586667	0.512	0.586667	0.56	0.56	0.571429	0.606897	**Percentage of regular working hours**		
0.458065	0.42069	0.406452	0.433333	0.406452	0.413333	0.488	0.413333	0.44	0.44	0.428571	0.393103	**Percentage of overtime hours**		
3031.277	2380.107	4723.339	3577.392	6136.19	5710.288	3462.768	4758.155	3799.931	5078.915	5244.712	3321.263	N	**pepsi**	product
2562.151	1728.411	3234.461	2735.653	4201.956	4023.157	3300.45	3352.336	2985.66	3990.576	3933.534	2151.273	O		
688.3073	160.9324	554.8274	536.2624	676.2728	663.0187	309.234	440.1333	710.3702	406.1476	694.0675	219.2497	N	**Mirnda**	
581.7836	116.8676	379.9362	410.083	463.0999	467.1268	294.7387	310.0939	558.148	319.116	520.5506	142.014	O		
847.7448	744.9773	1772.859	1367.861	1719.38	1927.355	1126.484	1102.608	1179.085	1900.986	1310.094	1102.516	N	**Seven up**	
716.5462	540.9954	1214.023	1046.012	1177.402	1357.909	1073.68	776.8375	926.424	1493.632	982.5701	714.1296	O		
0	212.8176	650.3779	350.5348	216.2998	427.8347	32.74473	324.64	733.0858	0	116.5714	277.1862	N	**Green apple**	
0	154.5461	445.3675	268.0561	148.1184	301.429	31.20982	228.7236	575.996	0	87.42857	179.5411	O		
0	348.7501	1121.256	224.8636	767.3987	467.2267	411.3548	224.8533	668.9404	575.8022	457.8494	175.1338	N	**Shani**	
0	253.259	767.8167	171.9545	525.5013	329.1824	392.0725	158.4194	525.596	452.416	343.387	113.4389	O		
0	210.0316	0	0	432.2813	170.752	0	209.6	353.3345	0	41.09091	65.87586	N	**Lemon**	
0	152.5229	0	0	296.0187	120.3025	0	147.6727	277.62	0	30.81818	42.66959	O		
4567.329	4057.616	8822.66	6056.915	9947.822	9366.475	5342.585	7059.989	7444.747	7961.851	7864.384	5161.225	N	**Total**	
3860.48	2946.602	6041.604	4631.758	6812.096	6599.107	5092.151	4974.083	5849.444	6255.74	5898.288	3343.066	O		

Table (11). Statistical models used to represent the demand and forecast its future values for each product

AIC	$\sqrt{MSE}$	Estimated parameters		The appropriate model	product
1156.113	2901.56	AR(1)	0.124865	SARIMA(1,0,0)x(2,0,0)2	Pepsi
		SAR(1)	0.10239		
		SAR(2)	0.120968		
		Constant	3762.39		
868.232	398.507	AR(1)	-0.0198591	SARIMA(1,0,0)x(1,0,0)2	Mirnda
		SAR(1)	0.0911998		
		Constant	701.95		
1037.481	1273.05	AR(1)	0.035999	SARIMA(1,0,0)x(2,0,0)2	Seven up
		SAR(1)	-0.0758764		
		SAR(2)	-0.129127		
		Constant	2866.35		
834.550	315.394	AR(1)	-0.107634	SARIMA(1,0,0)x(1,0,0)2	Green apple
		SAR(1)	-0.103428		
		Constant	516.841		
855.781	360.455	AR(1)	0.0948049	SARIMA(1,0,0)x(2,0,0)2	Shani
		SAR(1)	0.143061		
		SAR(2)	0.207756		
		Constant	341.279		
735.868	156.748	AR(1)	0.219731	SARIMA(1,0,0)x(2,0,0)2	Lemon
		SAR(1)	0.193886		
		SAR(2)	0.167199		
		Constant	72.9495		
1230.528	4932.81	AR(1)	0.0905548	SARIMA(1,0,0)x(1,0,0)2	Total
		SAR(1)	0.0650312		
		Constant	8439.96		

Table (12). Statistical models used to represent the production in regular time and forecast its future values for each product

AIC	$\sqrt{MSE}$	Estimated parameters		The appropriate model	product
1084.046	1783.68	AR(1)	0.0739811	SARIMA(1,0,0)x(1,0,0)2	Pepsi
		SAR(1)	0.129073		
		Constant	2551.15		
807.055	260.573	AR(1)	-0.107339	SARIMA(1,0,0)x(1,0,0)2	Mirnda
		SAR(1)	0.0479871		
		Constant	456.878		
968.914	790.778	AR(1)	0.0121666	SARIMA(1,0,0)x(2,0,0)2	Seven up
		SAR(1)	-0.0571747		
		SAR(2)	-0.121763		
		Constant	1642.7		
776.144	210.235	AR(1)	-0.172971	SARIMA(1,0,0)x(1,0,0)2	Green apple
		SAR(1)	-0.181965		
		Constant	335.039		
798.885	242.805	AR(1)	0.0554524	SARIMA(1,0,0)x(2,0,0)2	Shani
		SAR(1)	0.0151869		
		SAR(2)	0.21201		
		Constant	243.279		
846.836	343.483	AR(1)	0.0533575	SARIMA(1,0,0)x(1,0,0)2	Lemon
		SAR(1)	0.0525659		
		Constant	76.5246		

1159.329	3008.59	AR(1)	0.0694115	SARIMA(1,0,0)x(1,0,0)2	Total
		SAR(1)	0.0621425		
		Constant	4945.08		

Table (13). Statistical models used to represent the production in overtime and forecast its future values for each product

AIC	$\sqrt{\mathbf{MSE}}$	Estimated parameters		The appropriate model	product
1032.294	1228.01	AR(1)	0.0741917	SARIMA(1,0,0)x(2,0,0)2	Pepsi
		SAR(1)	0.156831		
		SAR(2)	0.149734		
		Constant	1508.98		
758.850	183.871	AR(1)	-0.134665	SARIMA(1,0,0)x(2,0,0)2	Mirnda
		SAR(1)	0.0677458		
		SAR(2)	0.179625		
		Constant	276.618		
917.789	562.205	AR(1)	0.0104805	SARIMA(1,0,0)x(1,0,0)2	Seven up
		SAR(1)	-0.0447293		
		Constant	1090.14		
730.942	151.476	AR(1)	-0.20674	SARIMA(1,0,0)x(2,0,0)2	Green apple
		SAR(1)	-0.192281		
		SAR(2)	0.132633		
		Constant	230.352		
749.494	172.305	AR(1)	0.047784	SARIMA(1,0,0)x(2,0,0)2	Shani
		SAR(1)	0.0267726		
		SAR(2)	0.266797		
		Constant	167.543		
814.444	274.293	AR(1)	0.0441565	SARIMA(1,0,0)x(1,0,0)2	Lemon
		SAR(1)	0.0571479		
		Constant	58.1526		
1107.144	2094.0	AR(1)	0.0891489	SARIMA(1,0,0)x(1,0,0)2	Total
		SAR(1)	0.104764		
		Constant	3452.77		

Table (14). Results of the Box-Pierce statistic to check demand models ability for forecasting

Critical value	Box-Pierce statistic value	The appropriate model	Product
0.765097	16.0808	SARIMA(1,0,0)x(2,0,0)2	Pepsi
0.983501	10.288	SARIMA(1,0,0)x(**1**,0,0)2	Mirnda
0.896982	13.3205	SARIMA(1,0,0)x(**2**,0,0)2	Seven up
0.983091	10.3269	SARIMA(1,0,0)x(**1**,0,0)2	Green apple
0.826958	14.9197	SARIMA(1,0,0)x(2,0,0)2	Shani
0.998833	6.57869	SARIMA(1,0,0)x(**2**,0,0)2	Lemon
0.922043	13.3792	SARIMA(1,0,0)x(1,0,0)2	Total

Table (15). Results of the Box-Pierce statistic to check ability of production models at regular time for forecasting

Critical value	Box-Pierce statistic value	The appropriate model	Product
0.672562	18.556	SARIMA(1,0,0)x(1,0,0)2	Pepsi
0.794738	16.4156	SARIMA(1,0,0)x(1,0,0)2	Mirnda
0.954864	11.3801	SARIMA(1,0,0)x(2,0,0)2	Seven up
0.960417	11.8491	SARIMA(1,0,0)x(1,0,0)2	Green apple
0.592852	18.8798	SARIMA(1,0,0)x(2,0,0)2	Shani
1.0	2.35727	SARIMA(1,0,0)x(1,0,0)2	Lemon
0.895132	14.1755	SARIMA(1,0,0)x(1,0,0)2	Total

Table (16). Results of the Box-Pierce statistic to check ability of production models at overtime for forecasting

Critical value	Box-Pierce statistic value	The appropriate model	Product
0.783868	15.7438	SARIMA(1,0,0)x(2,0,0)2	Pepsi
0.759108	16.1861	SARIMA(1,0,0)x(2,0,0)2	Mirnda
0.933972	12.9708	SARIMA(1,0,0)x(1,0,0)2	Seven up
0.964566	10.9071	SARIMA(1,0,0)x(2,0,0)2	Green apple
0.783719	15.7465	SARIMA(1,0,0)x(2,0,0)2	Shani
1.0	1.98465	SARIMA(1,0,0)x(1,0,0)2	Lemon
0.843194	15.4287	SARIMA(1,0,0)x(1,0,0)2	Total

Table (17). Forecasts of Demand for Months of 2018 from March to December for products under study and their total

Total	Lemon	Shani	Green apple	Seven up	Mirnda	pepsi	product / month
9916.352	134.806	575.875	420.44	2588.51	753.021	5443.7	**Mar.**
10080.38	132.998	596.149	420.546	2601.8	759.232	5569.65	**Apr.**
9858.606	142.76	570.194	423.135	2452.61	756.957	5512.95	**May**
9954.698	141.079	612.205	423.121	2452.12	757.523	5568.65	**Jun.**
9877.364	143.709	578.236	422.853	2453.05	757.316	5522.2	**Jul.**
9906.264	143.083	588.458	422.855	2451.37	757.368	5543.13	**Aug.**
9905.744	145.226	578.206	422.883	2470.56	757.349	5531.52	**Sep.**
9924.615	144.823	588.397	422.882	2470.76	757.353	5540.4	**Oct.**
9908.553	145.678	579.873	422.88	2469.18	757.352	5533.59	**Nov.**
9915.592	145.495	583.455	422.88	2469.38	757.352	5537.03	**Dec.**

Table (18). Forecasts of production at regular time for Months of 2018 from March to December for products under study and their total

Total	Lemon	Shani	Green apple	Seven up	Mirnda	pepsi	product / month
5725.016	85.4286	335.773	236.304	1489.45	431.17	3146.89	**Mar.**
5659.258	85.0745	265.059	234.418	1479.75	434.146	3160.81	**Apr.**
5647.262	85.3285	323.356	242.659	1401.5	433.278	3161.14	**May**
5698.993	85.3099	366.306	242.972	1408.04	433.425	3162.94	**Jun.**
5658.264	85.3233	333.66	241.478	1401.44	433.383	3162.98	**Jul.**
5644.912	85.3223	319.32	241.42	1402.25	433.39	3163.21	**Aug.**
5666.967	85.323	331.184	241.692	1412.16	433.388	3163.22	**Sep.**
5675.046	85.323	340.072	241.703	1411.31	433.388	3163.25	**Oct.**
5668.495	85.323	333.331	241.653	1411.55	433.388	3163.25	**Nov.**
5665.537	85.323	330.425	241.651	1411.5	433.388	3163.25	**Dec.**

Table (19). Forecasts of production at overtime for Months of 2018 from March to December for products under study and their total

Total	Lemon	Shani	Green apple	Seven up	Mirnda	pepsi	product / month
4069.959	64.6552	249.172	169.698	1053.64	284.174	2248.62	**Mar.**
4228.313	64.3089	186.123	143.624	1053.84	372.457	2407.96	**Apr.**
4188.43	64.5339	239.321	187.822	1054.55	315.933	2326.27	**May**
4314.102	64.5142	282.325	195.706	1054.54	332.897	2384.12	**Jun.**
4192.822	64.527	248.836	177.283	1054.51	316.236	2331.43	**Jul.**
4222.112	64.5259	233.166	172.306	1054.51	333.244	2364.36	**Aug.**
4213.042	64.5266	246.462	181.711	1054.51	321.962	2343.87	**Sep.**
4244.128	64.5266	257.516	183.714	1054.51	326.161	2357.7	**Oct.**
4216.431	64.5266	248.938	179.462	1054.51	322.404	2346.59	**Nov.**
4221.939	64.5266	245.053	178.416	1054.51	325.743	2353.69	**Dec.**

Table (20). The optimal solution (lowest possible cost) using the transportation model for Pepsi product

Supply from		Demand for											Total capacity available (supply)
		Period 1 March	**Period 2 April**	**Period 3 May**	**Period 4 June**	**Period 5 July**	**Period 6 August**	**Period 7 September**	**Period 8 October**	**Period 9 November**	**Period 10 December**	**Dummy**	
Beginning inventory		0 **2727.7**	3000	6000	9000	12000	15000	18000	21000	24000	27000	0	2727.7
Period 1	**Regular time**	660000 **2716**	663000 **430.89**	666000	669000	672000	675000	678000	681000	684000	687000	0	3146.89
	Over time	673000	676000	679000	682000	685000	688000	691000	694000	697000	700000	0 **2248.62**	2248.62
Period 2	**Regular time**	X	660000 **3160.81**	663000	666000	669000	672000	675000	678000	681000	684000	0	3160.81
	Over time	X	673000 **1977.95**	676000 **178.2**	679000	682000	685000	688000	691000	694000	697000	0 **251.81**	2407.96
Period 3	**Regular time**	X	X	660000 **3161.14**	663000	666000	669000	672000	675000	678000	681000	0	3161.14
	Over time	X	X	673000 **2173.61**	676000 **152.66**	679000	682000	685000	688000	691000	694000	0	2326.27
Period 4	**Regular time**	X	X	X	660000 **3162.94**	663000	666000	669000	672000	675000	678000	0	3162.94
	Over time	X	X	X	673000 **2253.05**	676000 **131.03**	679000	682000	685000	688000	691000	0	2384.12
Period 5	**Regular time**	X	X	X	X	660000 **3162.98**	663000	666000	669000	672000	675000	0	3162.98
	Over time	X	X	X	X	673000 **2228.15**	676000 **103.28**	679000	682000	685000	688000	0	2331.43
Period 6	**Regular time**	X	X	X	X	X	660000 **3163.21**	663000	666000	669000	672000	0	3163.21
	Over time	X	X	X	X	X	673000 **2276.64**	676000 **87.72**	679000	682000	685000	0	2364.36
Period 7	**Regular time**	X	X	X	X	X	X	660000 **3163.22**	663000	666000	669000	0	3163.22
	Over time	X	X	X	X	X	X	673000 **2280.58**	676000 **63.28**	679000	682000	0	2343.87
Period 8	**Regular time**	X	X	X	X	X	X	X	660000 **3163.25**	663000	666000	0	3163.25
	Over time	X	X	X	X	X	X	X	673000 **2313.86**	676000 **43.89**	679000	0	2357.7

Period 9	**Regular time**	X	X	X	X	X	X	X	X	660000 **3163.25**	663000	0	3163.25
	Over time	X	X	X	X	X	X	X	X	673000 **2326.5**	676000 **20.09**	0	2346.59
Period 10	**Regular time**	X	X	X	X	X	X	X	X	X	660000 **3163.25**	0	3163.25
	Over time	X	X	X	X	X	X	X	X	X	673000 **2353.69**	0	2353.69
Total Demand		5443.7	5569.65	5512.95	5568.65	5522.2	5543.13	5531.52	5540.4	5533.59	5537.03	2500.43	57803.25

1. The x sign means a very high cost, so that they are not taken into account (Heizer et al,2017:544) Optimal cost = 34975750000
2. dummy column is for balance purposes between production and demand

Table (21). The optimal solution (lowest possible cost) using the transportation model for Mirnda product

Supply from		Demand for											Total capacity available (supply)
		Period 1 March	**Period 2 April**	**Period 3 May**	**Period 4 June**	**Period 5 July**	**Period 6 August**	**Period 7 September**	**Period 8 October**	**Period 9 November**	**Period 10 December**	**Dummy**	
Beginning inventory		0 **279.2**	3000	6000	9000	12000	15000	18000	21000	24000	27000	0	279.2
Period 1	**Regular time**	660000 **431.17**	663000	666000	669000	672000	675000	678000	681000	684000	687000	0	431.17
	Over time	673000 **42.65**	676000	679000	682000	685000	688000	691000	694000	697000	700000	0 **241.52**	284.174
Period 2	**Regular time**	X	660000 **434.15**	663000	666000	669000	672000	675000	678000	681000	684000	0	434.146
	Over time	X	673000 **325.09**	676000 **7.75**	679000	682000	685000	688000	691000	694000	697000	0 **39.63**	372.457
Period 3	**Regular time**	X	X	660000 **433.28**	663000	666000	669000	672000	675000	678000	681000	0	433.278
	Over time	X	X	673000 **315.93**	676000	679000	682000	685000	688000	691000	694000	0	315.933
Period 4	**Regular time**	X	X	X	660000 **433.43**	663000	666000	669000	672000	675000	678000	0	433.425
	Over time	X	X	X	673000 **324.1**	676000 **7.7**	679000	682000	685000	688000	691000	0 **1.1**	332.897
Period 5	**Regular time**	X	X	X	X	660000 **433.38**	663000	666000	669000	672000	675000	0	433.383
	Over time	X	X	X	X	673000 **316.24**	676000	679000	682000	685000	688000	0	316.236

Period 6	**Regular time**	X	X	X	X	X	660000 **433.39**	663000	666000	669000	672000	0	433.39
	Over time	X	X	X	X	X	673000 **323.98**	676000 **2**	679000	682000	685000	0 **7.27**	333.244
Period 7	**Regular time**	X	X	X	X	X	X	660000 **433.39**	663000	666000	669000	0	433.388
	Over time	X	X	X	X	X	X	673000 **321.96**	676000	679000	682000	0	321.962
Period 8	**Regular time**	X	X	X	X	X	X	X	660000 **433.39**	663000	666000	0	433.388
	Over time	X	X	X	X	X	X	X	673000 **323.97**	676000 **1.56**	679000	0 **0.64**	326.161
Period 9	**Regular time**	X	X	X	X	X	X	X	X	660000 **433.39**	663000	0	433.388
	Over time	X	X	X	X	X	X	X	X	673000 **322.4**	676000	0	322.404
Period 10	**Regular time**	X	X	X	X	X	X	X	X	X	660000 **433.39**	0	433.388
	Over time	X	X	X	X	X	X	X	X	X	673000 **323.96**	0 **1.78**	325.743
Total Demand		753.021	759.232	756.957	757.523	757.316	757.368	757.349	757.353	757.352	757.352	291.932	7862.755

Optimal cost = 4850998000

Table (22). The optimal solution (lowest possible cost) using the transportation model for Seven up product

Supply from		Demand for											Total capacity available (supply)
		Period 1 March	**Period 2 April**	**Period 3 May**	**Period 4 June**	**Period 5 July**	**Period 6 August**	**Period 7 September**	**Period 8 October**	**Period 9 November**	**Period 10 December**	**Dummy**	
Beginning inventory		0 1598.53	3000	6000	9000	12000	15000	18000	21000	24000	27000	0	1598.53
Period 1	**Regular time**	660000 **989.98**	663000 **499.47**	666000	669000	672000	675000	678000	681000	684000	687000	0	1489.45
	Over time	673000	676000	679000	682000	685000	688000	691000	694000	697000	700000	0 **1053.64**	1053.64
Period 2	**Regular time**	X	660000 **1479.75**	663000	666000	669000	672000	675000	678000	681000	684000	0	1479.75
	Over time	X	673000 **622.58**	676000	679000	682000	685000	688000	691000	694000	697000	0 **431.26**	1053.84

Period 3	**Regular time**	X	X	660000 **1401.5**	663000	666000	669000	672000	675000	678000	681000	0	1401.5
	Over time	X	X	673000 **1051.11**	676000	679000	682000	685000	688000	691000	694000	0 **3.44**	1054.55
Period 4	**Regular time**	X	X	X	660000 **1408.04**	663000	666000	669000	672000	675000	678000	0	1408.04
	Over time	X	X	X	673000 **1044.08**	676000 **7.03**	679000	682000	685000	688000	691000	0 **3.43**	1054.54
Period 5	**Regular time**	X	X	X	X	660000 **1401.44**	663000	666000	669000	672000	675000	0	1401.44
	Over time	X	X	X	X	673000 **1044.58**	676000 **9.93**	679000	682000	685000	688000	0	1054.51
Period 6	**Regular time**	X	X	X	X	X	660000 **1402.25**	663000	666000	669000	672000	0	1402.25
	Over time	X	X	X	X	X	673000 **1039.19**	676000 **15.32**	679000	682000	685000	0	1054.51
Period 7	**Regular time**	X	X	X	X	X	X	660000 **1412.16**	663000	666000	669000	0	1412.16
	Over time	X	X	X	X	X	X	673000 **1043.08**	676000 **11.43**	679000	682000	0	1054.51
Period 8	**Regular time**	X	X	X	X	X	X	X	660000 **1411.31**	663000	666000	0	1411.31
	Over time	X	X	X	X	X	X	X	673000 **1048.02**	676000 **6.49**	679000	0	1054.51
Period 9	**Regular time**	X	X	X	X	X	X	X	X	660000 **1411.55**	663000	0	1411.55
	Over time	X	X	X	X	X	X	X	X	673000 **1051.14**	676000 **3.37**	0	1054.51
Period 10	**Regular time**	X	X	X	X	X	X	X	X	X	660000 **1411.5**	0	1411.50
	Over time	X	X	X	X	X	X	X	X	X	673000 **1054.51**	0	1054.51
Total Demand		2588.51	2601.8	2452.61	2452.12	2453.05	2451.37	2470.56	2470.76	2469.18	2469.38	1491.77	26371.11

Optimal cost = 15484670000

Table (23). The optimal solution (lowest possible cost) using the transportation model for Green apple product

Supply from		Demand for											Total capacity available (supply)
		Period 1 March	**Period 2 April**	**Period 3 May**	**Period 4 June**	**Period 5 July**	**Period 6 August**	**Period 7 September**	**Period 8 October**	**Period 9 November**	**Period 10 December**	**Dummy**	
Beginning inventory		0 **420.44**	3000 **138.6**	6000	9000	12000	15000	18000	21000	24000	27000	0	559.04
Period 1	**Regular time**	660000	663000 **236.3**	666000	669000	672000	675000	678000	681000	684000	687000	0	236.304
	Over time	673000	676000	679000	682000	685000	688000	691000	694000	697000	700000	0 **169.7**	169.698
Period 2	**Regular time**	X	660000 **45.64**	663000 **188.78**	666000	669000	672000	675000	678000	681000	684000	0	234.418
	Over time	X	673000	676000	679000	682000	685000	688000	691000	694000	697000	0 **143.62**	143.624
Period 3	**Regular time**	X	X	660000 **234.36**	663000 **8.3**	666000	669000	672000	675000	678000	681000	0	242.659
	Over time	X	X	673000	676000	679000	682000	685000	688000	691000	694000	0 **187.82**	187.822
Period 4	**Regular time**	X	X	X	660000 **242.97**	663000	666000	669000	672000	675000	678000	0	242.972
	Over time	X	X	X	673000 **171.85**	676000 **14.74**	679000	682000	685000	688000	691000	0 **9.11**	195.706
Period 5	**Regular time**	X	X	X	X	660000 **241.48**	663000	666000	669000	672000	675000	0	241.478
	Over time	X	X	X	X	673000 **166.63**	676000 **10.65**	679000	682000	685000	688000	0	177.283
Period 6	**Regular time**	X	X	X	X	X	660000 **241.42**	663000	666000	669000	672000	0	241.42
	Over time	X	X	X	X	X	673000 **170.78**	676000 **1.52**	679000	682000	685000	0	172.306
Period 7	**Regular time**	X	X	X	X	X	X	660000 **241.69**	663000	666000	669000	0	241.692
	Over time	X	X	X	X	X	X	673000 **179.67**	676000 **2.04**	679000	682000	0	181.711
Period 8	**Regular time**	X	X	X	X	X	X	X	660000 **241.7**	663000	666000	0	241.703
	Over time	X	X	X	X	X	X	X	673000 **179.14**	676000 **4.58**	679000	0	183.714

Period 9	**Regular time**	X	X	X	X	X	X	X	X	660000 **241.65**	663000	0	241.653
	Over time	X	X	X	X	X	X	X	X	673000 **176.65**	676000 **2.81**	0	179.462
Period 10	**Regular time**	X	X	X	X	X	X	X	X	X	660000 **241.65**	0	241.651
	Over time	X	X	X	X	X	X	X	X	X	673000 **178.42**	0	178.416
Total Demand		420.44	420.546	423.135	423.121	422.853	422.855	422.883	422.882	422.88	422.88	510.257	4734.732

Optimal cost = 2437385000

Table (24). The optimal solution (lowest possible cost) using the transportation model for Shani product

Supply from		Demand for											Total capacity available (supply)
		Period 1 **March**	**Period 2** **April**	**Period 3** **May**	**Period 4** **June**	**Period 5** **July**	**Period 6** **August**	**Period 7** **September**	**Period 8** **October**	**Period 9** **November**	**Period 10** **December**	**Dummy**	
Beginning inventory		0 **457.81**	3000	6000 **7.52**	9000	12000	15000	18000	21000	24000	27000	0	465.33
Period 1	**Regular time**	660000 **118.06**	663000 **217.71**	666000	669000	672000	675000	678000	681000	684000	687000	0	335.773
	Over time	673000	676000	679000	682000	685000	688000	691000	694000	697000	700000	0 **249.172**	249.172
Period 2	**Regular time**	X	660000 **265.06**	663000	666000	669000	672000	675000	678000	681000	684000	0	265.059
	Over time	X	673000 **113.38**	676000	679000	682000	685000	688000	691000	694000	697000	0 **72.74**	186.123
Period 3	**Regular time**	X	X	660000 **323.36**	663000	666000	669000	672000	675000	678000	681000	0	323.356
	Over time	X	X	673000 **239.32**	676000	679000	682000	685000	688000	691000	694000	0	239.321
Period 4	**Regular time**	X	X	X	660000 **366.31**	663000	666000	669000	672000	675000	678000	0	366.306
	Over time	X	X	X	673000 **245.9**	676000 **32.27**	679000	682000	685000	688000	691000	0 **4.15**	282.325
Period 5	**Regular time**	X	X	X	X	660000 **333.66**	663000	666000	669000	672000	675000	0	333.66
	Over time	X	X	X	X	673000 **212.3**	676000 **36.53**	679000	682000	685000	688000	0	248.836

Period 6	**Regular time**	X	X	X	X	X	660000 **319.32**	663000	666000	669000	672000	0	319.32
	Over time	X	X	X	X	X	673000 **232.61**	676000 **0.56**	679000	682000	685000	0	233.166
Period 7	**Regular time**	X	X	X	X	X	X	660000 **331.18**	663000	666000	669000	0	331.184
	Over time	X	X	X	X	X	X	673000 **246.46**	676000	679000	682000	0	246.462
Period 8	**Regular time**	X	X	X	X	X	X	X	660000 **340.07**	663000	666000	0	340.072
	Over time	X	X	X	X	X	X	X	673000 **248.33**	676000 **5.58**	679000	0 **3.61**	257.516
Period 9	**Regular time**	X	X	X	X	X	X	X	X	660000 **333.33**	663000	0	333.331
	Over time	X	X	X	X	X	X	X	X	673000 **240.96**	676000 **7.98**	0	248.938
Period 10	**Regular time**	X	X	X	X	X	X	X	X	X	660000 **330.43**	0	330.425
	Over time	X	X	X	X	X	X	X	X	X	673000 **245.053**	0	245.053
Total Demand		575.875	596.149	570.194	612.205	578.236	588.458	578.206	588.397	579.873	583.455	329.68	6180.728

Optimal cost = 3582915000

Table (25)

Supply from		Demand for											Total capacity available (supply)
		Period 1 March	**Period 2 April**	**Period 3 May**	**Period 4 June**	**Period 5 July**	**Period 6 August**	**Period 7 September**	**Period 8 October**	**Period 9 November**	**Period 10 December**	**Dummy**	
Beginning inventory		0 134.8	3000 **133**	6000 **35.2**	9000	12000	15000	18000	21000	24000	27000	0	303
Period 1	**Regular time**	660000	663000	666000 **51.32**	669000	672000	675000	678000	681000	684000	687000	0 **34.11**	85.4286
	Over time	673000	676000	679000	682000	685000	688000	691000	694000	697000	700000	0 **64.66**	64.6552
Period 2	**Regular time**	X	660000	663000 **56.25**	666000 **28.83**	669000	672000	675000	678000	681000	684000	0	85.0745
	Over time	X	673000	676000	679000	682000	685000	688000	691000	694000	697000	0 **64.31**	64.3089

Period 3	**Regular time**	X	X	660000	663000 **85.33**	666000	669000	672000	675000	678000	681000	0	85.3285
	Over time	X	X	673000	676000	679000	682000	685000	688000	691000	694000	0 **64.53**	64.5339
Period 4	**Regular time**	X	X	X	660000 **26.92**	663000 **58.39**	666000	669000	672000	675000	678000	0	85.3099
	Over time	X	X	X	673000	676000	679000	682000	685000	688000	691000	0 **64.51**	64.5142
Period 5	**Regular time**	X	X	X	X	660000 **85.32**	663000	666000	669000	672000	675000	0	85.3233
	Over time	X	X	X	X	673000	676000	679000	682000	685000	688000	0 **64.53**	64.527
Period 6	**Regular time**	X	X	X	X	X	660000 **85.32**	663000	666000	669000	672000	0	85.3223
	Over time	X	X	X	X	X	673000 **57.76**	676000	679000	682000	685000	0 **6.77**	64.5259
Period 7	**Regular time**	X	X	X	X	X	X	660000 **85.32**	663000	666000	669000	0	85.323
	Over time	X	X	X	X	X	X	673000 **59.9**	676000	679000	682000	0 **4.62**	64.5266
Period 8	**Regular time**	X	X	X	X	X	X	X	660000 **85.32**	663000	666000	0	85.323
	Over time	X	X	X	X	X	X	X	673000 **59.5**	676000	679000	0 **5.03**	64.5266
Period 9	**Regular time**	X	X	X	X	X	X	X	X	660000 **85.32**	663000	0	85.323
	Over time	X	X	X	X	X	X	X	X	673000 **60.36**	676000	0 **4.17**	64.5266
Period 10	**Regular time**	X	X	X	X	X	X	X	X	X	660000 **85.32**	0	85.323
	Over time	X	X	X	X	X	X	X	X	X	673000 **60.17**	0 **4.35**	64.5266
Total Demand		134.806	132.998	142.76	141.079	143.709	143.083	145.226	144.823	145.678	145.495	381.5936	1801.2506

Optimal cost = 742554400

Table (26). The optimal solution (lowest possible cost) using the transportation model for Total product

Supply from		Demand for											Total capacity available (supply)
		Period 1 March	**Period 2 April**	**Period 3 May**	**Period 4 June**	**Period 5 July**	**Period 6 August**	**Period 7 September**	**Period 8 October**	**Period 9 November**	**Period 10 December**	**Dummy**	
Beginning inventory		0 **5618.49**	3000 **271.6**	6000 **42.72**	9000	12000	15000	18000	21000	24000	27000	0	5932.8
Period 1	**Regular time**	660000 **4255.21**	663000 **1384.37**	666000 **51.32**	669000	672000	675000	678000	681000	684000	687000	0 **34.11**	5725.016
	Over time	673000 **42.65**	676000	679000	682000	685000	688000	691000	694000	697000	700000	0 **4027.312**	4069.959
Period 2	**Regular time**	X	660000 **5385.41**	663000 **245.03**	666000 **28.83**	669000	672000	675000	678000	681000	684000	0	5659.258
	Over time	X	673000 **3039**	676000 **185.95**	679000	682000	685000	688000	691000	694000	697000	0 **1003.37**	4228.313
Period 3	**Regular time**	X	X	660000 **5553.64**	663000 **93.63**	666000	669000	672000	675000	678000	681000	0	5647.262
	Over time	X	X	673000 **3779.97**	676000 **152.66**	679000	682000	685000	688000	691000	694000	0 **255.79**	4188.43
Period 4	**Regular time**	X	X	X	660000 **5640.61**	663000 **58.39**	666000	669000	672000	675000	678000	0	5698.993
	Over time	X	X	X	673000 **4038.98**	676000 **192.77**	679000	682000	685000	688000	691000	0 **82.3**	4314.102
Period 5	**Regular time**	X	X	X	X	660000 **5658.26**	663000	666000	669000	672000	675000	0	5658.264
	Over time	X	X	X	X	673000 **3967.9**	676000 **160.39**	679000	682000	685000	688000	0 **64.53**	4192.822
Period 6	**Regular time**	X	X	X	X	X	660000 **5644.91**	663000	666000	669000	672000	0	5644.912
	Over time	X	X	X	X	X	673000 **4100.96**	676000 **107.12**	679000	682000	685000	0 **14.04**	4222.112
Period 7	**Regular time**	X	X	X	X	X	X	660000 **5666.96**	663000	666000	669000	0	5666.967
	Over time	X	X	X	X	X	X	673000 **4131.65**	676000 **76.75**	679000	682000	0 **4.62**	4213.042
Period 8	**Regular time**	X	X	X	X	X	X	X	660000 **5675.04**	663000	666000	0	5675.046
	Over time	X	X	X	X	X	X	X	673000 **4172.82**	676000 **62.1**	679000	0 **9.28**	4244.128

Period 9	**Regular time**	X	X	X	X	X	X	X	X	660000 **5668.49**	663000	0	5668.495
	Over time	X	X	X	X	X	X	X	X	673000 **4178.01**	676000 **34.25**	0 **4.17**	4216.431
Period 10	**Regular time**	X	X	X	X	X	X	X	X	X	660000 **5665.54**	0	5665.537
	Over time	X	X	X	X	X	X	X	X	X	673000 **4215.803**	0 **6.13**	4221.939
Total Demand		9916.35 2	10080.38	9858.606	9954.698	9877.364	9906.264	9905.744	9924.615	9908.553	9915.592	5505.66	104753.828

Optimal cost = 62074272400

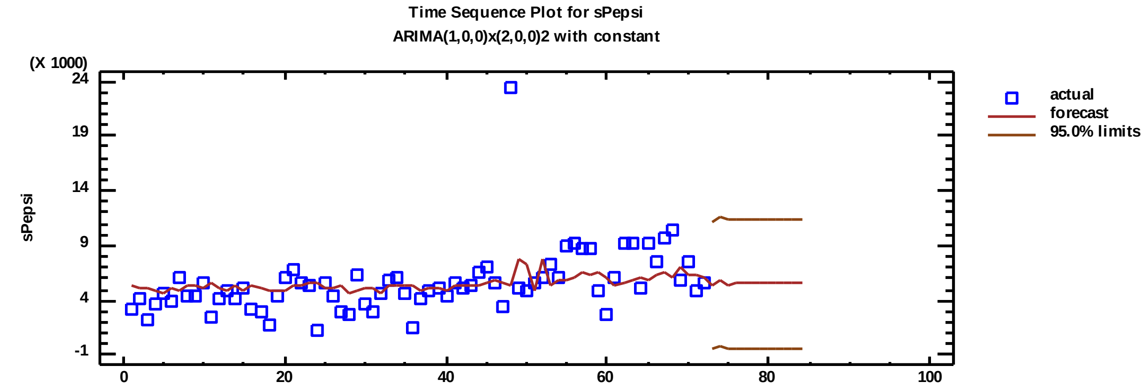

Figure (1). Actual demand series on Pepsi and forecasts for 2018

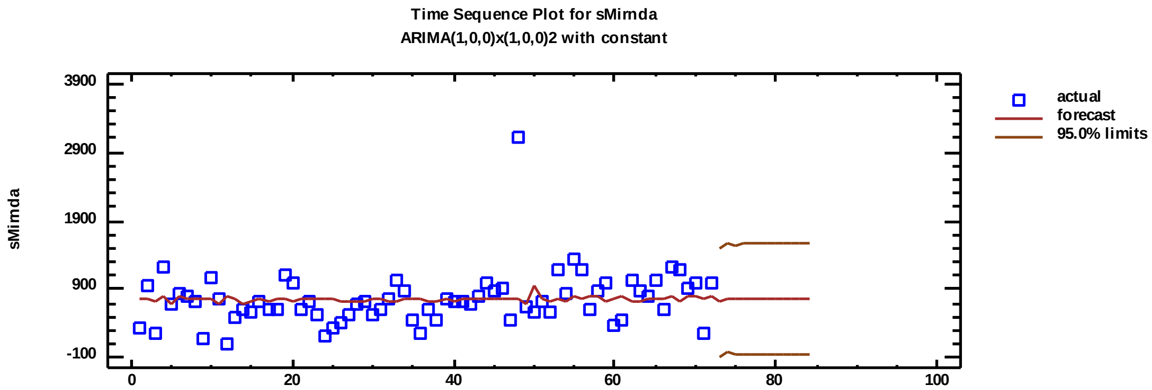

Figure (2). Actual demand series on Mirnda and forecasts for 2018

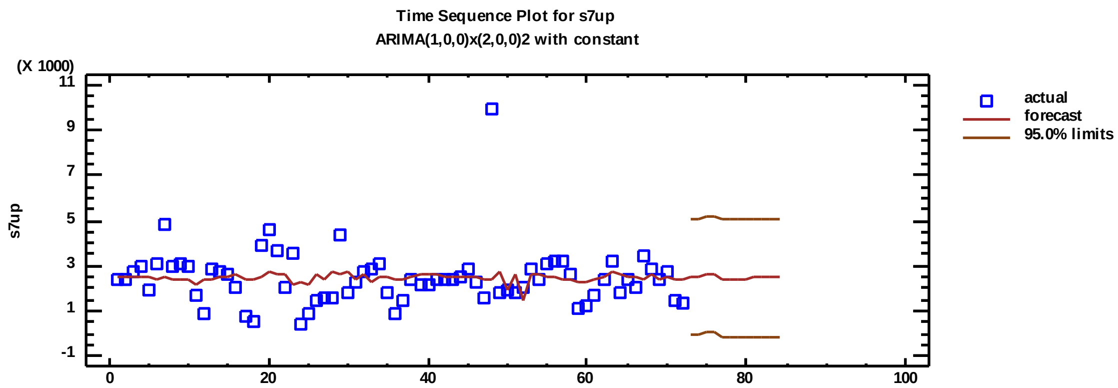

Figure (3). Actual demand series on Seven up and forecasts for 2018

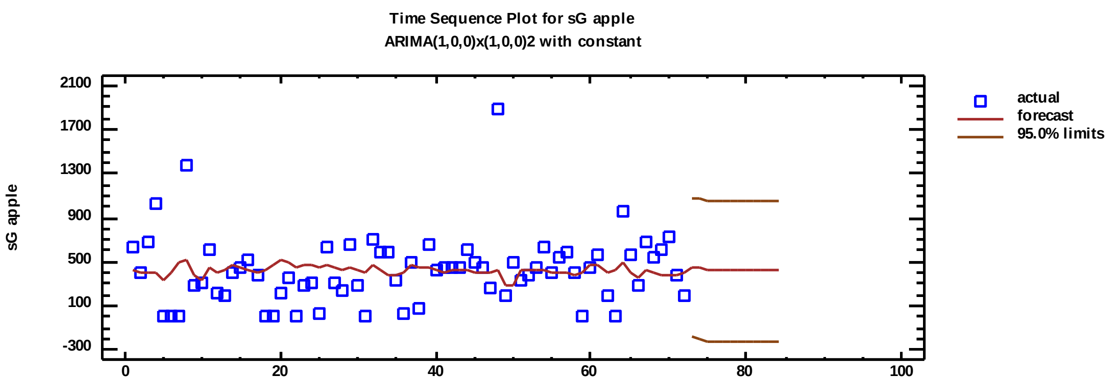

Figure (4). Actual demand series on Green apple and forecasts for 2018

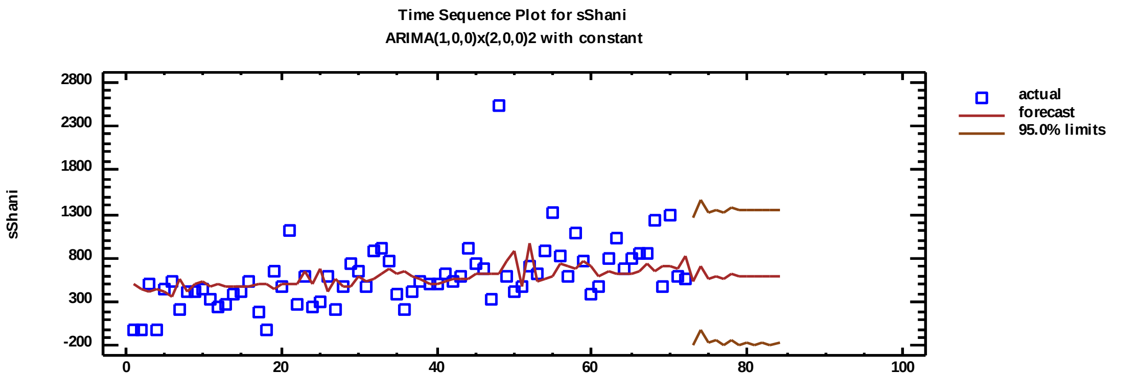

Figure (5). Actual demand series on Shani and forecasts for 2018

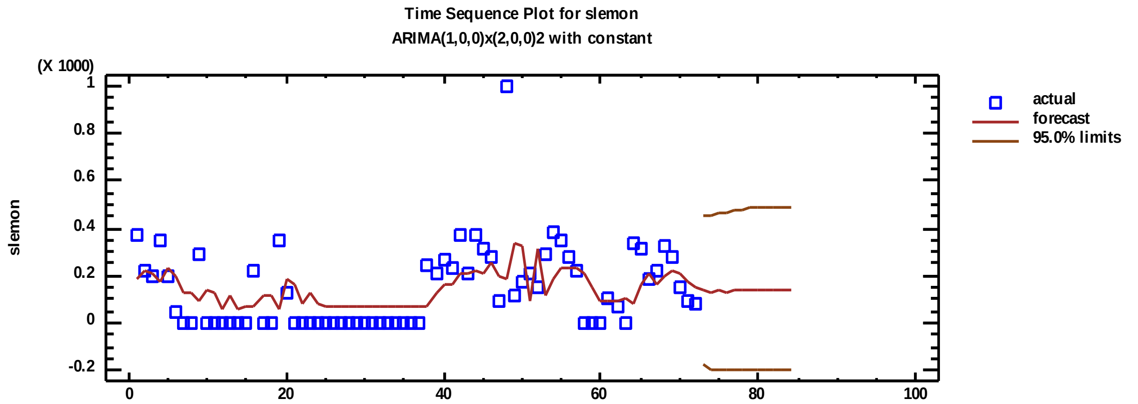

Figure (6). Actual demand series on Lemon and forecasts for 2018

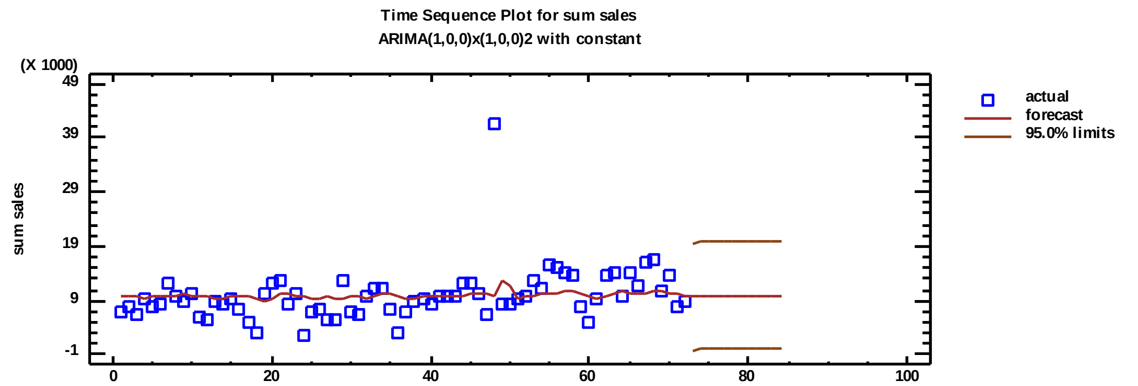

Figure (7). Total actual demand series and forecasts for 2018

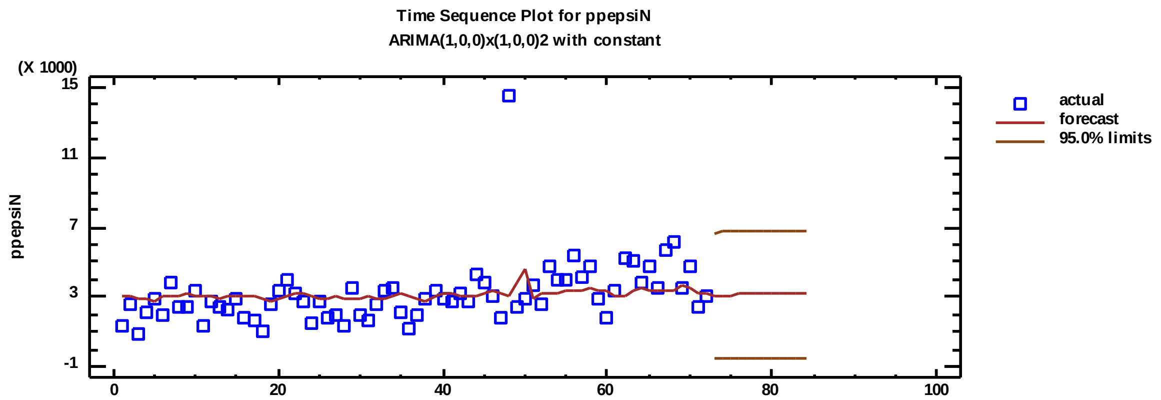

Figure (8). Pepsi production series and forecasts for 2018 in regular time

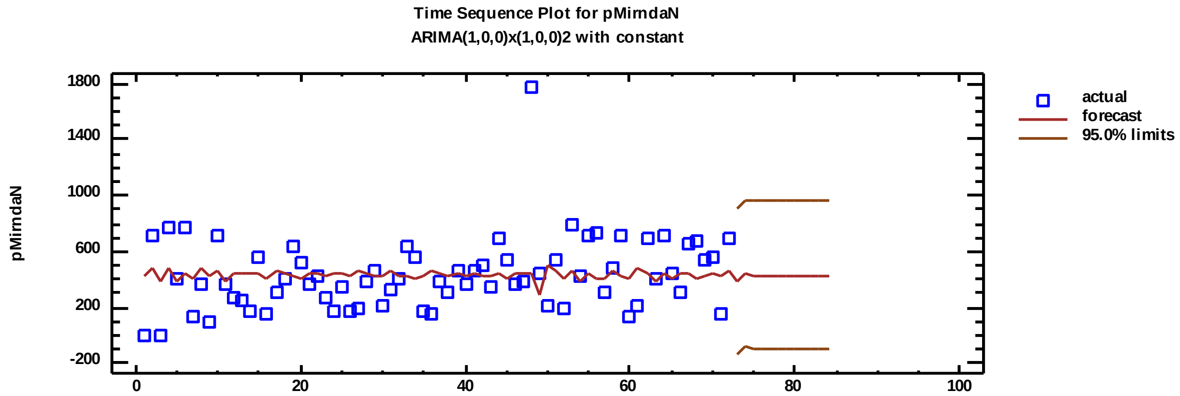

Figure (9). Mirnda production series and forecasts for 2018 in regular time

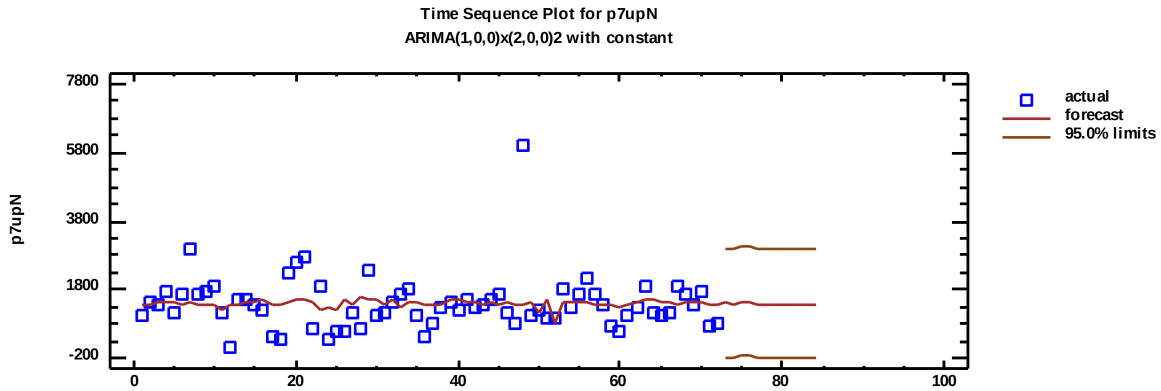

Figure (10). Seven up production series and forecasts for 2018 in regular time

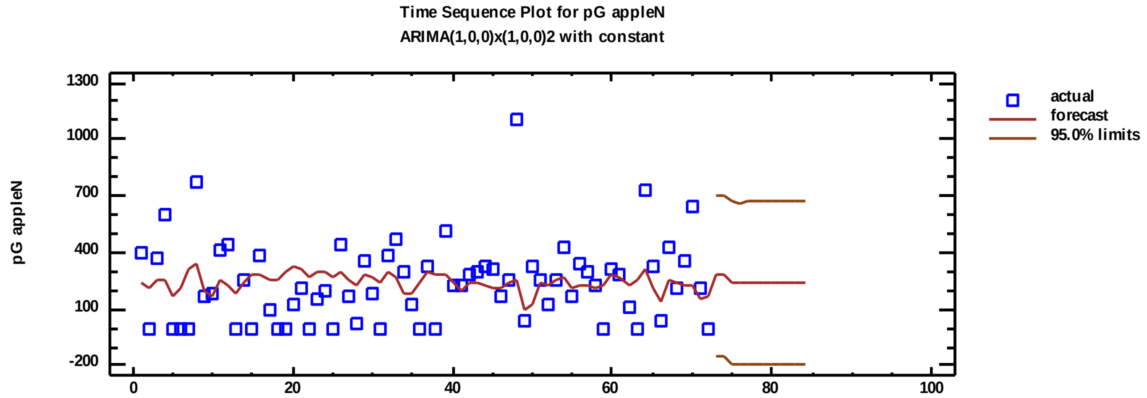

Figure (11). Green apple production series and forecasts for 2018 in regular time

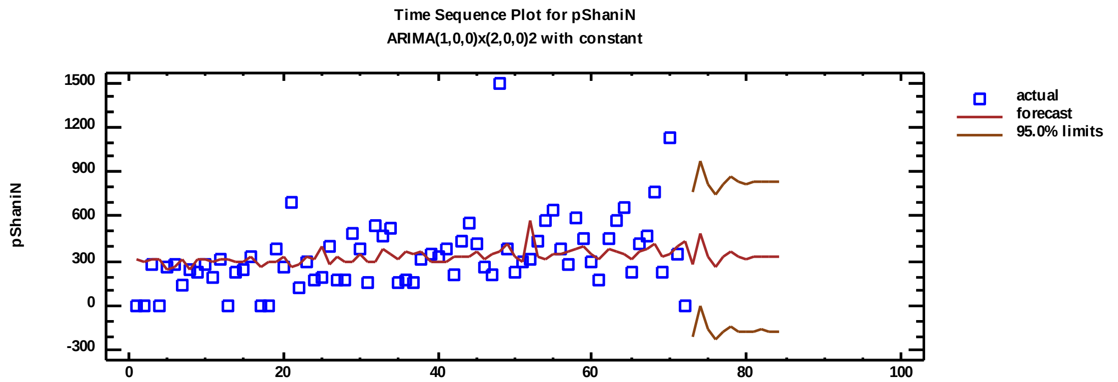

Figure (12). Shani production series and forecasts for 2018 in regular time

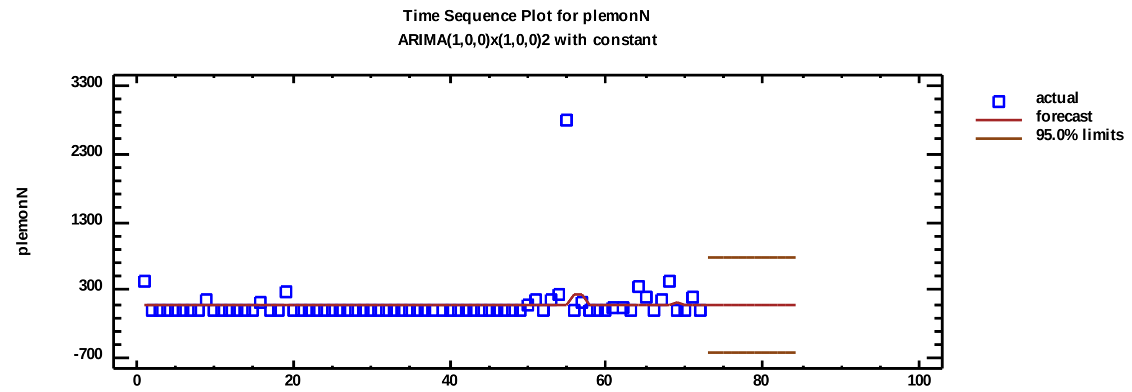

Figure (13). Lemon production series and forecasts for 2018 in regular time

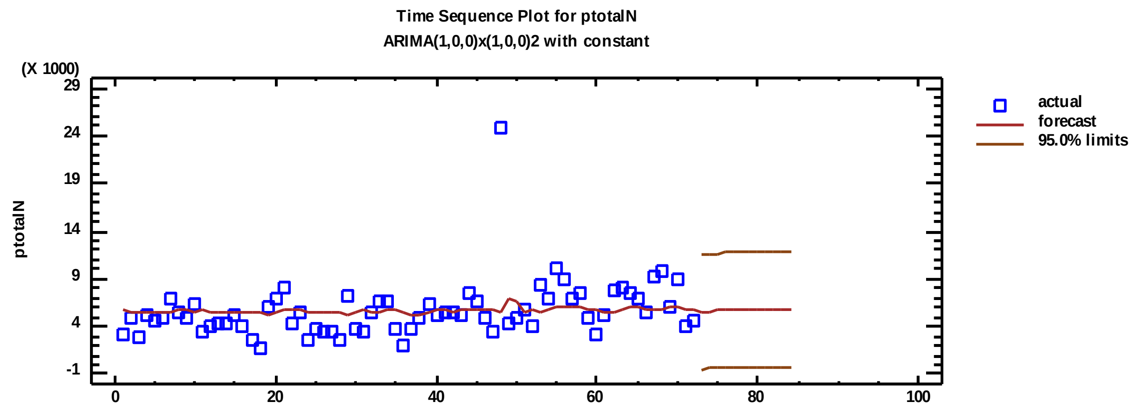

Figure (14). Total production series and forecasts for 2018 in regular time

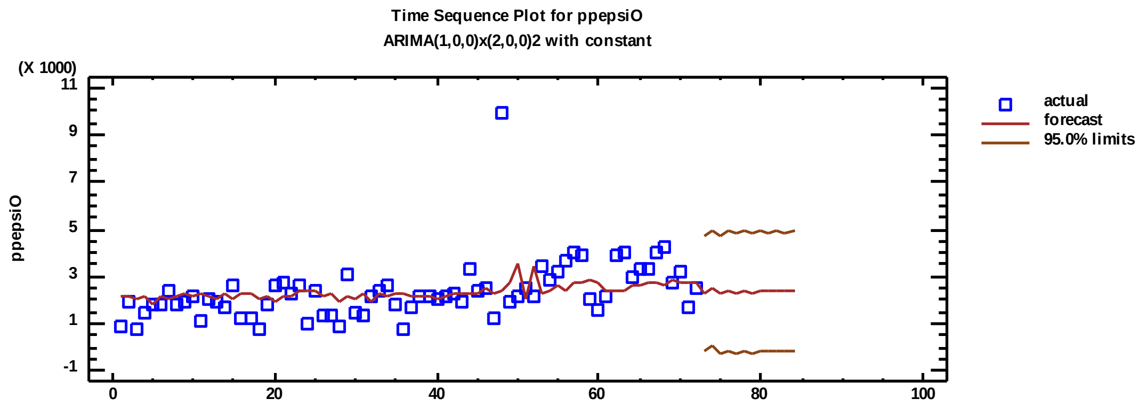

Figure (15). Pepsi production series and forecasts for 2018 in overtime

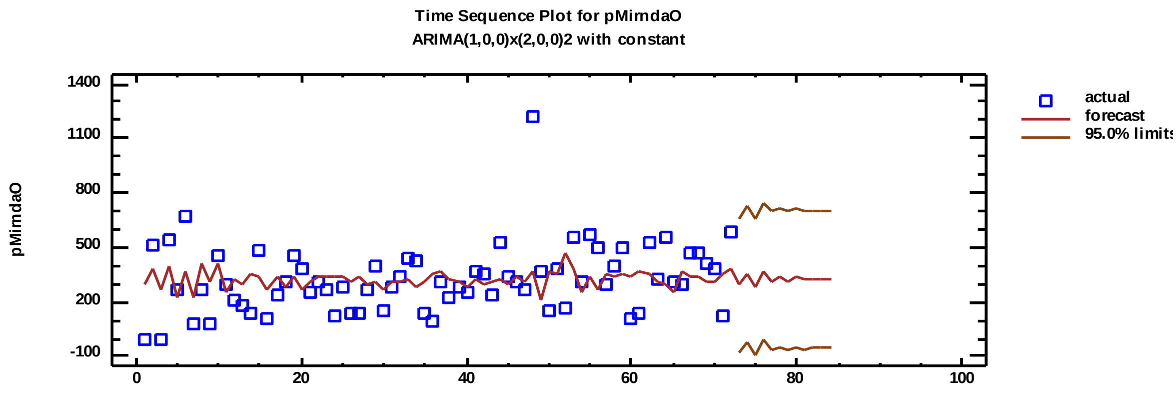

Figure (16). Mirnda production series and forecasts for 2018 in overtime

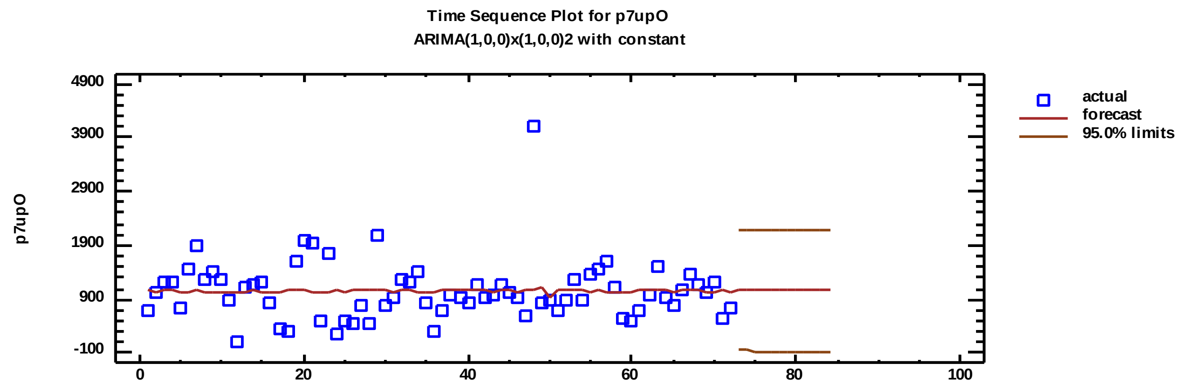

Figure (17). Seven up production series and forecasts for 2018 in overtime

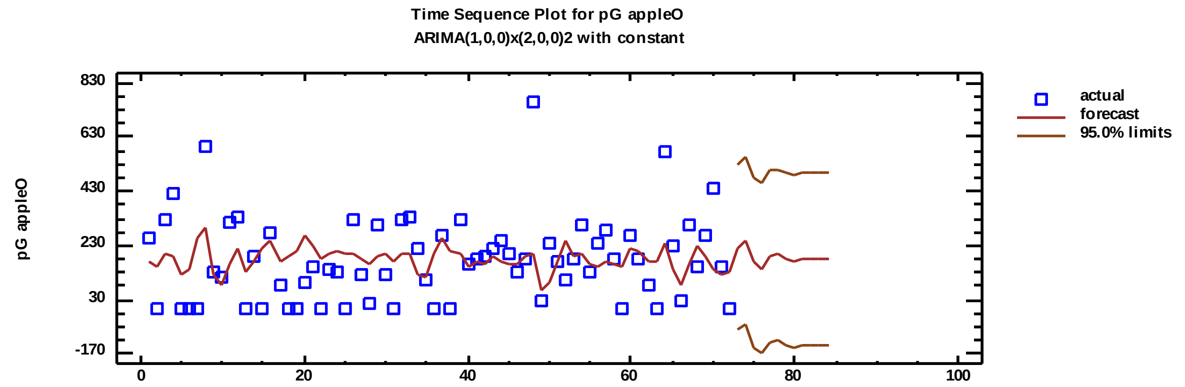

Figure (18). Green apple production series and forecasts for 2018 in overtime

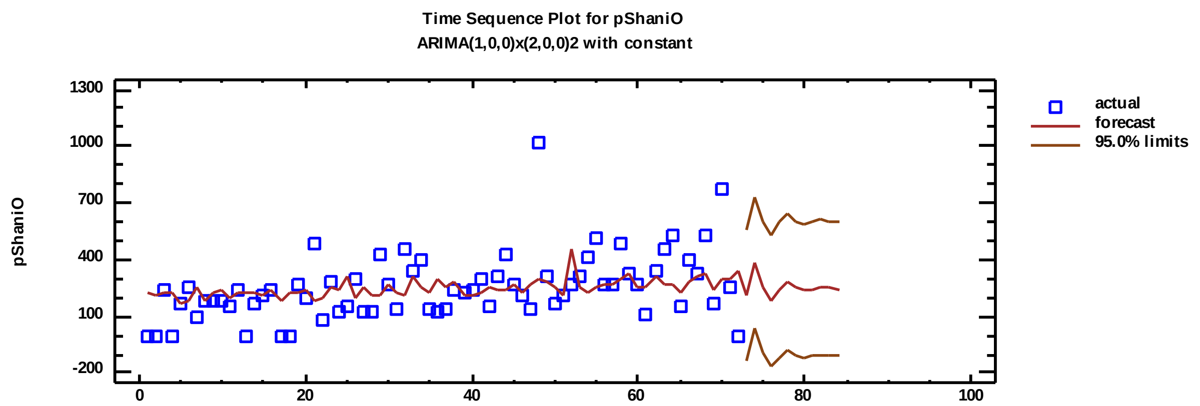

Figure (19). Shani production series and forecasts for 2018 in overtime

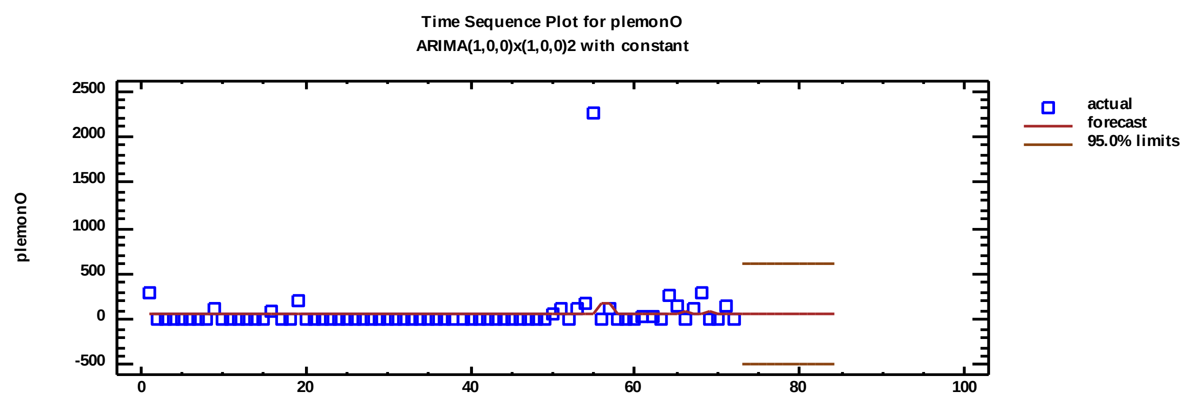

Figure (20). Lemon production series and forecasts for 2018 in overtime

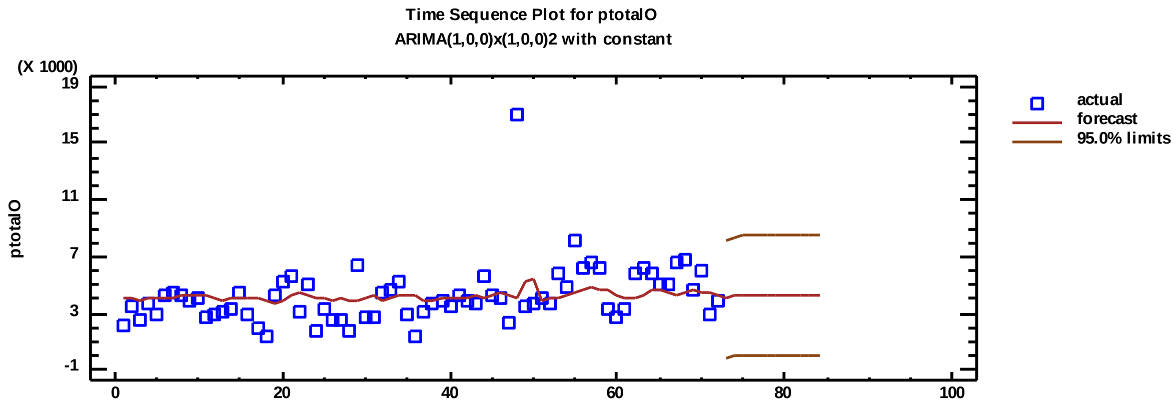

Figure (21). Total production series and forecasts for 2018 in overtime

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