MHD Effects on Porosity and Concentration in an Unsteady Flow in a Vertical Channel

Areo A. O.[1], Akinpelu F. O.[1], Adetunde I. A.[2,*]

[1]Ladoke Akintola University of Technology, Department of Pure and Applied Mathematics, Ogbomoso, Oyo State, Nigeria
[2]Prof I. A. Adetunde Consults, 8 Westmount Close, Collyhurst Street, Manchester, UK

Abstract This article investigates MHD effects on porosity and concentration in an unsteady flow in a vertical channel. The nonlinear governing equations of motion with concentration equation incorporate variable porosity were employed and solved analytically using a perturbation technique. The effects of the flow were determined through the physical parameters; Womersley parameter (AI), Eckert number (Ec), Reynolds number (Re), Prandlt number (Pr), Grashof number (Gr), Schmidt number (Sc), Chemical reaction parameter (K_1), and the Mass Grashof number (Gc), were investigated on the temperature, velocity and concentration. Graphical results are presented and discussed quantitatively. From the course of the investigation, it was revealed that fluid temperature increase within the channel with increase porosity.

Keywords MHD, Porosity, Concentration, Physical Parameters, Vertical Channel

1. Introduction

The study of flow and heat transfer in a porous media has also received much attention in the last few decades due to its ever-increasing applications in industries and in contemporary technology. Phenomena of transport in porous media are encountered in many Engineering disciplines. Convective flow through porous medium has applications in the field of Chemical Engineering for filtration and purification processes. In Petroleum Technology, it is used to study the movement of natural gas, oil and water through oil channels or reservoirs. Agricultural Engineering deals with the movement of water and studies in the root zone in the soil. Reservoir engineers deals with the flow of oil, water and gas in petroleum reservoirs.

Many researchers have worked on MHD flows, notably among them are Mhone and Makinde [5] have studied unsteady MHD flow with heat transfer in a diverging channel. Ahammad and Mollah [1] studied the MHD free convection flow and mass transfer problem over a stretching sheet considering Dufour and Soret effects with magnetic field. Hazarika and Jadav [4] worked on effects of variable viscosity and thermal conductivity on MHD free convective flow along a vertical porous plate with viscous dissipation. Seddeek and Salama [7] carried out the effects of temperature dependent viscosity and thermal conductivity with variable suction on unsteady MHD convective heat

transfer past a vertical moving porous plate. Chakraborty [3] studied MHD flow and heat transfer of a dusty viscoelastic stratified fluid down an inclined channel in porous medium under variable viscosity. Soundalgekar et al [8] discussed MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field. Taneja and Jain [9] looked at the unsteady MHD flow in a porous medium in the presence of radiative heat where they obtained expressions for velocity, temperature and rate of heat transfer. Attia [2] have investigated an unsteady MHD couette flow and heat transfer of dust fluid with variable physical properties.

Several studies involving heat and mass transfer in Newtonian fluid flows through porous media have been undertaken. However, the combine effects of porosity and viscosity with viscous dissipation have not been incorporated. This work is set out to investigate the effect of porosity and viscosity on unsteady flow of MHD with viscous dissipation and concentration in a porous vertical channel.

2. Mathematical Formulation

The effect of a uniform transverse magnetic field B on unsteady two-dimensional electric conducting fluid flows are considered and its velocities are given as

$$q = u(x, y, t)i + v(x, y, t)j \qquad (a)$$

through a symmetric vertical channel ($D : -\infty \angle x \angle +\infty$, $-b(x) \angle y \angle b(x)$) where (x, y) are Cartesian co-ordinates such that ox is the axis of symmetry of the channel and $y = \pm b(x)$ are the rigid and impermeable walls of the

channel. The walls of the channel are kept at a constant temperature T_w. The fluid is incompressible with uniform properties i.e. density ρ, kinematic viscosity v and electrical conductivity σ. A volume flux with oscillating frequency δ and pulse m is prescribed as in figure 1 below:
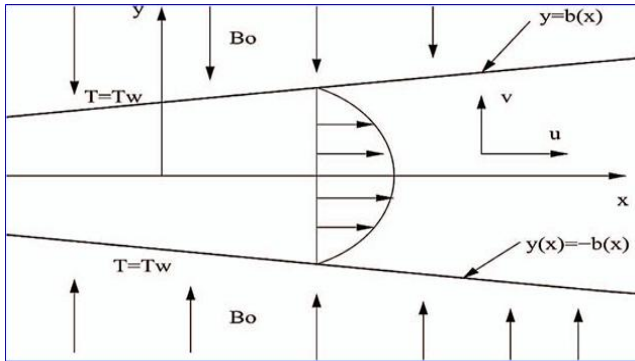

Figure 1. Problem Geometry (Source Mhone and Makinde, 2006)

$$\int_0^{b(x)} u\,dy = Q(1+me^{i\delta t}) \qquad (b)$$

A uniform magnetic force is applied in the $y-direction$. A very small magnetic Reynolds number is assumed and therefore the induced magnetic field is neglected. Two key physical effects occur when the fluid moves into the magnetic field; the first one is that an electric field E is induced in the flow. There is no excess charge density and then $\nabla.E = 0$. Neglecting the induced magnetic field implies that $\nabla \times B = 0$ and therefore the induced electric field is negligible. The second key effect is dynamical i.e. a Lorentz force ($J \times B$), where J is the current density acts on the fluid and modifies its motion. Therefore, there is a transfer of energy ($J.E$) from the electromagnetic field to the fluid. In this study, relativistic effects are neglected, and J is given by Ohm's law:

$$J = \sigma(q \times B) \qquad (c)$$

Within the framework of these assumptions the magneto-hydrodynamic flow relevant to the problem is governed by the set of equations.

3. Mathematical Analysis

The governing equations for motion under the auspices of the Continuity equation, Momentum equation, Energy equation and Concentration equation are considered, following Hazarika and Jadov [4] and Mhone and Makinde [5] resulted into the following equations;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\nabla^2 u - \frac{\sigma B_0^2 u}{\rho} - \frac{bu^2}{k} + g\beta(T-T_0) + g\beta(C-C_0) \qquad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + v\nabla^2 v \qquad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} =$$

$$\frac{k}{\rho Cp}\nabla^2 T + 2\frac{v}{Cp}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] + \frac{\sigma B_0^2 u^2}{\rho Cp} + g\beta(T-T_0) + g\beta(C-C_0) \qquad (4)$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_1\frac{\partial^2 T}{\partial y^2} + K_L(C-C_0) \qquad (5)$$

With the following conditions:

$$\text{Symmetry: } \frac{\partial u}{\partial y} = 0,\, v = 0,\, \frac{\partial T}{\partial y} = 0,\, C = 0 \text{ on } y = 0 \qquad (6)$$

$$\text{Non-slip: } u + v\frac{db}{dx} = 0,\; T = T_w,\; C = C_w \text{ on } y = b(x) \qquad (7)$$

It is convenient to introduce the stream function ψ defined by

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad (8)$$

So that

$$\frac{\partial u}{\partial x} = \frac{\partial^2 \psi}{\partial x \partial y} \qquad \frac{\partial v}{\partial x} = -\frac{\partial^2 \psi}{\partial x^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial^2 \psi}{\partial y^2} \qquad \frac{\partial v}{\partial y} = -\frac{\partial^2 \psi}{\partial x \partial y} \tag{9}$$

Substituting equation (9) into (1) gives

$$\left[\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} \right] = 0 \quad \text{It satisfied continuity equation.} \tag{10}$$

But $\left[\dfrac{\partial^2 \psi}{\partial x^2} + \dfrac{\partial^2 \psi}{\partial y^2} \right] = \omega$ Which can be written as

$$\nabla^2 \psi = \omega \tag{11}$$

To eliminate the pressure term from (2) and (3), differentiate (2) with respect to y and (3) with respect to x gives

$$\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right] = -\frac{1}{\rho}\frac{\partial}{\partial y}\left[\frac{\partial p}{\partial x} \right] + v\nabla^2 \frac{\partial u}{\partial y} - \frac{\sigma B_0^2}{\rho}\frac{\partial u}{\partial y} - \frac{2b}{k}\frac{\partial u}{\partial y} + \frac{\partial}{\partial y}\left[g\beta(T-T_0) + g\beta(C-C_0) \right] \tag{12a}$$

$$\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right] = -\frac{1}{\rho}\frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x} \right] + v\nabla^2 \frac{\partial v}{\partial x} \tag{12b}$$

Subtract equation (12b) from (12a) to give

$$\frac{\partial}{\partial t}\left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right] + \frac{\partial}{\partial y}\left[v\frac{\partial u}{\partial y} - v\frac{\partial v}{\partial x} \right] + \frac{\partial}{\partial x}\left[u\frac{\partial u}{\partial y} - u\frac{\partial v}{\partial x} \right] = v\nabla^2 \left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right]$$

$$\tag{13}$$

$$-\frac{\sigma B_0^2}{\rho}\frac{\partial u}{\partial y} - \frac{2b}{k}\frac{\partial u}{\partial y} + \frac{\partial}{\partial y}\left[g\beta(T-T_0) + g\beta(C-C_0) \right]$$

Substituting (9) into (13) yields

$$\frac{\partial}{\partial t}\left[\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2} \right] + \frac{\partial}{\partial y}\left[-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial x^2} \right] + \frac{\partial}{\partial x}\left[\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x^2} \right] =$$

$$\tag{14}$$

$$v\nabla^2 \left[\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2} \right] - \frac{\sigma B_0^2}{\rho}\frac{\partial^2 \psi}{\partial y^2} - \frac{2b}{k}\frac{\partial^2 \omega}{\partial y^2} + \frac{\partial}{\partial y}\left[g\beta(T-T_0) + g\beta(C-C_0) \right]$$

Simplifying (14) further to obtain

$$\frac{\partial \omega}{\partial t} + \frac{\partial(\omega,\psi)}{\partial(y,x)} = v\nabla^2 \omega - \frac{\sigma B_0^2}{\rho}\frac{\partial^2 \psi}{\partial y^2} - \frac{2b}{k}\frac{\partial^2 \omega}{\partial y^2} + \frac{\partial}{\partial y}\left[g\beta(T-T_0) + g\beta(C-C_0) \right] \tag{15}$$

where $\dfrac{\partial}{\partial y}\left[-\dfrac{\partial \psi}{\partial y}\dfrac{\partial^2 \psi}{\partial x^2} - \dfrac{\partial \psi}{\partial x}\dfrac{\partial^2 \psi}{\partial y^2} \right] + \dfrac{\partial}{\partial x}\left[\dfrac{\partial \psi}{\partial y}\dfrac{\partial^2 \psi}{\partial x \partial y} - \dfrac{\partial \psi}{\partial x}\dfrac{\partial^2 \psi}{\partial y \partial x} \right] = \dfrac{\partial(\omega,\psi)}{\partial(y,x)}$ $\tag{16}$

Substituting (9) into (4) leads to

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{k}{\rho Cp}\nabla^2 T + 2\frac{v}{Cp}\left[\left(\frac{\partial^2 \psi}{\partial x \partial y} \right)^2 + \left(-\frac{\partial^2 \psi}{\partial x \partial y} \right)^2 + \frac{1}{2}\left(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} \right)^2 \right]$$

$$\tag{17}$$

$$+ \frac{\sigma B_0^2 \left(\dfrac{\partial \psi}{\partial y} \right)^2}{\rho Cp} + g\beta(T-T_0) + g\beta(C-C_0)$$

Further simplifying of (17) resulted into

$$\frac{\partial T}{\partial t} + \frac{\partial (T,\psi)}{\partial (y,x)} = \frac{k}{\rho Cp}\nabla^2 T + \frac{v}{Cp}\left(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2}\right)^2 + \frac{4v}{Cp}\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2 + \frac{\sigma B_0^2}{\rho Cp}\left(\frac{\partial \psi}{\partial y}\right)^2$$

(18)

$$+ g\beta(T_\omega - T_0) + g\beta(C_\omega - C_0)$$

Similarly substituting (9) into (5) using the same approach as in above leads to

$$\frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_1\frac{\partial^2 T}{\partial y^2} + K_L(C - C_0)$$

(19)

Simplifying equation (19) further to obtain

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial}\left(\frac{\psi,C}{y,x}\right) = D\frac{\partial^2 C}{\partial y^2} + D_1\frac{\partial^2 T}{\partial y^2} + K_L(C - C_0)$$

(20)

The corresponding boundary conditions are:

$$\frac{\partial^2 \psi}{\partial y^2} = 0, \quad \psi = 0, \quad \frac{\partial T}{\partial y} = 0 \quad \text{On } y = 0$$

(21)

$$\frac{d\psi}{dy} = \frac{\partial \psi}{\partial x}\frac{db}{dx} = 0, \quad T = T_\omega, \quad \psi = Q(1 + me^{i\sigma t}) \text{ on } y = b(x).$$

(22)

The function b(x) is assumed to depend upon a small parameter ε such that

$$b(x,\varepsilon) = a_0 S\left(\frac{\varepsilon x}{a_o}\right)\left(0 < \varepsilon = \frac{a_0}{L} << 1\right)$$

(23)

where a_0 the characteristics constant half width of the channel, L is the characteristics constant length of the channel and S is the function describing the channel wall divergence geometry. This assumption helps us to simplify the problem by writing the equations in non-dimensional form.

To achieve this, T_0 is defined as the reference temperature and the following non-dimensional qualities were introduced.

$$\omega' = \frac{a_0^2 \omega}{Q}, \quad x' = \frac{\varepsilon x}{a_0}, \quad y' = \frac{y}{a_0}, \quad \psi' = \frac{\psi}{Q}, \quad t' = \delta t,$$

$$\theta = \frac{T - T_0}{T_\omega - T_0}, \quad p' = \frac{\varepsilon a_0^2}{\rho v Q}P, \quad \phi = \frac{C - C_0}{C_\omega - C_0}$$

(24)

Differentiating (24) with respect to x, y and t the non-dimensional quantities and Substituting into (15, 17, 18, 19) and after neglecting terms of order ε^2 and higher order as well as the primes for charity to obtain

$$\frac{\partial^2 \omega}{\partial y^2} - \alpha\frac{\partial \omega}{\partial t} = \text{Re}\left[\frac{\partial (\omega,\psi)}{\partial (y,x)} + \Omega\frac{\partial^2 \psi}{\partial y^2} + Da\frac{\partial^2 \psi_{0s}}{\partial y^2} - Gr\frac{\partial \theta}{\partial y} - Gc\frac{\partial \phi}{\partial y}\right]$$

(25)

$$\frac{\partial^2 \theta}{\partial y^2} - \alpha\Pr\frac{\partial \theta}{\partial t} = \text{Re}\Pr\left[\frac{\partial}{\partial}\left(\frac{\theta,\omega}{y,x}\right) - \Omega E_c\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 - Gr\theta - Gc\phi\right] - \Pr E_c\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2$$

(26)

$$\frac{\partial \phi}{\partial t} + \text{Re}\frac{1}{\alpha}\frac{\partial}{\partial}\left(\frac{\psi,\phi}{y,x}\right) = Sc\frac{\partial^2 \phi}{\partial y^2} + Sc_1\frac{\partial^2 \theta}{\partial y^2} + K_1\phi$$

(27)

Where $Gr = g\beta\frac{a_0 L}{Q}(T_w - T_0)$ thermal Grashof number, $Gc = g\beta\frac{a_0 L}{Q}(C_w - C_0)$ mass Grashof number,

$\Omega = \frac{\sigma B_0^2 a_0 L}{\rho Q}$ is the magnetic field intensity parameter $\alpha = \frac{\delta a_0^2}{v}$ the womersley number, $\text{Re} = \frac{Q\varepsilon}{v}$ the effective

Reynolds number $E_c = \dfrac{Q^2}{a_0^2(T_w - T_0)C_p}$ is the Eckert number $Sc = Da_0\delta$ Schmitt number and $k_1 = k_L\delta$ chemical

reaction number and $Pr = \rho C_p \dfrac{v}{k}$ the prandlt number.

The boundary conditions are:

$$\frac{\partial^2 \psi}{\partial y^2} = 0, \quad \psi = 0, \frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = 0 \qquad \text{On } y = 0 \tag{28}$$

$$\frac{\partial \psi}{\partial y} = 0, \quad \phi = 1, \quad \theta = 1, \quad \psi = 1 + me^{it} \qquad \text{On } y = S(x) \tag{29}$$

Due to the nonlinear nature of the equations, it is convenient to adopt a power series expansion with the effective flow Reynolds number (Re) as follows:

$$\psi = \sum_{j=o}^{\infty} \text{Re}^j (\psi_{js} + me^{it}\psi_j), \qquad \omega = \sum_{j=o}^{\infty} \text{Re}^j (\omega_{js} + me^{it}\omega_j)$$

$$\theta = \sum_{j=o}^{\infty} \text{Re}^j (\theta_{js} + me^{it}\theta_j), \qquad \phi = \sum_{j=o}^{\infty} \text{Re}^j (\phi_{js} + me^{it}\phi_j) \tag{30}$$

where $\psi_{js}, \psi_j, \omega_{js}, \omega_j, \theta_{js}, \theta_j, \phi_{js}$ and ϕ_j are functions of $S(x)$ and y. it is important to note that the real part of the equation (30) forms the solution of the problem which is physically meaningful. Substituting equation (30) into equations (25-27) and collecting terms of like order of Re and me^{it}, gives zero order:

$$\sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial^2 \omega_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \omega_j}{\partial y^2} \right) - \alpha \left(\sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial \omega_{js}}{\partial t} + mie^{it}\omega_j \right) \right) =$$

$$\text{Re}\left[\begin{array}{l} 0 + \Omega \sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial^2 \psi_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \psi_j}{\partial y^2} \right) + Da\left(\frac{\partial^2 \psi_{0s}}{\partial y^2} + me^{it}\frac{\partial^2 \psi_0}{\partial y^2} + \text{Re}\frac{\partial^2 \psi_{1s}}{\partial y^2} + \text{Re}\,me^{it}\frac{\partial^2 \psi_1}{\partial y^2} \right) \\ + Gr\left(\frac{\partial \theta_{js}}{\partial y} + me^{it}\frac{\partial \theta_j}{\partial y} \right) - Gc\left(\frac{\partial \phi_{js}}{\partial y} + me^{it}\frac{\partial \phi_j}{\partial y} \right) \end{array} \right] \tag{31}$$

$$\sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial^2 \theta_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \theta_j}{\partial y^2} \right) - \alpha \Pr\left(\sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial \theta_{js}}{\partial t} + mie^{it}\theta_j \right) \right) =$$

$$\text{Re}\Pr\left[0 - \Omega E_c \left(\sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial^2 \psi_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \psi_j}{\partial y^2} \right) \right) - Gr \sum_{j=o}^{\infty} \text{Re}^j (\theta_{js} + me^{it}\theta_j) \right]$$

$$- Gc \sum_{j=o}^{\infty} \text{Re}^j (\phi_{js} + me^{it}\phi_j) - E_c \Pr\left(\sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial^2 \psi_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \psi_j}{\partial y^2} \right) \right)^2 \tag{32}$$

$$\sum_{j=0}^{\infty} \text{Re}^j (\phi_{js} + mie^{it}\phi_j) + \text{Re}\frac{1}{\alpha}(0) = Sc\left(\sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial^2 \phi_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \phi_j}{\partial y^2} \right) \right)$$

$$+ Sc_1\left(\sum_{j=0}^{\infty} \text{Re}^j \left(\frac{\partial^2 \theta_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \theta_j}{\partial y^2} \right) \right) + K_1\left(\sum_{j=o}^{\infty} \text{Re}^j (\phi_{js} + me^{it}\phi_j) \right) \tag{33}$$

when j = 0, and collecting terms of like order of me^{it}, lead to Zero order (31), (32) and (33) yields

$$\frac{\partial^2 \omega_0}{\partial y^2} = \lambda_1^2 \omega_0, \tag{34}$$

$$\frac{\partial^2 \psi_0}{\partial y^2} = -\omega_0 \tag{35}$$

$$\frac{\partial^2 \theta_0}{\partial y^2} - \lambda_2^2 \theta_0 = -2 E_c \Pr \frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial^2 \psi_{0s}}{\partial y^2} \tag{36}$$

$$\frac{\partial^2 \phi_0}{\partial y^2} + \frac{1}{Sc}[K_1 - i]\phi_0 = -\frac{Sc_1}{Sc} \frac{\partial^2 \theta_0}{\partial y^2} \tag{37}$$

Considering the term js in (32), (32) and (33) the following set of equations were obtained.

$$\frac{\partial^2 \theta_{0s}}{\partial y^2} = E_c \Pr \left(\frac{\partial \psi_{0s}}{\partial y} \right)^2 \tag{38}$$

$$\frac{\partial^2 \omega_{0s}}{\partial y^2} = 0 \tag{39}$$

$$\frac{\partial^2 \psi_{0s}}{\partial y^2} = -\omega_{0s} \tag{40}$$

$$\frac{\partial^2 \phi_{os}}{\partial y^2} - \frac{k_1}{Sc}\phi_{0s} = \frac{Sc_1}{Sc} \frac{\partial^2 \theta_{0s}}{\partial y^2} \tag{41}$$

The boundary conditions are

$$\frac{\partial^2 \psi_0}{\partial y^2} = \frac{\partial^2 \psi_{0s}}{\partial y^2} = 0, \ \phi_0 = 0, \ \psi_0 = \psi_{0s} = 0, \ \frac{\partial \theta_0}{\partial y} = \frac{\partial \theta_{0s}}{\partial y} = 0, \ \frac{\partial \phi_0}{\partial y} = \frac{\partial \phi_{0s}}{\partial y} = 0, \ on \ y = 0 \tag{42}$$

$$\frac{\partial \psi_0}{\partial y} = \frac{\partial \psi_{0s}}{\partial y} = 0, \quad \psi_0 = \psi_{0s} = 1 \quad \theta_0 = 0 \quad \theta_{0s} = 1, \ \phi_0 = 0 \quad \phi_{0s} = 1, \quad on \quad y = S(x) \tag{43}$$

When j = 1 order 1 gives:

$$\frac{\partial^2 \psi_1}{\partial y^2} = -\omega_1 \tag{44}$$

$$\frac{\partial^2 \omega_1}{\partial y^2} - \lambda_1^2 \omega_1 = \frac{\partial (\omega_0, \psi_{0s})}{\partial (y, x)} + \frac{\partial (\omega_{0s}, \psi_0)}{\partial (y, x)} + \Omega \frac{\partial^2 \psi_0}{\partial y^2} - Gr \frac{\partial \theta_0}{\partial y} - Gc \frac{\partial \phi_0}{\partial y} \tag{45}$$

$$\frac{\partial^2 \theta_1}{\partial y^2} - \alpha \Pr \theta_1 = \Pr \left(\frac{\partial (\psi_0, \theta_{0s})}{\partial (y, x)} + \frac{\partial (\psi_{0s}, \omega_0)}{\partial (y, x)} + 2\Omega E_c \frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial^2 \psi_{0s}}{\partial y^2} \right)$$
$$- Gr \Pr \theta_0 - Gc \Pr \phi_0 - 2Ec \Pr \left(\frac{\partial^2 \psi_0}{\partial y^2} \frac{\partial^2 \psi_{1s}}{\partial y^2} + \frac{\partial^2 \psi_1}{\partial y^2} \frac{\partial^2 \psi_{0s}}{\partial y^2} \right) \tag{46}$$

$$\frac{\partial^2 \phi_1}{\partial y^2} - \frac{[1 + K_1]}{Sc}\phi_1 = \frac{Sc_1}{Sc} \frac{\partial^2 \theta_1}{\partial y^2} \tag{47}$$

Considering the term js the following set of equations were obtained.

$$\frac{\partial^2 \psi_{1s}}{\partial y^2} = -\omega_{1s} \tag{48}$$

$$\frac{\partial^2 \omega_{1s}}{\partial y^2} = \frac{\partial (\psi_{0s}, \omega_{0s})}{\partial (y,x)} - \Omega \frac{\partial^2 \psi_{0s}}{\partial y^2} - Gr \frac{\partial \theta_{0s}}{\partial y} - Gc \frac{\partial \phi_{0s}}{\partial y} \tag{49}$$

$$\frac{\partial^2 \theta_{1s}}{\partial y^2} = \Pr \frac{\partial (\psi_{0s}, \theta_{0s})}{\partial (y,x)} - \Omega E_c \left(\frac{\partial \psi_{0s}}{\partial y}\right)^2 - Gr\theta_{os} - Gc\phi_{os} - 2E_c \Pr \frac{\partial^2 \psi_{0s}}{\partial y^2} \frac{\partial^2 \psi_{1s}}{\partial y^2} \tag{50}$$

$$\frac{\partial^2 \phi_{1s}}{\partial y^2} + \frac{k_1}{Sc} \phi_{1s} = -\frac{Sc_1}{Sc} \frac{\partial^2 \theta_{1s}}{\partial y^2} \tag{51}$$

We now solving the following equations: (34), (35), (36), (37), (38), (39), (40), (41), (42), (43), (44), (45), (46) (47), (48), (49), (50) and (51) and the stream function ψ, vorticity ω, temperature distribution θ and volume fraction ϕ were obtained thus:

$$\omega_0 = \frac{1 + me^{it}}{\sinh \lambda_1 s(x)} \cosh \lambda_1 y \tag{52}$$

$$\omega_{0s} = \frac{1}{s(x)} y \tag{53}$$

$$\psi_0 = \frac{(1 + me^{it})}{\lambda_1^2} \left[\frac{(\lambda_1^2 + 1)y}{S(x)} - \frac{\sinh \lambda_1 y}{\sinh \lambda_1(x)} \right] \tag{54}$$

$$\psi_{os} = \frac{S(x)y}{2} - \frac{y^3}{6S(x)} \tag{55}$$

$$\theta_o = \frac{2E_c \Pr(1 + me^{it})}{(\lambda_1^2 - \lambda_2^2)S(x)\sinh \lambda_1 S(x)} \left[\frac{2\lambda_1 \cosh \lambda_1 S(x) + S(x)\sinh \lambda_1 S(x)}{\cosh \lambda_2 S(x)} \cosh \lambda_2 y - y\sinh \lambda_1 y \right] \tag{56}$$

$$\theta_{os} = 1 + \frac{E_c \Pr(y^4 - S(x)^4)}{12S(x)^2} \tag{57}$$

$$\phi_o = \left(\frac{2Sc_1 E_c \Pr(1 + me^{it})}{Sc(\lambda_1^2 - \lambda_2^2)S(x)} \right)$$
$$\left[\frac{\left[\frac{S(x)\sinh \lambda_1 S(x)}{\cosh \lambda_2 S(x)} \lambda_2^2 \cosh \lambda_2 S(x) - y\lambda_1^2 \sinh \lambda_1 S(x) - 2\lambda_1 \cosh \lambda_1 S(x) \right] \cos d_1 y}{\cosh d_1 S(x)\sinh \lambda_1 S(x)} + \left(\frac{2\lambda_1 Sc}{\sinh \lambda_1 S(x)\left(\lambda_2^2 + \frac{1}{Sc}(K_1 - i) \right)} - \frac{2\lambda_1^3}{\sinh \lambda_1 S(x)\left(\lambda_2^2 + \frac{1}{Sc}(K_1 - i) \right)^2} \right) \cosh \lambda_1 y - \frac{\lambda_2^2 \cosh \lambda_2 y}{\cosh \lambda_2 S(x)\left(\lambda_2^2 + \frac{1}{Sc}(K_1 - i) \right)} + \frac{\lambda_1^2 y\sinh \lambda_1 y}{\left(\lambda_2^2 + \frac{1}{Sc}(K_1 - i) \right)\sinh \lambda_1 S(x)} \right] \tag{58}$$

$$\phi_{os} = -\frac{Sc_1 \Pr Ec}{\cosh\left(\pm\sqrt{\frac{k_1}{Sc}} \right) S(x)(Sc - K_1)} \cosh\left(\pm\sqrt{\frac{k_1}{Sc}} \right) y - \frac{Ec \Pr Sc_1}{S^2(x)(2Sc - K_1)} y^2 \tag{59}$$

$$\omega_1 = -\left(\begin{array}{l} 2\left(\dfrac{2E_c Gr\,\mathrm{Pr}(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\right)+ \\[2ex] \left(\dfrac{2Sc_1 GcE_c\,\mathrm{Pr}(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right) \\[2ex] \dfrac{\left[\dfrac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x)-y\lambda_1^2\sinh\lambda_1 S(x)-2\lambda_1\cosh\lambda_1 S(x)\right]d_1}{\cosh\left(\dfrac{2Sc_1 GcE_c\,\mathrm{Pr}(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right)} \\[3ex] \left[\dfrac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x)-y\lambda_1^2\sinh\lambda_1 S(x)-2\lambda_1\cosh\lambda_1 S(x)\right]\left(\dfrac{2Sc_1 GcE_c\,\mathrm{Pr}(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right) \\[3ex] \dfrac{\left[\dfrac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x)-y\lambda_1^2\sinh\lambda_1 S(x)-2\lambda_1\cosh\lambda_1 S(x)\right]\left(\sqrt{\dfrac{1}{Sc}}(K_1-i)\right)}{\cosh d_1 S(x)\sinh\lambda_1 S(x)} \\[3ex] \dfrac{}{\cosh\left(\sqrt{\dfrac{1}{Sc}}(K_1-i)\right)S(x)\sinh\lambda_1 S(x)} \\[2ex] S(x)\sinh\lambda_1 S(x) \\[2ex] \left(\sqrt{\dfrac{1}{Sc}}(K_1-i)\right) \end{array}\right)$$

$$\sinh\lambda_1 y$$

$$\left(\begin{array}{l} 2\left(\dfrac{2E_c Gr\,\mathrm{Pr}(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\right)+\left(\begin{array}{l}\left(\dfrac{2Sc_1 GcE_c\,\mathrm{Pr}(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right)\\[2ex] \dfrac{\left[\begin{array}{l}\dfrac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x)\\[1ex] -y\lambda_1^2\sinh\lambda_1 S(x)-2\lambda_1\cosh\lambda_1 S(x)\end{array}\right]\left(\sqrt{\dfrac{1}{Sc}}(K_1-i)\right)}{\cosh d_1 S(x)\sinh\lambda_1 S(x)}\end{array}\right) \\[6ex] \left(\left(\sqrt{\dfrac{1}{Sc}}(K_1-i)\right)\right) \end{array}\right)$$

$$\sinh\lambda_1 S(x)-\dfrac{(1+me^{it})(\lambda_1^2+1)S'(x)}{\lambda_1^2\left(S(x)\right)^2}+\dfrac{S'(x)}{2}+\dfrac{S'(x)}{\left(S(x)\right)^2}$$

$$-\left(\begin{array}{l} -\dfrac{\Omega(1+me^{it})}{\sinh\lambda_1 S(x)}+\dfrac{2E_c Gr\,\mathrm{Pr}(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}+ \\[3ex] \left(\dfrac{2Sc_1 GcE_c\,\mathrm{Pr}(1+me^{it})\lambda_1}{Sc(\lambda_1^2-\lambda_2^2)S(x)\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)}\right)\left(2\lambda_1 Sc-\dfrac{2\lambda_1^3}{\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)}+1\right) \end{array}\right)\sinh\lambda_1 S(x)$$

$$-\frac{(1+me^{it})\lambda_1\left(\cosh\lambda_1 S(x)+\dfrac{S'(x)}{\lambda_1^2}\right)}{\left(\sinh\lambda_1 S(x)\right)^2}\cosh\lambda_1 S(x)$$

$$-\left(\frac{2E_c Gr\Pr(1+me^{it})\lambda_2^3}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\left(\frac{2\lambda_1\cosh\lambda_1 S(x)+S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\right)\right)\sinh\lambda_2 S(x)$$

$$-\left(\frac{\lambda_1 2Sc_1 GcE_c\Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)}\right)S(x)\sinh\lambda_1 S(x)-$$

$$\left(\frac{2E_c Gr\Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\right)S(x)\cosh\lambda_1 S(x)-\left(-\frac{S'(x)}{2\left(S(x)\right)^2}\right)S(x)^2$$

$$-\left(\frac{\left(\dfrac{2Sc_1 GcE_c\Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right)}{\dfrac{\left[\dfrac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x)-y\lambda_1^2\sinh\lambda_1 S(x)-2\lambda_1\cosh\lambda_1 S(x)\right]\left(\sqrt{\dfrac{1}{Sc}(K_1-i)}\right)}{\cosh\left(\sqrt{\dfrac{1}{Sc}(K_1-i)}\right)S(x)\sinh\lambda_1 S(x)}}\right)\sinh\lambda_1 y$$

$$\sinh\left(\sqrt{\frac{1}{Sc}(K_1-i)}\right)S(x)$$

$$+\left(\frac{(1+me^{it})(\lambda_1^2+1)S'(x)}{\lambda_1^2\left(S(x)\right)^2}+\frac{S'(x)}{2}+\frac{S'(x)}{\left(S(x)\right)^2}\right)$$

$$-\left(\begin{array}{l}\dfrac{\Omega(1+me^{it})}{\sinh\lambda_1 S(x)}-\dfrac{2E_c Gr\Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}-\\[2mm]\left(\dfrac{2Sc_1 GcE_c\Pr(1+me^{it})\lambda_1}{Sc(\lambda_1^2-\lambda_2^2)S(x)\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)}\right)\left(2\lambda_1 Sc-\dfrac{2\lambda_1^3}{\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)}+1\right)\end{array}\right)\sinh\lambda_1 y$$

$$+\left(\frac{(1+me^{it})\lambda_1\left(\cosh\lambda_1 S(x)+\dfrac{S'(x)}{\lambda_1^2}\right)}{\left(\sinh\lambda_1 S(x)\right)^2}\right)\cosh\lambda_1 y$$

$$+\left(\frac{2E_c Gr\Pr(1+me^{it})\lambda_2^3}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\left(\frac{2\lambda_1\cosh\lambda_1 S(x)+S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\right)\right)\sinh\lambda_2 y$$

$$+\left(\frac{\lambda_1 2Sc_1 GcE_c\Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)}\right)y\sinh\lambda_1 y$$

$$+\left(\frac{2E_c Gr\Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\right)y\cosh\lambda_1 y-\left(\frac{S'(x)}{2\left(S(x)\right)^2}\right)y^2 \tag{60}$$

$$\psi = \frac{(1+me^{it})}{\lambda_1^2}\left(\frac{(\lambda_1^2+1)y}{S(x)} - \frac{\sinh\lambda_1 y}{\sinh\lambda_1(x)}\right) + me^{it}\left(\frac{(1+me^{it})}{\lambda_1^2}\left(\frac{(\lambda_1^2+1)y}{S(x)} - \frac{\sinh\lambda_1 y}{\sinh\lambda_1(x)}\right)\right)$$

$$+ \mathrm{Re}\left(\begin{array}{c} \dfrac{S'(x)y^4}{48} + \dfrac{y^6}{720} + \Omega\left(\dfrac{S(x)y^2}{12} - \dfrac{y^5}{120S(x)}\right) + Gr\left(\dfrac{y^2}{2} + \dfrac{Ec\,\mathrm{Pr}\,y^7}{3240S(x)^2}\right) \\[2ex] + Gc\left(\dfrac{b_{24}d_7\sinh d_7 y}{d_7^2} + \dfrac{b_{26}y^5}{60}\right) - \dfrac{b_{39}y^3}{6} - \dfrac{b_{40}y^2}{2} - b_{41}y - b_{42} \end{array}\right)$$

$$+ \mathrm{Re}\,me^{it}\left(\begin{array}{c} (b_{34}+2b_{36}+b_{38}d_6)\dfrac{\sinh\lambda_1 y}{\lambda_1^2} + b_{29}\dfrac{\cosh\lambda_1 y}{\lambda_1^2} + b_{30}\dfrac{y^2}{2} + b_{31}\dfrac{\sinh\lambda_1 y}{\lambda_1^2} \\[2ex] + b_{32}\dfrac{\cosh\lambda_1 y}{\lambda_1^2} + b_{33}\dfrac{\sinh\lambda_2 y}{\lambda_2^2} + b_{34}\dfrac{\cosh\lambda_2 y}{\lambda_2^2} + b_{35}\left(\dfrac{y\cosh\lambda_1 y}{\lambda_1^2} - \dfrac{2\sinh\lambda_1 y}{\lambda_1^3}\right) \\[2ex] + b_{36}\left(\dfrac{y\sinh\lambda_1 y}{\lambda_1^2} - \dfrac{2\cosh\lambda_1 y}{\lambda_1^3}\right) + b_{37}\dfrac{y^4}{12} + b_{38}\dfrac{\sinh d_6 y}{d_6^2} + b_{39}y + b_{40} \end{array}\right) \quad (61)$$

$$\omega = \frac{1}{s(x)}y + me^{it}\left(\frac{1+me^{it}}{\sinh\lambda_1 s(x)}\sinh\lambda_1 y\right)$$

$$+ \mathrm{Re}\left(\begin{array}{c} \dfrac{S'(x)y^2}{4} - \dfrac{y^4}{24} - \Omega\left(\dfrac{S(x)y}{2} - \dfrac{y^3}{6S(x)}\right) - Gr\left(1 + \dfrac{Ec\,\mathrm{Pr}\left(\dfrac{y^5}{5} - S(x)^4\right)}{12S(x)^2}\right) \\[3ex] - Gc\left(b_{24}d_7\sinh d_7 y + \dfrac{b_{26}y^3}{3}\right) + b_{39}y + b_{40} \end{array}\right) \quad (62)$$

$$+ \mathrm{Re}\,me^{it}\left(\begin{array}{c} b_{28}\sinh\lambda_1 y + b_{29}\cosh\lambda_2 y + b_{30} + b_{31}\sinh\lambda_1 y + \\[1ex] b_{32}\cosh\lambda_1 y + b_{33}\sinh\lambda_2 y + b_{34}\cosh\lambda_2 y + b_{35}y\sinh\lambda_1 y \\[1ex] + b_{36}y\cosh\lambda_1 y + b_{37}y^2 + b_{38}\sinh d_6 y \end{array}\right)$$

$$\phi = b_{24}\cosh d_7 y + b_{26}y^2$$

$$+ me^{it}\left(\left(\begin{array}{c} \left(\dfrac{2Sc_1 E_c\,\mathrm{Pr}(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right) \\[3ex] \left[\dfrac{\left[S(x)\sinh\lambda_1 S(x)\lambda_2^2 - y\lambda_1^2\sinh\lambda_1 S(x) - 2\lambda_1\cosh\lambda_1 S(x)\right]\cos d_6 y}{\cosh d_6 S(x)\sinh\lambda_1 S(x)} + \right. \\[3ex] \left(\dfrac{2\lambda_1 Sc}{\sinh\lambda_1 S(x)\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)} - \dfrac{2\lambda_1^3}{\sinh\lambda_1 S(x)\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)^2}\right) \\[3ex] \cosh\lambda_1 y - \\[2ex] \left.\dfrac{\lambda_2^2\cosh\lambda_2 y}{\cosh\lambda_2 S(x)\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)} + \dfrac{\lambda_1^2 y\sinh\lambda_1 y}{\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)}\right] \end{array}\right)\right)$$

$$+ \mathrm{Re}\left(b_{76} \sinh d_{10}y + b_{77} \cosh d_{10}y + b_{79} \cosh d_7 y + b_{83} y^4 + b_{84} y^3 + b_{86} y + b_{87}\right)$$

$$+ \mathrm{Re}\, me^{it}\begin{pmatrix} b_{62} \sinh d_6 y + b_{63} \cosh d_9 y + b_{64} \sinh d_8 y + b_{65} \cosh d_8 y + \\[2mm] b_{66} \sinh \lambda_1 y + b_{67} \cosh \lambda_1 y + b_{68} \sinh \lambda_2 y + b_{69} \cosh \lambda_2 y + b_{70} \cosh d_6 y \\[2mm] + b_{71} y \sinh \lambda_1 y + b_{72} y^5 + b_{73} y^4 + b_{74} y^3 + b_{75} y^2 + b_{76} y + b_{77} \end{pmatrix} \qquad (63)$$

$$\theta = 1 + \frac{E_c \,\mathrm{Pr}(y^4 - S(x)^4)}{12 S(x)^2}$$

$$+ me^{it}\left(\frac{2 E_c \,\mathrm{Pr}(1 + me^{it})}{(\lambda_1^2 - \lambda_2^2) S(x) \sinh \lambda_1 S(x)} \left[\begin{array}{c} \dfrac{2\lambda_1 \cosh \lambda_1 S(x) + S(x) \sinh \lambda_1 S(x)}{\cosh \lambda_2 S(x)} \\[3mm] \cosh \lambda_2 y - y \sinh \lambda_1 y \end{array} \right] \right)$$

$$+ \mathrm{Re}\begin{pmatrix} \mathrm{Pr}\,\dfrac{1}{24}\left(\dfrac{6 S^2(x) S'(x) y^2 + y^4 S'(x)}{S^2(x)} \right) + \dfrac{E_c \,\mathrm{Pr}\, y^5 S'(x)}{30 S(x)^3} \\[4mm] - \dfrac{\Omega}{8}\left(S(x)^2 S' y^2 - \dfrac{y^4}{3} + \dfrac{y^6}{15 S(x)^2} \right) - Gr\left(\dfrac{y^2}{2} + \dfrac{E_c \,\mathrm{Pr}(\frac{y^6}{15} - S(x)^4 y^2)}{24 S^2(x)} \right) - \\[4mm] Gc\left(b_{24}(\dfrac{\cosh d_7 y}{d_7^2})^2 + b_{26}\dfrac{y^4}{12} \right) - 2 E_c \,\mathrm{Pr}\left(\dfrac{\left(1 + me^{it}\right)\sinh \lambda_1 y}{\lambda_1^2 \sinh \lambda_1 S(x)} \right) \\[4mm] \left(\dfrac{y^4}{24} - \Omega \dfrac{y^3}{6 S(x)} + Gr\left(\dfrac{Ec \,\mathrm{Pr}\, 4 y^5}{240 S(x)^2} \right) + Gc\left(b_{24} \sinh d_7 y + \dfrac{b_{26} y^3}{3} \right) \right) + b_{43} y + b_{44} \end{pmatrix} \qquad (64)$$

$$+ \mathrm{Re}\, me^{it}\begin{pmatrix} b_{62} \sinh d_6 y + b_{63} \cosh d_9 y + b_{64} \sinh d_8 y + b_{65} \cosh d_8 y + b_{66} \sinh \lambda_1 y + \\[2mm] b_{67} \cosh \lambda_1 y + b_{68} \sinh \lambda_2 y + b_{69} \cosh \lambda_2 y + b_{70} \cosh d_6 y + b_{71} y \sinh \lambda_1 y \\[2mm] + b_{72} y^5 + b_{73} y^4 + b_{74} y^3 + b_{75} y^2 + b_{76} y + b_{77} \end{pmatrix}$$

where:

$$b_1 = 0 \qquad (65)$$

$$b_2 = \frac{1 + mie^{it}}{\sinh \lambda_1 S(x)} \qquad (66)$$

$$b_3 = \frac{1}{s(x)} \qquad (67)$$

$$b_4 = 0 \qquad (68)$$

$$b_5 = \frac{1 + me^{it}(1 + \dfrac{1}{\lambda_1^2})}{S(x)} \qquad (69)$$

$$b_6 = 0 \qquad (70)$$

$$b_7 = \frac{S(x)}{2} \tag{71}$$

$$b_8 = 0 \tag{72}$$

$$b_9 = 0 \tag{73}$$

$$b_{10} = -\frac{2E_c \Pr(1+me^{it})\sinh\lambda_1 S(x)}{(\lambda_1^2 - \lambda_2^2)S(x)\sinh\lambda_1 S(x)\cosh\lambda_2 S(x)} \tag{74}$$

$$b_{11} = 0 \tag{75}$$

$$b_{12} = 0 \tag{76}$$

$$b_{13} = -\frac{2E_c \Pr(1+me^{it})y}{(\lambda_1^2 - \lambda_2^2)S(x)\sinh\lambda_1 S(x)}\sinh\lambda_1 y \tag{77}$$

$$b_{14} = 0 \tag{78}$$

$$b_{15} = 1 - \frac{E_c \Pr S^2(x)}{12} \tag{79}$$

$$b_{16} = 0 \tag{80}$$

$$b_{17} = \frac{2Sc_1 E_c \Pr(1+me^{it})}{S(x)Sc(\lambda_1^2 - \lambda_2^2)\cosh d_1 S(x)\sinh\lambda_1 S(x)}\left[\begin{array}{c} \dfrac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x) \\ -y\lambda_1^2\sinh\lambda_1 S(x) - 2\lambda_1\cosh\lambda_1 S(x) \end{array}\right] \tag{81}$$

$$b_{18} = 0 \tag{82}$$

$$b_{19} = \frac{4\lambda_1 Sc_1 E_c \Pr(1+me^{it})}{ScS(x)\sinh\lambda_1 S(x)(\lambda_1^2 - \lambda_2^2)\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1 - i)\right)} - \frac{2a_{22}\lambda_1}{\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1 - i)\right)} \tag{83}$$

$$b_{20} = 0 \tag{84}$$

$$b_{21} = -\frac{2Sc_1 E_c \Pr(1+me^{it})\lambda_2^2}{Sc\cosh\lambda_2 S(x)\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1 - i)\right)(\lambda_1^2 - \lambda_2^2)} \tag{85}$$

$$b_{22} = \frac{2Sc_1 E_c \Pr(1+me^{it})\lambda_1^2}{Sc\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1 - i)\right)(\lambda_1^2 - \lambda_2^2)S(x)\sinh\lambda_1 S(x)} \tag{86}$$

$$b_{23} = 0 \tag{87}$$

$$b_{24} = -\frac{Sc_1 \Pr Ec}{\cosh d_2 S(x)(Sc - K_1)} \tag{88}$$

$$b_{25} = 0 \tag{89}$$

$$b_{26} = -\frac{Ec \Pr Sc_1}{S^2(x)(2Sc - K_1)} \tag{90}$$

$$b_{27} = 0 \tag{91}$$

$$b_{28} = -b_{34} - 2b_{36} - b_{38}d_6 \tag{92}$$

$$b_{29} = \left(b_{34} + 2b_{36} + b_{38}d_6\right)\sinh\lambda_1 S(x) - b_{30} - b_{31}\sinh\lambda_1 S(x) - b_{32}\cosh\lambda_1 S(x) - b_{33}\sinh\lambda_2 S(x)$$
$$- b_{34}\cosh\lambda_2 S(x) - b_{35}S(x)\sinh\lambda_1 S(x) - b_{36}S(x)\cosh\lambda_1 S(x) - b_{37}S(x)^2 - b_{38}\sinh d_6 S(x) \tag{93}$$

$$b_{30} = \frac{(1+me^{it})(\lambda_1^2+1)S'(x)}{\lambda_1^2\left(S(x)\right)^2} + \frac{S'(x)}{2} + \frac{S'(x)}{\left(S(x)\right)^2} \tag{94}$$

$$b_{31} = -\frac{(\Omega+Da)(1+me^{it})}{\sinh\lambda_1 S(x)} + \frac{2E_c Gr\Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)} +$$

$$\left(\frac{2Sc_1 GcE_c \Pr(1+me^{it})\lambda_1}{Sc(\lambda_1^2-\lambda_2^2)S(x)\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)}\right)\left(2\lambda_1 Sc - \frac{2\lambda_1^3}{\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)}+1\right) \tag{95}$$

$$b_{32} = \frac{(1+me^{it})\lambda_1\left(\cosh\lambda_1 S(x)+\dfrac{S'(x)}{\lambda_1^2}\right)}{\left(\sinh\lambda_1 S(x)\right)^2} \tag{96}$$

$$b_{33} = \frac{2E_c Gr\Pr(1+me^{it})\lambda_2^3}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\left(\frac{2\lambda_1\cosh\lambda_1 S(x)+S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\right) \tag{97}$$

$$b_{34} = 0 \tag{98}$$

$$b_{35} = \frac{\lambda_1 2Sc_1 GcE_c \Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)\left(\lambda_2^2+\dfrac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)} \tag{99}$$

$$b_{36} = \frac{2E_c Gr\Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)} \tag{100}$$

$$b_{37} = -\frac{S'(x)}{2\left(S(x)\right)^2} \tag{101}$$

$$b_{38} = \left(\frac{2Sc_1 GcE_c \Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right)$$

$$\frac{\left[\dfrac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x)-y\lambda_1^2\sinh\lambda_1 S(x)-2\lambda_1\cosh\lambda_1 S(x)\right]d_6}{\cosh d_1 S(x)\sinh\lambda_1 S(x)} \tag{102}$$

$$b_{39} = \frac{(\Omega+Da)S(x)}{2} + Gcb_{24}d_7^2 \tag{103}$$

$$b_{40} = \frac{S(x)^4}{24} + \frac{(\Omega+Da)S(x)^2}{3} + Gr\left(1+Ec\Pr\left(\frac{S(x)^3}{60}-\frac{S(x)^2}{12}\right)\right)$$

$$+ Gc\left(d_7\sinh d_7 S(x) - b_{32}S(x)^3 - b_{34}S(x)\right) \tag{104}$$

$$b_{41} = 0 \tag{105}$$

$$b_{42} = \frac{S'(x)S(x)^4}{48} + \frac{S(x)^6}{720} + \Omega\left(\frac{S(x)^3}{12}-\frac{S(x)^4}{120}\right) + Gr\left(\frac{S(x)^2}{2}+\frac{Ec\Pr S(x)^5}{3240}\right)$$

$$+ Gc\left(\frac{b_{24}\sinh d_7 S(x)}{d_7}+\frac{b_{26}S(x)^5}{60}\right) - \frac{b_{39}S(x)^3}{6} - \frac{b_{40}S(x)^2}{2} \tag{106}$$

$$b_{43} = 0 \tag{107}$$

$$b_{44} = -\Pr\frac{7S^2(x)S'(x)}{24} - \frac{E_c\Pr S(x)^2 S'(x)}{30} + \frac{\Omega}{8}\left(S(x)^4 S'(x) - \frac{S(x)^4}{3} + \frac{S(x)^4}{15}\right) +$$
$$Gr\left(\frac{S(x)^2}{2} - \frac{E_c\Pr S(x)^4}{360}\right) + Gc\left(b_{24}\frac{\cosh d_7 S(x)}{d_7^2}\cosh d_7 S(x) + b_{26}\frac{S(x)^4}{12}\right) +$$
$$2E_c\Pr\left(\frac{(1+me^{it})}{\lambda_1^2}\right)\left(\begin{array}{c}\dfrac{S(x)^4}{24} - \Omega\dfrac{S(x)^2}{6} + Gr\left(\dfrac{Ec\Pr 4S(x)^3}{240}\right)\\[2ex] -Gc\left(b_{24}\sinh d_7 S(x) + \dfrac{b_{26}S(x)^3}{3}\right)\end{array}\right) \tag{108}$$

$$b_{45} = \frac{b_{47}\lambda_1 - b_{50}\lambda_2}{d_8} \tag{109}$$

$$b_{46} = -\frac{\left(\begin{array}{c}(\dfrac{b_{47}\lambda_1 - b_{50}\lambda_2}{d_8})\sinh d_8 S(x) + b_{47}\sinh \lambda_1 S(x) + b_{48}\cosh \lambda_1 S(x)\\[1ex] + b_{50}\cosh \lambda_2 S(x) + +b_{52}\cosh d_6 S(x) + b_{53}S(x)\sinh \lambda_1 S(x) + b_{54}S(x)^5\\[1ex] + b_{55}S(x)^4 + b_{56}S(x)^3 + b_{57}S(x)^2 + b_{59}\end{array}\right)}{\cosh d_8 S(x)} \tag{110}$$

$$b_{47} = \frac{2Ec\Pr(1+me^{it})b_{40}}{\sinh \lambda_1 S(x)} \tag{111}$$

$$b_{48} = \frac{\Pr(1+me^{it})S'(x)}{\lambda_1 \sinh \lambda_1 S(x)\left(\lambda_1^2 - \lambda_2\right)} - \frac{Gr\Pr}{\left(\lambda_1^2 - \lambda_2\right)\sinh \lambda_1 S(x)\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)}$$
$$\left(2\lambda_1 Sc - \frac{2\lambda_1^3}{\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)}\right) \tag{112}$$

$$b_{49} = 0 \tag{113}$$

$$b_{50} = -Gr\Pr\left(\frac{2E_c\Pr(1+me^{it})\left(2\lambda_1\cosh \lambda_1 S(x) + S(x)\sinh \lambda_1 S(x)\right)}{\left(\lambda_2^2 - \lambda_2\right)(\lambda_1^2 - \lambda_2^2)S(x)\sinh \lambda_1 S(x)\cosh \lambda_2 S(x)}\right)$$
$$- \frac{\lambda_2^2 Gc\Pr}{\cosh \lambda_2 S(x)\left(\lambda_2^2 - \lambda_2\right)\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)} \tag{114}$$

$$b_{51} = 0 \tag{115}$$

$$b_{52} = \frac{-Gc\Pr\left(\dfrac{S(x)\sinh \lambda_1 S(x)}{\cosh \lambda_2 S(x)}\lambda_2^2\cosh \lambda_2 S(x) - y\lambda_1^2\sinh \lambda_1 S(x) - 2\lambda_1\cosh \lambda_1 S(x)\right)}{\left(d_6^2 - \lambda_2\right)\cosh d_6 S(x)\sinh \lambda_1 S(x)} \tag{116}$$

$$b_{53} = -\frac{\Pr}{\sinh \lambda_1 S(x)(2\lambda_1 + \lambda_1^2 - \lambda_2)}$$
$$\left(\frac{\lambda_1 Gc}{\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)}\left(\lambda_1^3 - 2Sc - \frac{2\lambda_1^2}{\left(\lambda_2^2 + \dfrac{1}{Sc}(K_1-i)\right)}\right) + \frac{2E_c Gr\Pr(1+me^{it})}{(\lambda_1^2 - \lambda_2^2)S(x)}\right) \tag{117}$$

$$b_{54} = \frac{E_c \,(\mathrm{Pr})^2 \, S'(x)}{S(x)^3} \left(\frac{1}{10} - 4E_c b_{30} \right) + \frac{\lambda_1 b_{56} \,\mathrm{Pr}}{20} \tag{118}$$

$$b_{55} = \frac{-4(Ec\,\mathrm{Pr})^2 \, S'(x) b_{37}}{(12 - \lambda_2) S(x)^3} \tag{119}$$

$$b_{56} = \frac{-4(Ec\,\mathrm{Pr})^2 \, S'(x) b_{37}}{(6 - \lambda_2) S(x)^3} \tag{120}$$

$$b_{57} = \frac{\mathrm{Pr}}{(2 - \lambda_2)} \left(\frac{(1 + me^{it})}{\lambda_1^2} \left(\frac{\left(\lambda_1^2 + 1\right) S'(x)}{S(x)^2} \right) + \frac{S'(x)}{2} - \frac{S'(x)}{S(x)^2} \right) \tag{121}$$

$$b_{58} = 0 \tag{122}$$

$$b_{59} = \frac{-\mathrm{Pr}}{\lambda_2} \left(\frac{(1 + me^{it})\left(\lambda_1^2 + 1\right) S'(x)}{\lambda_1^2 S(x)^2} + \frac{S'(x)}{2} \right) \tag{123}$$

$$b_{60} = \frac{-(b_{74} + b_{62} d_8 + b_{64}\lambda_1 + b_{66}\lambda_2)}{d_9} \tag{124}$$

$$b_{61} = - \left(\frac{\begin{array}{l}(-\dfrac{(b_{74} + b_{74}d_8 + b_{74}\lambda_1 + b_{74}\lambda_2)}{d_4})\sinh d_9 S(x) + b_{62}\sinh d_8 S(x) + b_{63}\cosh d_8 S(x) + \\ b_{64}\sinh\lambda_1 S(x) + b_{65}\cosh\lambda_1 S(x) + b_{66}\sinh\lambda_2 S(x) + b_{67}\cosh\lambda_2 S(x) + b_{68}\cosh d_6 S(x) \\ +b_{69}\,y\sinh\lambda_1 S(x) + b_{70}S(x)^5 + b_{71}S(x)^4 + b_{72}S(x)^3 + b_{73}S(x)^2 + b_{74}S(x) + b_{75}\end{array}}{\cosh d_9 S(x)} \right) \tag{125}$$

$$b_{62} = \frac{Sc_1 d_8 \left(b_{47}\lambda_1 - b_{50}\lambda_2\right)}{Scd_8^2 - K_1 - 1} \tag{126}$$

$$b_{63} = \frac{Sc_1 d_8^2 b_{46}}{Scd_8^2 - K_1 - 1} \tag{127}$$

$$b_{64} = \frac{Sc_1 \lambda_1^2 b_{47}}{Sc\lambda_1^2 - K_1 - 1} \tag{128}$$

$$b_{65} = \frac{Sc_1 \lambda_1^2 b_{48}}{Sc\lambda_1^2 - K_1 - 1} \tag{129}$$

$$b_{66} = 0 \tag{130}$$

$$b_{67} = \frac{Sc_1 \lambda_2^2 b_{50}}{Sc\lambda_2^2 - K_1 - 1} \tag{131}$$

$$b_{68} = \frac{Sc_1 d_b^2 b_{52}}{Scd_6^2 - K_1 - 1} \tag{132}$$

$$b_{69} = \frac{Sc_1 \lambda_1^2 b_{53}}{Sc\lambda_1^2 - K_1 - 1} \tag{133}$$

$$b_{70} = \frac{20 Sc_1 b_{54}}{20 Sc - K_1 - 1} \tag{134}$$

$$b_{71} = \frac{12 Sc_1 b_{55}}{12 Sc - K_1 - 1} \tag{135}$$

$$b_{72} = \frac{6 Sc_1 b_{55} + 20 b_{54}}{6 Sc - K_1 - 1} \tag{136}$$

$$b_{73} = \frac{2 Sc_1 b_{57} + 12 b_{55}}{2 Sc - K_1 - 1} \tag{137}$$

$$b_{74} = \frac{-6 Sc_1 b_{56}}{K_1 + 1} \tag{138}$$

$$b_{75} = \frac{-2 Sc_1 b_{57}}{K_1 + 1} \tag{139}$$

$$b_{76} = \frac{-b_{83} - b_{78} d_7}{d_{10}} \tag{140}$$

$$b_{77} = - \left(\frac{\begin{array}{c} (\frac{-b_{83} - b_{78} d_7}{d_{10}}) \sinh d_{10} S(x) + b_{78} \sinh d_7 S(x) + b_{79} \cosh d_7 S(x) + b_{80} \sinh \lambda_1 S(x) + \\ b_{81} \cosh \lambda_1 S(x) + b_{82} S(x)^5 + b_{83} S(x)^4 + b_{84} S(x)^3 + b_{85} S(x)^2 + b_{86} S(x) + b_{87} \end{array}}{\cosh d_{10} y} \right) \tag{141}$$

$$b_{78} = 0 \tag{142}$$

$$b_{79} = \frac{Gc Sc_1 b_{24}}{Sc} \tag{143}$$

$$b_{80} = 0 \tag{144}$$

$$b_{81} = 0 \tag{145}$$

$$b_{82} = 0 \tag{146}$$

$$b_{83} = \frac{Sc_1}{4 S(x)^2 Sc} \left(\Omega - \frac{Gr Ec \, Pr}{6} \right) \tag{147}$$

$$b_{84} = \frac{2 Ec \, Pr \, S'(x) Sc_1}{3 S(x)^3 Sc} \tag{148}$$

$$b_{85} = \frac{Sc_1}{Sc} \left(\frac{Pr \, S'(x)}{S(x)^2} - 4\Omega - Gc b_{26} \right) \tag{149}$$

$$b_{86} = 0 \tag{150}$$

$$b_{87} = \frac{Sc_1}{Sc} \left(\frac{Pr \, S'(x)}{2} - \frac{\Omega S(x)^2 S'(x)}{4} - Gr - \frac{Ec \, Pr \, S(x)^2}{12} \right) \tag{151}$$

$$d_6 = \sqrt{\frac{1}{Sc}(K_1 - i)} \tag{152}$$
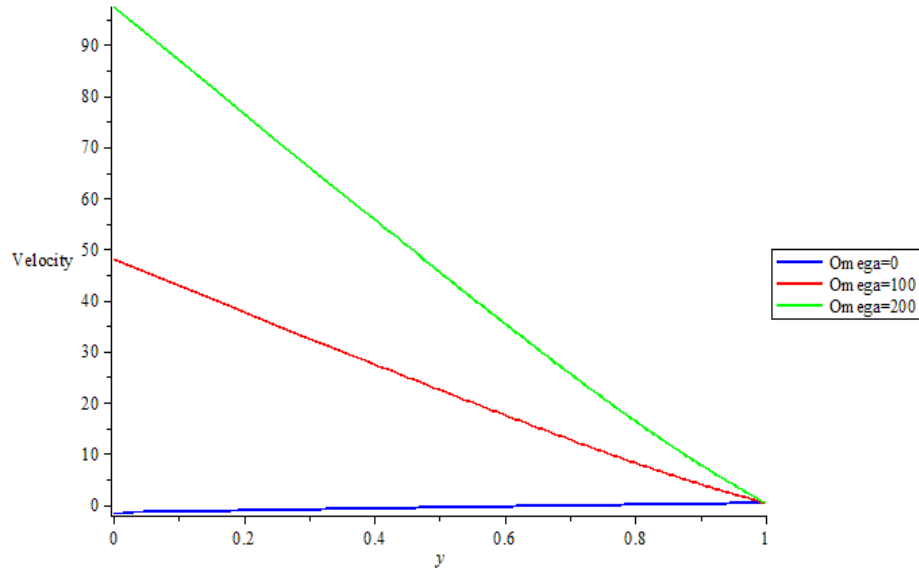
$$d_7 = \pm \sqrt{\frac{k_1}{Sc}} \tag{153}$$

$$\mathrm{d}_8 = \pm\sqrt{\alpha \Pr} \tag{154}$$

$$d_9 = \pm\sqrt{\frac{1+K_1}{Sc}} \tag{155}$$
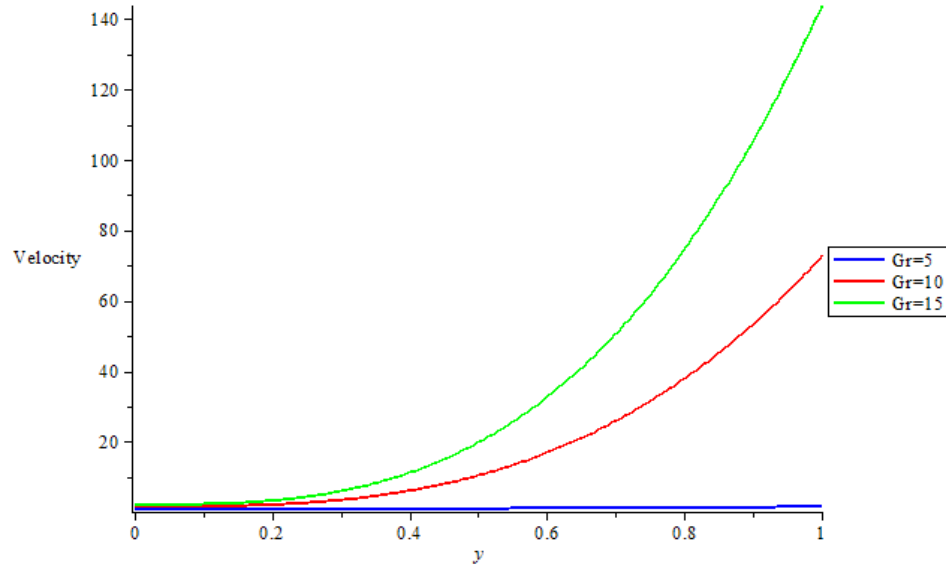
$$d_{10} = \pm\sqrt{\frac{k_1}{Sc}}i \tag{156}$$
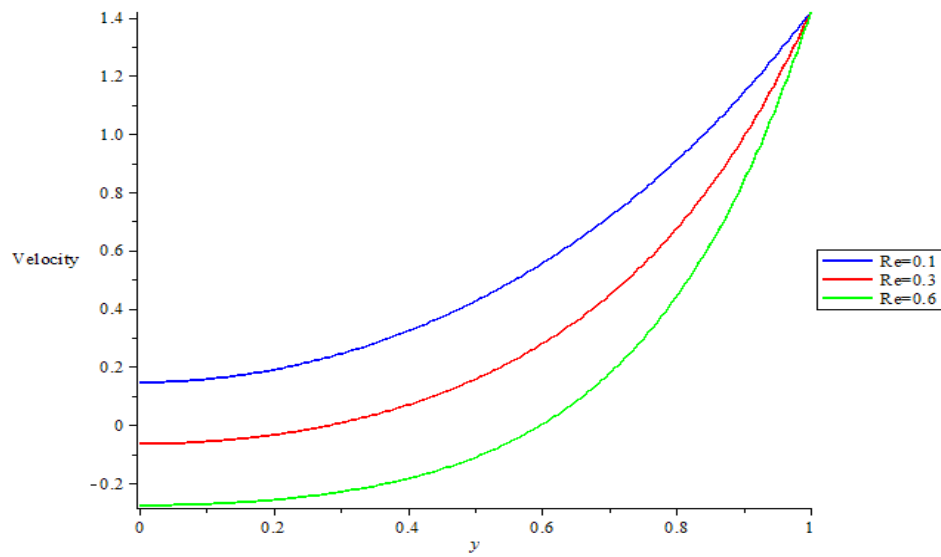
4. Method of Solution

   The problems of consideration are solved analytically which resulted into the graphical results under results and discussion.
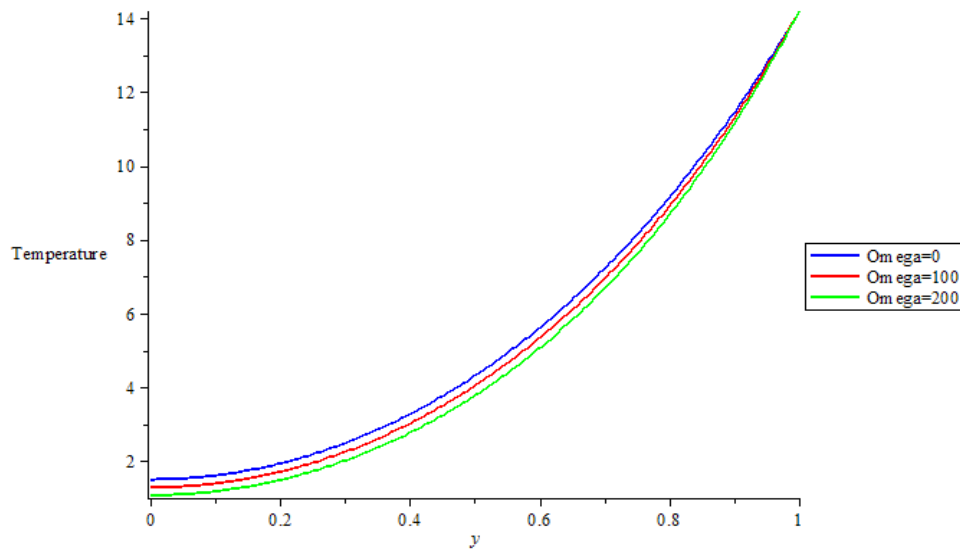

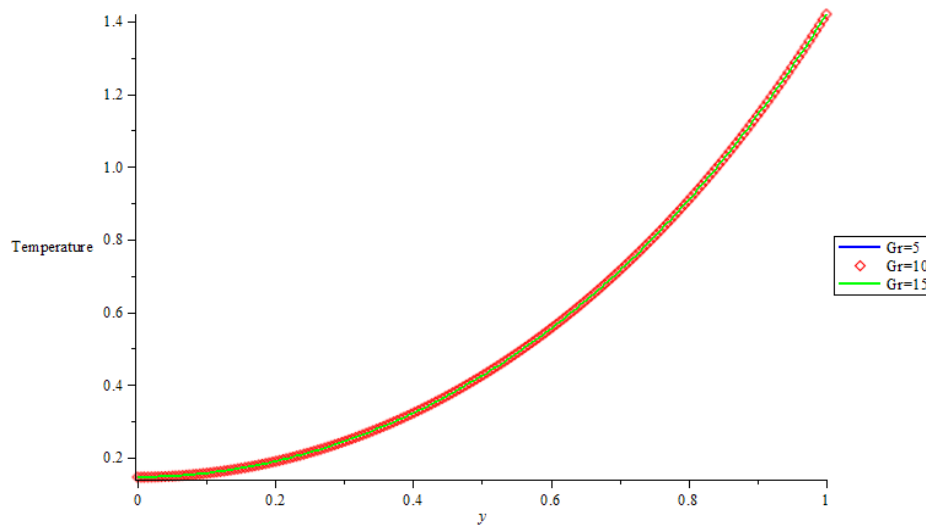**Figure 1.**   Velocity profile for different value of magnetic field intensity parameters

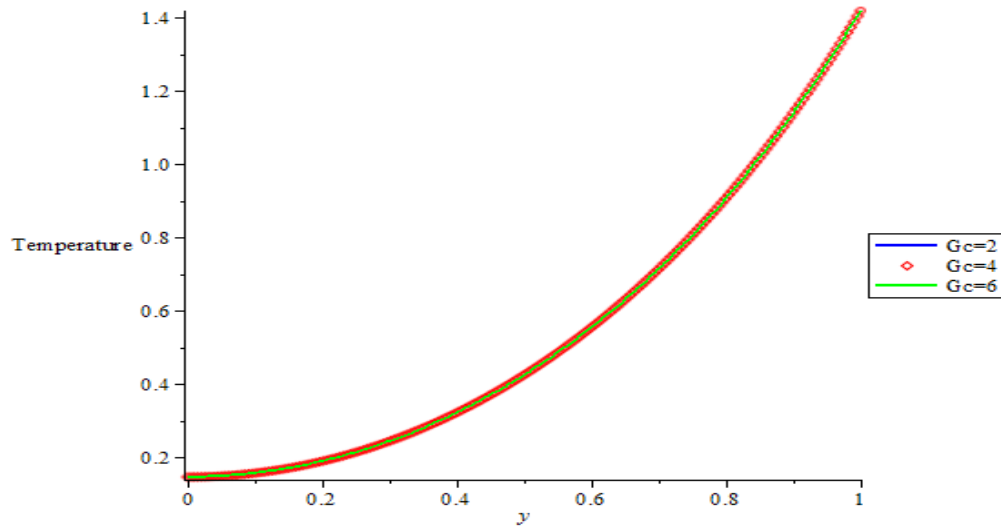**Figure 2.**   Velocity profile for different value of thermal Grashof numbers

Figure 3.   Velocity profile for different value of Reynolds numbers
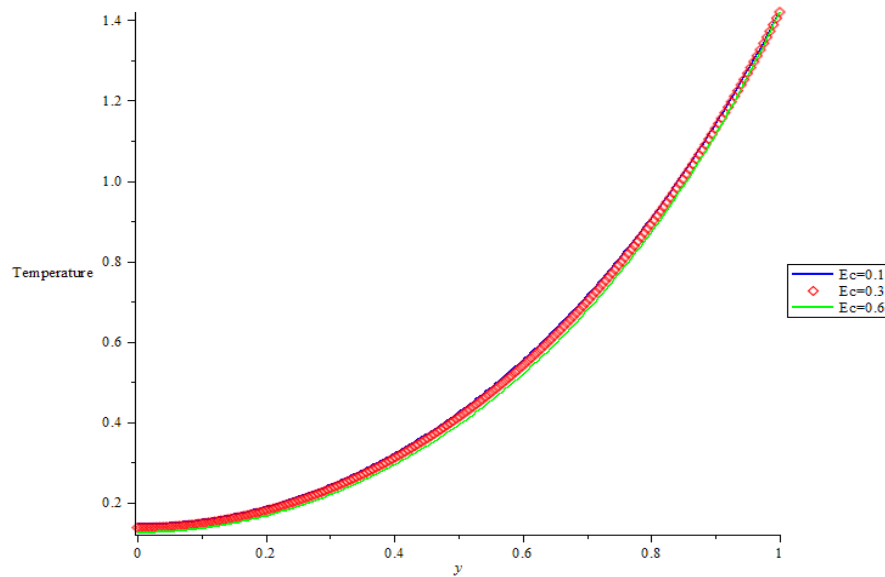

Figure 4.   Temperature profile for different value of magnetic field intensity parameters

Figure 5.   Temperature profile for different value of thermal Grashof numbers

Figure 6.   Temperature profile for different value of Mass Grashof numbers

Figure 7.   Temperature profile for different value of Eckert numbers

5. Results and Discussion

In order to study the behaviour of velocity, temperature and concentration profile, a comprehensive numerical computation using mathematical software Maple 12 was carried out for various values of the parameters that describe the flow characteristics, and the results were reported in terms of graphs as shown in figures (1)- (7). The pulse m was chosen small so that the ensuring flow is a small oscillatory disturbance about the steady flow. We have observed that for every m, the womersley parameter α can be varied only for a range of values, hence we set $\alpha = 0.5$. The effect of varying the Reynolds number Re, the Eckert number E_c and the Prandtl number Pr to flow structure were observed. Therefore we set $\text{Re} = 0.1$, $E_c = 1$, $\text{Pr} = 7.1$ in the ensuring analysis, we assume that m and Ω can be varied while keeping Re, E_c, and Pr fixed. This assumption is valid because the physics of the problem and the range of values of

m and Ω that are involved. When the porosity was considered, figure1 shows that the effect of increasing values of Ω on steady flow is to dampen the velocity profile. This is well known for Hartmann flow. And with increase in Magnetic field intensity parameter (omega) numbers there is Velocity increase which pushed the flow to the wall of the channel that leads to increase in heat along the wall of the channel thereby increasing the velocity in the boundary layer. This is depicted in Figure 2 with increase in thermal Grashof numbers the velocity increase and the flow were pushed away from the wall. This suggests that unsteadiness has the effect of cooling the fluid. For increase in Reynolds number velocity decreases as shown in figure 3. This suggests that in this model, increasing Reynolds number enhances unsteadiness. The pressure gradient, which is trying to accelerate the fluid, is counteracted by the magnetic drag. Figure 4 shows that there is slight decrease in temperature as Magnetic field intensity parameter (omega) increases. The effect of thermal Grashof number and mass Grashof number

in figure 5 but not well felt on temperature in figure 6 there is a slight increase in temperature as Eckert number increase in figure 7.

6. Conclusions

A model was formulated with the inclusion of concentration equation. The study added porosity parameter to explain viscous dissipation in a porous vertical channel. Approximate numerical solutions were found using regular perturbation technique together with their boundary condition. The outcome of study showed that increase in viscous dissipation led to decrease in Temperature and velocity profile but not in concentration. The porosity effect on heat and mass transfer were clearly exposed with the significance of the parameters introduced.

Overall observations based on the problem formulated and analysed upon which conclusions were drawn and listed as:

  i. evaluated the effect of porosity on heat and mass transfer of natural convection fluid flow in porous media;
  ii. assessed the impacts of porosity on heat and mass transfer of MHD fluid flow in porous media;
  iii. established the influence of MHD fluid flow on vertically porous channel; and
  iv. determined the effects of porosity on heat and mass transfer of MHD fluid flow in a vertically porous channel. Limitation exhibited by the methods used was that: The porosity effect on heat and mass transfer over a vertical porous channel was considered. The resulting governing equations were simplified and solved using perturbation technique. The results are presented in graphical forms.

The impact of variations of velocity, temperature and concentration parameters on non-dimensional variables of the heat and mass transfer was established to explain porosity heat and mass transfer of MHD fluid flow in porous vertical channel. The influence of porosity effects were also noticed along with other listed parameters and these contributed chronologically to MHD fluid flow thereby explaining heat and mass transfer over porous vertical channel.

The study concluded that increase in mass Grashof, thermal Grashof, magnetic parameter; womersley parameters, Reynolds, Eckert, Schmidt and chemical reaction numbers had significant effects on the MHD fluid flow in porous vertical channel.

Nomenclature

υ = The kinematics Viscosity coefficient

ρ = The fluid density

σ = Electrical conductivity of the fluid

ψ = Stream function

C = Concentration of the fluid within the boundary layer.

∂ = A volume flux with oscillating frequency

m = Pulse

Ω = Magnetic field intensity parameter (Omega)

Re = Effective flow Reynolds number

A_1 = Womersley number

Gr = Thermal Grashof number

Gc = Mass Grashof number

C_p = Specific heat at constant pressure

T = Temperature in the boundary layer

θ = Non-dimensional temperature

E_c = Eckert number

P = Pressure

U = Velocity component in the x-direction

V = Velocity component in the y-direction

t = Time

Sc = Schmidt number

K_1 = Chemical reaction number

u, v, w = Cartesian velocity components

x, y, z = Cartesian coordinates

Q = The flux rate across any section of the channel.

K = The wall slip parameter.

B_0 = The electromagnetic induction

μ = The magnetic permeability

H_0 = The intensity of magnetic field

A = The characteristic half-width of the channel

ε = A small dimensionless parameter that specifies the slow variation in the cross-section of the channel

L = The channel characteristic length

Y = ±b(x) are the rigid and impermeable walls of the channel

Pr = Prandlt number

REFERENCES

[1] Ahammad M.U. Mollah Md. S.H. (2011) Numerical sturdy of MHD free convection flow and mass transfer over a stretching sheet considering Dufour and Soret effects in the presence of magnetic field, international Journal of Engineering and Technology, 11(05): 4-11.

[2] Attia H.A. (2006) Unsteady MHD couette flow and heat transfer of dusty with variable physical properties Applied Mathematics and Computation, 177 308-318.

[3] Chakraborty S. (2001) MHD flow and heat transfer of a dusty viscoelastic stratified fluid down an inclined channel in porous medium under variables viscosity. Theoretical and Applied Mechanics Vol.26, pp. 1-14. 4.

[4] Hazarika G.C. and Jadav Konch (2014), Effects of variable viscosity and thermal conductivity on MHD free convective flow along a vertical porous plate with viscous dissipation "International journal of mathematics trends and technology –volume 15 number 1.

[5] Mhone, P.Y. and Makinde, O.D (2006). "Unsteady MHD flow with heat transfer in a diverging channel" Rom. journ Phys. Vol. 51. Nos. 9-10, pp.967-979, Bucharest.

[6]   Palani G. and Ganesan P. (2007), Heat Transfer Effects on Dusty Gas Flow past a Semi- Infinite Inclined plate forsch Ingenieurwes, Vol. 71, pp 223-230.

[7]   Seddeek, M. A. and Salama, F. A. (2007), The effects of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Computational Materials Science, Vol. 40(2), pp. 186-192.

[8]   Soundalgekar V.M., Gupta SK. and Birajdar NS. (1979), Effects of Mass transfer and free convection currents on MHD Stokes problem for a vertical plate. Nuclear Engineering Design, vol.53, pp.339-346.

[9]   Taneja, R and Jain, N. C (Jan 2004): MHD flow with slips effects and temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. Def. Sc. J., 21 - 29.