Stress Distribution Dynamic Study Due to the Bending Mode of the Wind Turbine Blade

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Abstract   In this work, we study the dynamic phenomenon soliciting blades. The study is results of a simulation tool that shows the distribution of stresses due to the bending mode of the blade during its rotation taking into account both the mechanical forces (centrifugal force, inertial) and secondly , aerodynamic effects. Modeling is made via the finite elements methods, applied to two nodes to estimate the elimination of any kind of damage linked to this mode.

Keywords   Stress Distribution, Bending Mode, Finite Element Method

1. Introduction

Wind energy is one of the oldest energy sources used in the world. The wind turbine blades are the most flexible part of this machine, and their modal behavior has a great influence on the overall dynamics and calculating stresses due to the bending mode during the rotation of the blades. Recently more attention has been paid to modal analysis. This reflected by the number of experimental and numerical study conducted on this subject [1]. Indeed, the system bending equation has no exact solution to enable us calculating stress, in addition it is characterized by long complex boundary hampering any digital solution [4], using a finite element modeling with Ansys software, applied to a complex blade shape [2, 5].

In this paper, the modeling transverse strain is made applying the finite elements methods based on modal analysis using Matlab code. This method consists in discretization of the blade in small beam elements using the second law of dynamics [3]

$$EI_y \frac{\partial^4 Z}{\partial x^4} - G \frac{\partial^2 Z}{\partial x^2} + m \frac{\partial^2 Z}{\partial t^2} = \frac{\partial F}{\partial x} \qquad (1)$$

E : Young's modulus
I : Blade cross section moments of inertia
G : Centrifugal force
m : Mass per unit length of blade

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This article includes II paragraph and a paragraph for the description of the results In paragraph I We are interested in the dynamical study of the vibratory movement of the rotating blade. It deals with the forced movement of the blade [8, 9], that is to say the movement of the blade under the effect of aerodynamic loads [12, 13]. The resolution of the forced motion equation is done using the method of modal superposition [12]. This allows us to find the deflection at the free end of the blade and also the stresses acting on the blade that is dealt with in paragraph II.
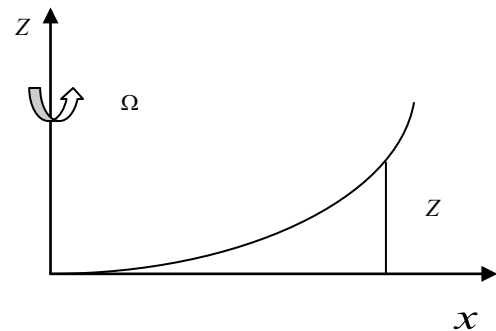

Figure 1.   Deflexion of the blade

2. Solving the Equation of the Blade Bending Mode by the Finite Element Method

Distribution of Z is represented by the nodal displacement by using the interpolation function [7]:

$$Z(x) = \{H\}\{\Delta_i\}, \qquad (2)$$

Where $\{H\}$ is the interpolation vector that expresses:

$$\{H\} = \begin{Bmatrix} 1 - 3\left(\dfrac{x}{l}\right)^2 + 2\left(\dfrac{x}{l}\right)^3 \\[2mm] \left(\dfrac{x}{l}\right) - 2\left(\dfrac{x}{l}\right)^2 + \left(\dfrac{x}{l}\right)^3 \\[2mm] 3\left(\dfrac{x}{l}\right)^2 - 2\left(\dfrac{x}{l}\right)^3 \\[2mm] -\left(\dfrac{x}{l}\right)^2 - \left(\dfrac{x}{l}\right)^3 \end{Bmatrix} \qquad (3)$$

and the vector of degrees of freedom of an element is defined as:

$$\{\Delta_i\} = \begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix} \qquad (4)$$
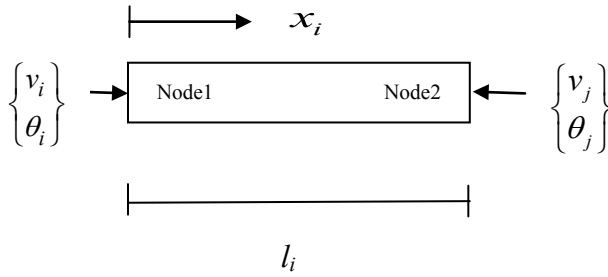

Figure 2. A finite element showing nodal degrees of freedom

By substituting the formula (2) in (1) and applying the method of weighted residues Galerkin, we obtain the equation for the next element
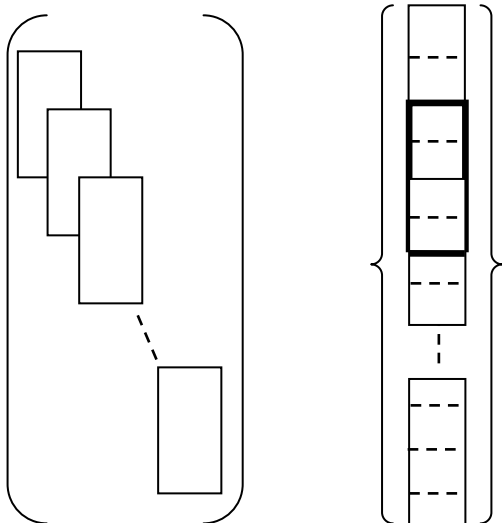
$$M_e\ddot{\Delta}_i + K_e\Delta_i = F_e \qquad (5)$$

Figure 3. The assembly process of elementary matrices and elementary vectors forces

M_e : Elementary mass matrix

K_e : Elementary stiffness matrix

F_e : Elementary force vector

The overall equation of the system is obtained by the assembly of the elementary matrices. The result is shown in figure 2.

Then there is an equation of motion of the blade in the absence of damping by an equation of the same form as that of an element [10]

$$M\ddot{\Delta} + K\Delta = F \qquad (6)$$

2.1. Boundary Conditions

In This case, we studied a free embedded blade. This condition of embedding one end, involves the cancellation of the potential and kinetic energies at this point, this is equivalent to removing 2 rows and 2 columns of the mass and stiffness matrices in the overall equation of the system.

2.2. Solution Procedure

2.2.1. Study of Free Motion

The motion is free when the blade is not required by external forces, which means that equation (6) becomes equal to zero.

$$M\ddot{\Delta} + K\Delta = 0 \qquad (7)$$

In this case the solution takes the form:

$$\Delta(x,t) = x\sin(\omega t + \varphi) \qquad (8)$$

Substituting (9) in the equation (8) we get:

$$(K - \omega_i^2 M)x_i = 0 \qquad (9)$$

Properties eigenmodes

$$X_m^T M X_n = \delta_{mn} \qquad (10)$$

$$X_m^T K X_n = \delta_{mn} \qquad (11)$$

The orthogonality property of the modes will be operated during the study of forced vibrations.

2.2.2. Study of Forced Vibrations

• A modal superposition method

In order to decouple the equations, we use the orthogonality property of the eigenmodes. Therefore, it is interesting to rewrite this equation in the base of the natural modes [14]. The displacement Δ is written in matrix form:

$$\Delta(t) = P\varphi(t) \qquad (12)$$

Equation (7) allows having a 2n equation which represents a system with one degree of freedom:

$$\ddot{\varphi}_k(t) + \omega_k^2\varphi_k(t) = C_k \qquad (13)$$

Note that the terms of the first part of the equation for a given mode are independent of the other modes, while the

second member (aerodynamic load) depends on all modes since the solution of total strain equation (14) is:

$$\varphi_k(t) = \frac{C_k}{\omega_k^2}\left(1-\cos(\omega_k t)\right) + \varphi_0 \cos(\omega_k t) + \frac{\dot{\varphi}_0}{\omega_k}\sin(\omega_k t) \quad (14)$$
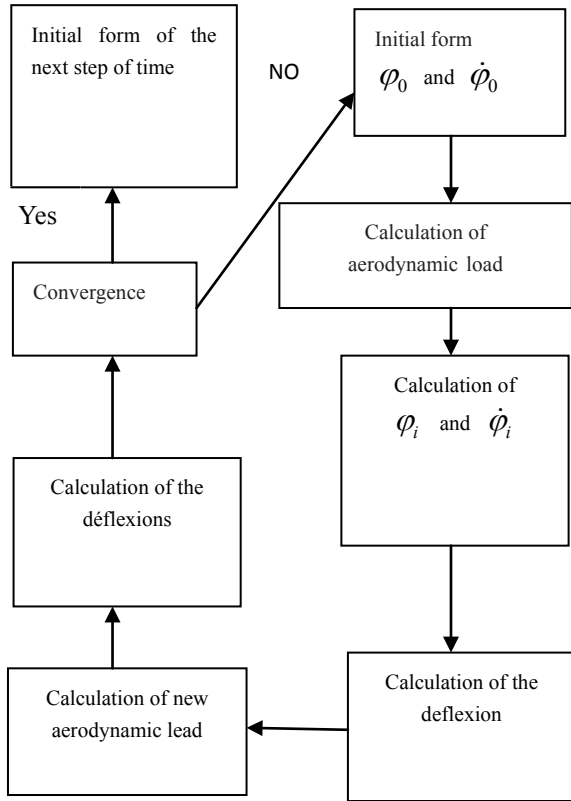

Figure 4. The flowchart summarizing the method for solving equation (6)

3. Design Stress

3.1. The Normal Bending Stress

The blade wind turbine is considered as an elastic beam rotating with a constant angular velocity. The blade is treated as a Bernoulli beam clamped-free right. The design stress is due to the bending moment. The blade is considered as a beam Bernoulli clamped-free right to study the stresses acting on the cross section S in a point at a distance Y from the neutral axis. Its equation is:

$$\sigma_x = -\frac{\mathfrak{M}_f Y}{I} \quad (15)$$

The bending moment Mf is defined as

$$\mathfrak{M}_f = EI\frac{\partial^2 z}{\partial x^2} \quad (16)$$

Then from (16) we have

$$\sigma_x = -EY\frac{\partial^2 z}{\partial x^2} \quad (17)$$

So the constraint of the maximum bending moment is expressed as follows:

$$\sigma_{x\max} = -EY_{\max}\frac{\partial^2 z}{\partial x^2} \quad (18)$$

$\mathfrak{M}_f$: The bending moment to which it is subject.

$Y_{\max}$: Half the thickness of the blade section

$$\sigma_{x\max} = -EY_{\max}\frac{2}{l^2}\left[6\frac{x}{l}-3 \quad l(3\frac{x}{l}-2) \quad 3-6\frac{x}{l} \quad l(3-6\frac{x}{l})\right]\begin{bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{bmatrix}$$
$$(19)$$

For half element we obtain:

$$\sigma_{x\max} = -EY_{\max}/l\left[\theta_i - \theta_j\right] \quad (20)$$

3.2. Normal Stress due to the Centrifugal Force

The expression of the normal stress due to centrifugal force is as follows [6]:

$$\sigma_G = \frac{G}{S} \quad (21)$$

with

S is the cross section of the blade

$$G = \frac{1}{2}m\Omega^2(L^2 - x^2) \quad (22)$$

The resulting normal stress:

$$\sigma_N = \sigma_x + \sigma_G \quad (23)$$

The bending moment is accompanied in most cases with a shearing force. This generates a shear stress. This shear stress generates only little risk of breakage and is therefore generally neglected (Bernoulli model).

The tangential stresses is neglected compared to the normal bending stresses for a beam whose length is large compared to the transverse dimensions as the case of the blade.

4. Resultants and Discussion

For numerical simulation of presented below of normal stress, we will take a blade that has the following characteristics hollow section NACA 2414.

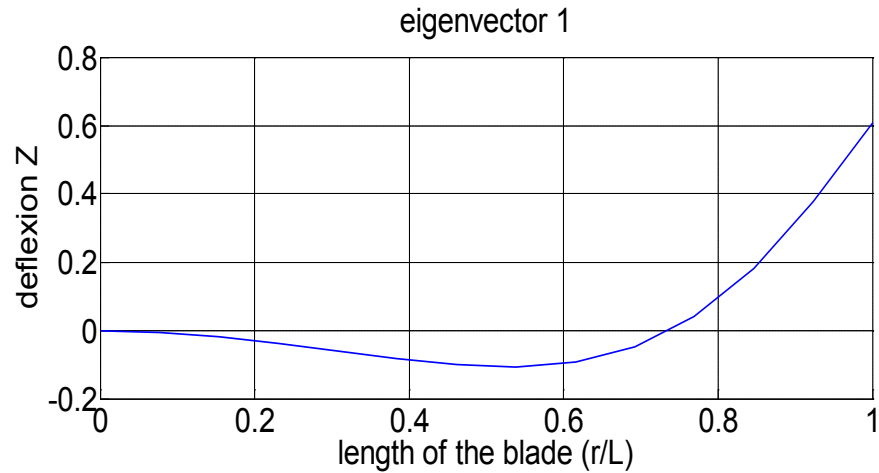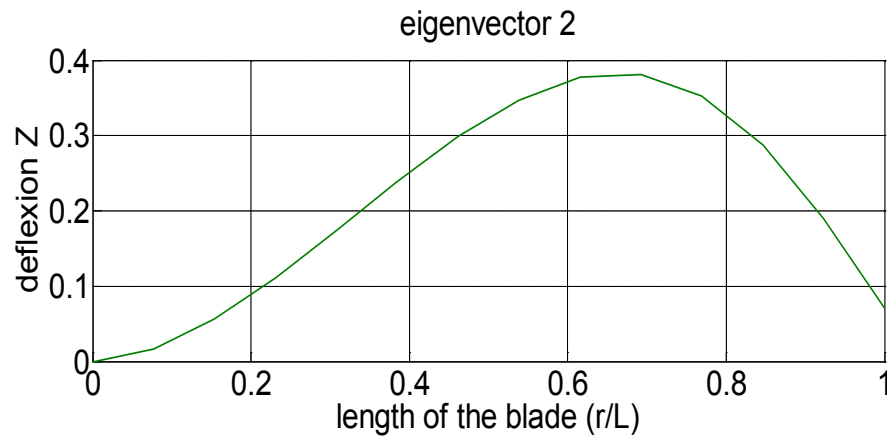|	
E	$5.8.10^9\,N/m^2$
I	$9.5.10^{-7}\,m^4$
L	$8m$
c	$0.4m$
λ	8
m	6.69 kg/m

Figure 5. Eigenmode 1
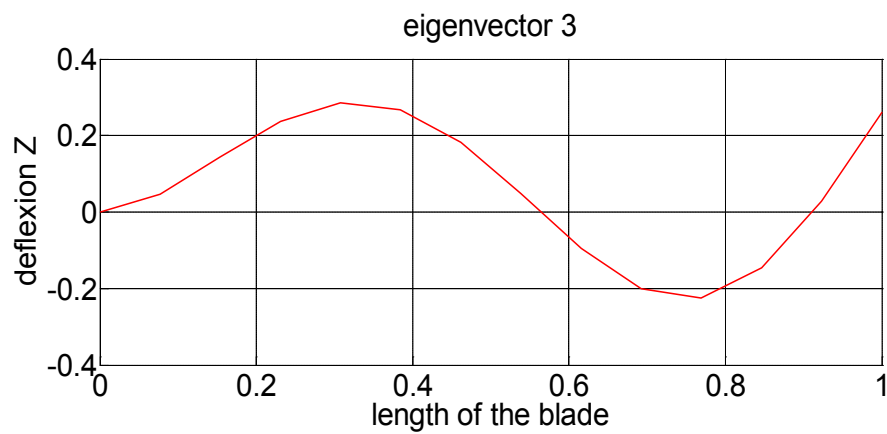
Figure 6. Eigenmode 2

Figure 7. Eigenmode 3

Of all the mechanical vibration problems, the lowest frequency is the most interesting, for this reason, the first three modes are taken into account.
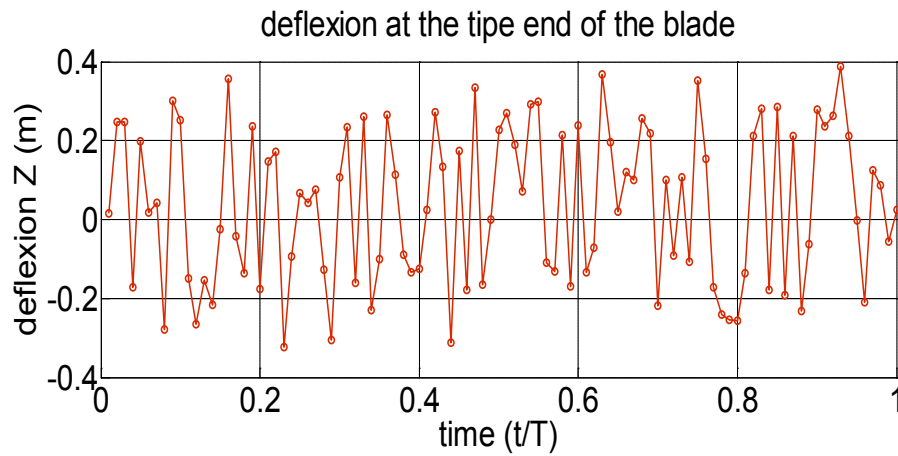
deflexion at the tipe end of the blade

Figure 8. Deflexion of the tip end of the blade (Ω=12rad/s)

We can see that the defluxion maximal of the tip end of the blade for Ω = 12rad/s is 0.3875m.

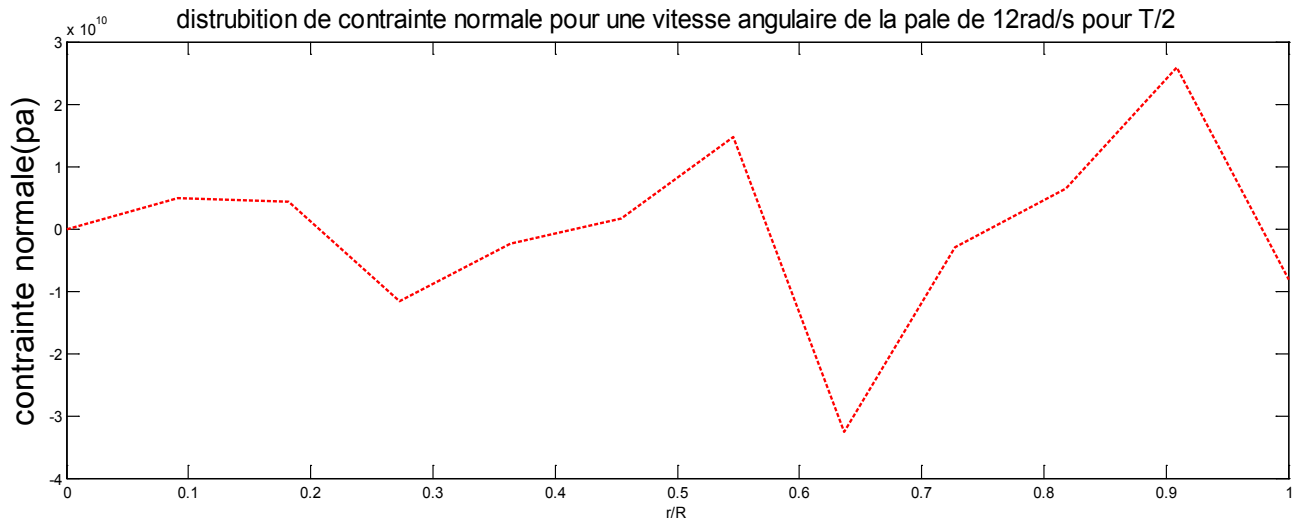**distrubition de contrainte normale pour une vitesse angulaire de la pale de 12rad/s pour T/2**

Figure 9. Normal stress along the blade, Ω=12rad/s, T/2

distribution de contrainte normale pour une vitesse angulaire 12rad/s pourT/4

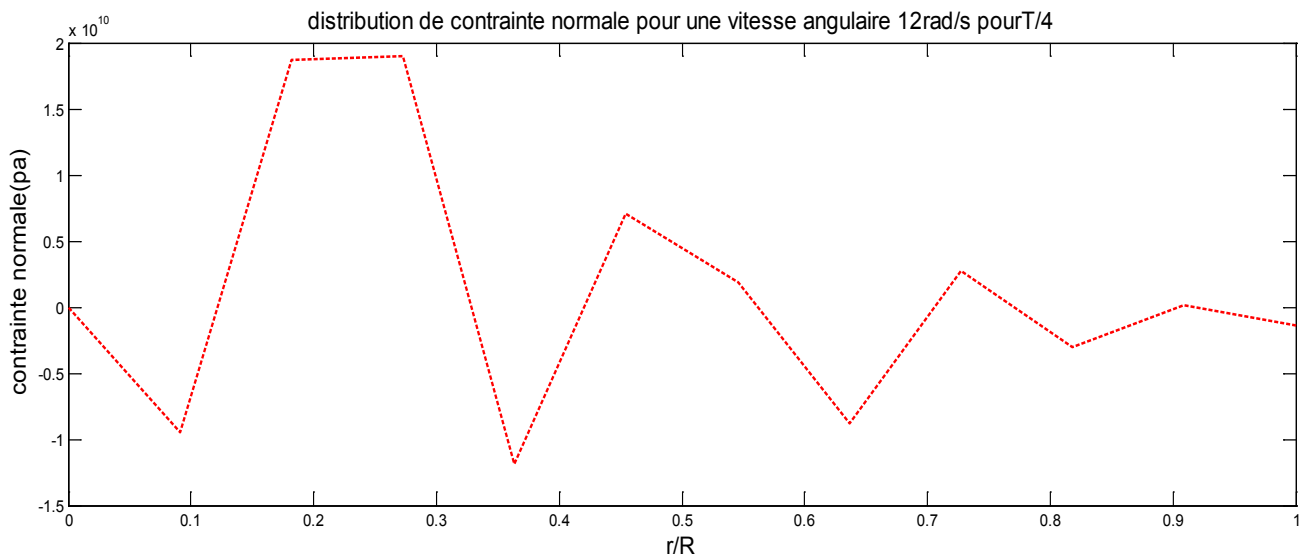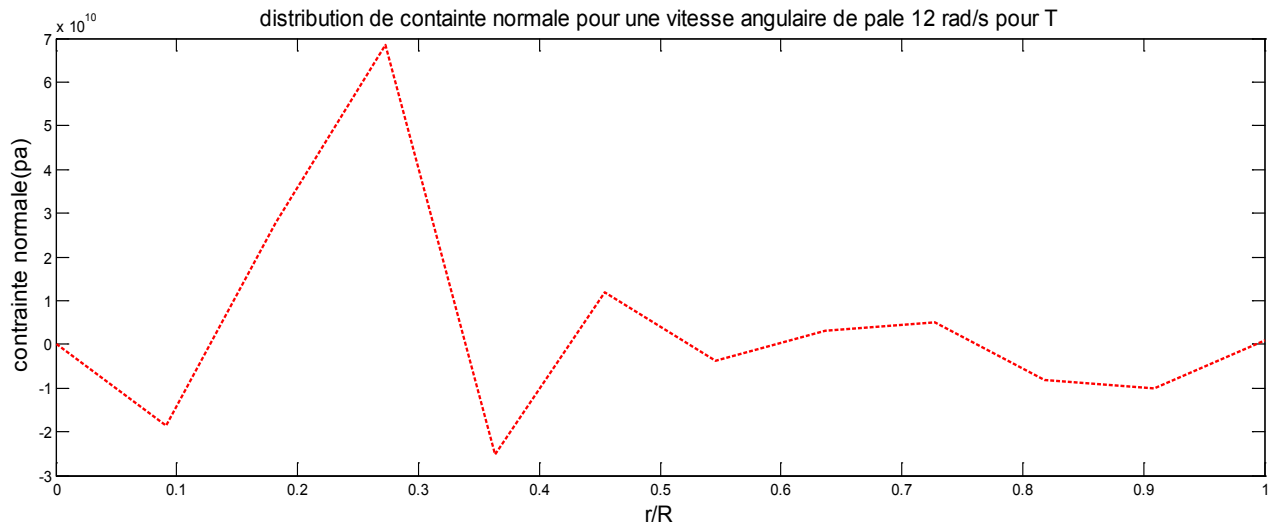**Figure 10.** Normal stress along the blade, Ω=12rad/s, T/4

Figure 11.   Normal stress along the blade, Ω=12rad/s, T

For a blade of a constant angular velocity (12rad/s), the maximum stress is located at the penultimate element of the period (T/2). For period (T/4), maximum stress is located in items 2 and 3. For period (T), the normal stress is in the element 3.

5. Conclusions and Perspectives

The aim of this work is to develop a mathematical model of analysis to solving the equation of the bending of the blade and that Allows to study the normal stress of wind turbine blade subjected to aerodynamic loads. This profound study, allows us to calculate slowness of the blade and to estimate its lifetime.

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